STRICT HIERARCHY BETWEEN PARALLEL, SEQUENTIAL, AND INDEFINITE-CAUSAL-ORDER STRATEGIES

FOR CHANNEL DISCRIMINATION

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IQI Vienna

ÖAW ÖSTERREICHISCHE AKADEMIE DER WISSENSCHAFTEN

THE TASK:
MINIMUM-ERROR CHANNEL DISCRIMINATION

STATE DISCRIMINATION

STATE DISCRIMINATION

CANDIDATES:

$$\rho_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \rho_2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

STATE DISCRIMINATION

CANDIDATES:

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INPUT:

STATE DISCRIMINATION

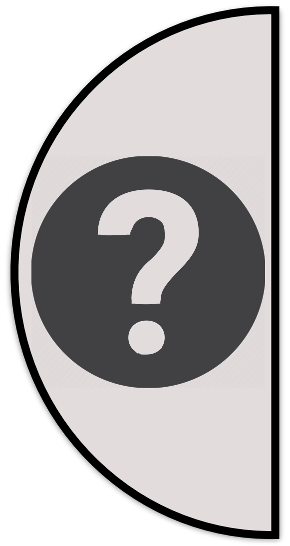
CANDIDATES:

$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$,

$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

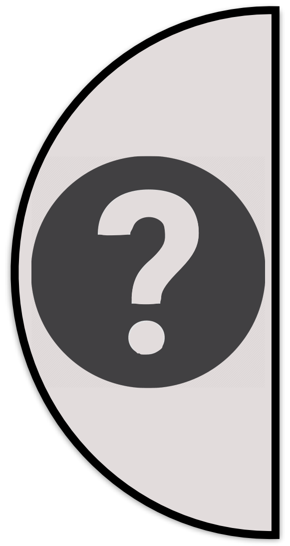
INPUT:

PROMISSE:



$=$

(with probability $p_1 = \frac{1}{3}$) ,

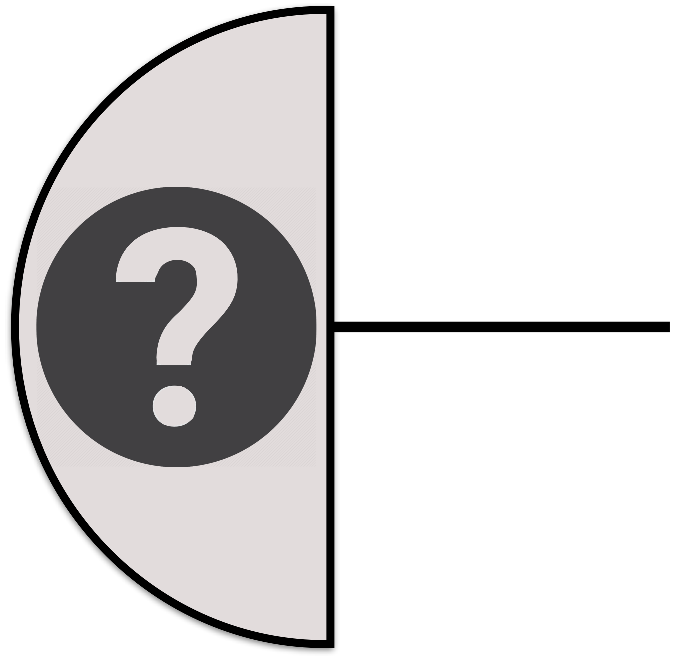


$=$

(with probability $p_2 = \frac{2}{3}$)

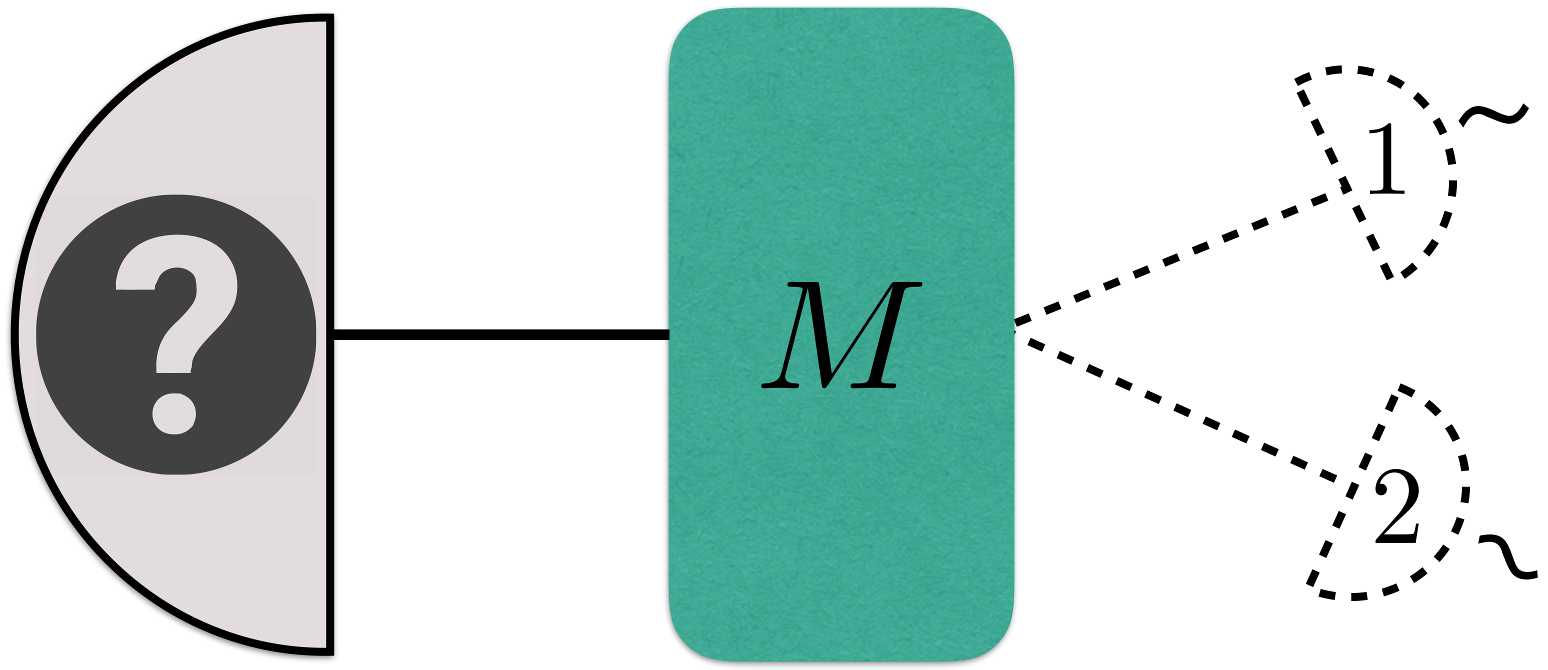
STRATEGY

ONE COPY!



STRATEGY

ONE COPY!



STRATEGY

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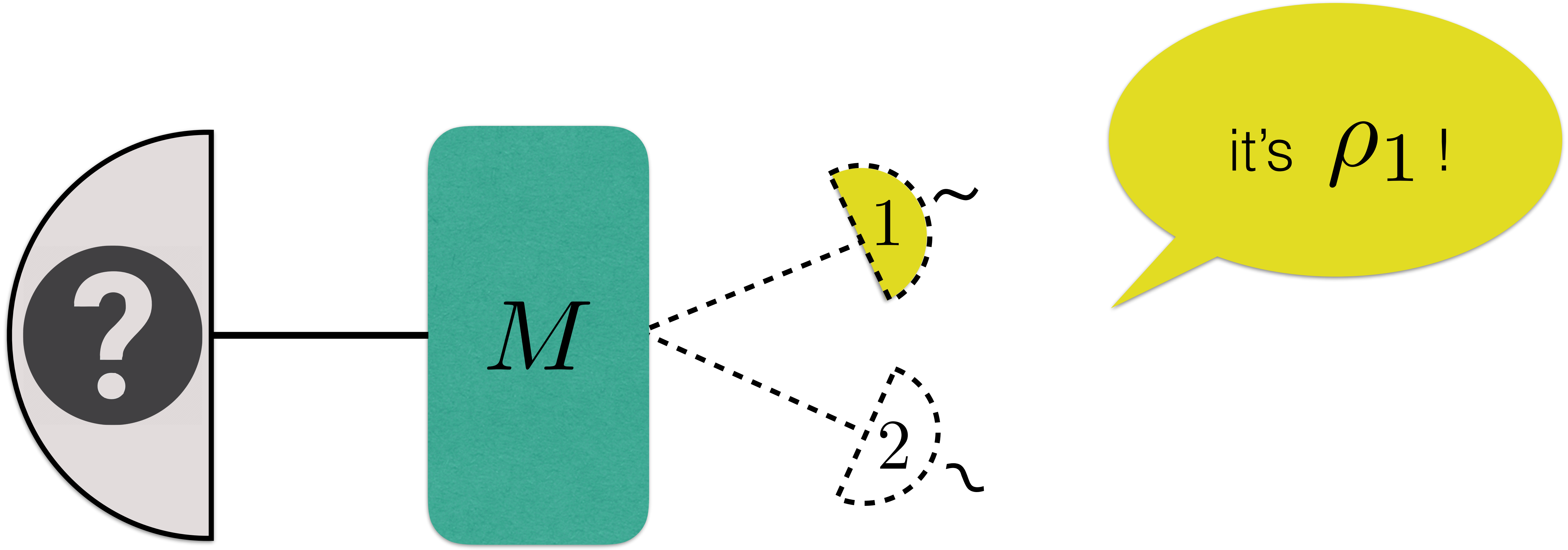

STRATEGY

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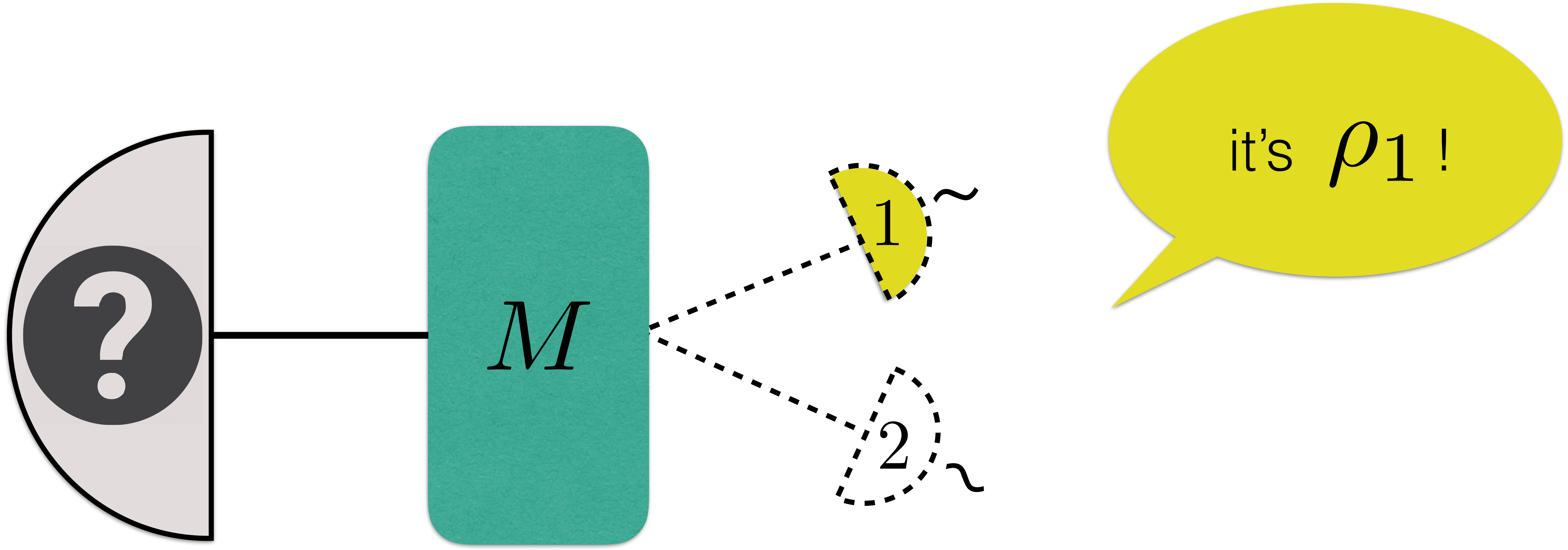
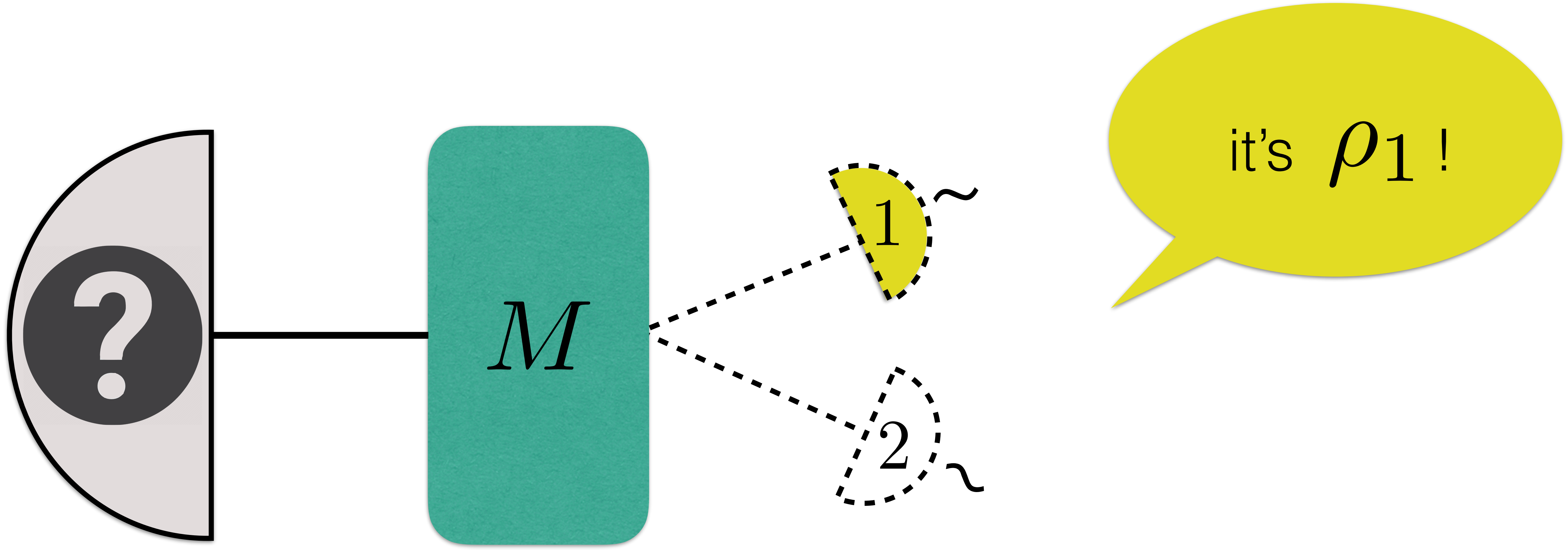


$$p_{\text{succ}} = p_1 p(1|\rho_1, M) + p_2 p(2|\rho_2, M)$$

$$= p_1 \text{tr}(M_1 \rho_1) + p_2 \text{tr}(M_2 \rho_2)$$

STRATEGY

ONE COPY!



$$p_{\text{succ}} = p_1 p(1|\rho_1, M) + p_2 p(2|\rho_2, M)$$

$$= p_1 \text{tr}(M_1\rho_1) + p_2 \text{tr}(M_2\rho_2)$$

$$p^*_{\text{succ}} = \max_{\{M_1, M_2\}} \; p_1 \text{Tr}(M_1\rho_1) + p_2 \text{Tr}(M_2\rho_2)$$

THE ACTUAL TASK:

MINIMUM-ERROR CHANNEL DISCRIMINATION

CHANNEL DISCRIMINATION

CANDIDATES: $\widetilde{C}_1$, $\widetilde{C}_2$, $\widetilde{C}_3$

CHANNEL DISCRIMINATION

CANDIDATES: $\widetilde{C}_1$, $\widetilde{C}_2$, $\widetilde{C}_3$

INPUT:

CHANNEL DISCRIMINATION

CANDIDATES: $\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3$

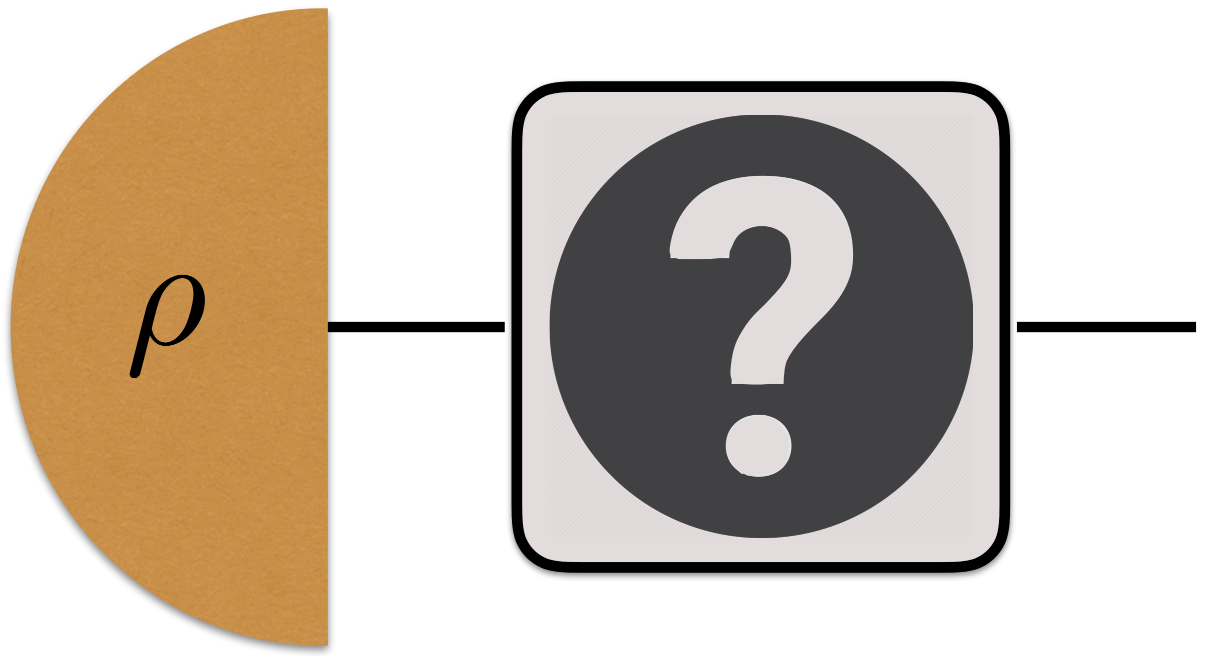
INPUT: ?

PROMISSE: $? = \widetilde{C}_1$, $? = \widetilde{C}_2$, $? = \widetilde{C}_3$

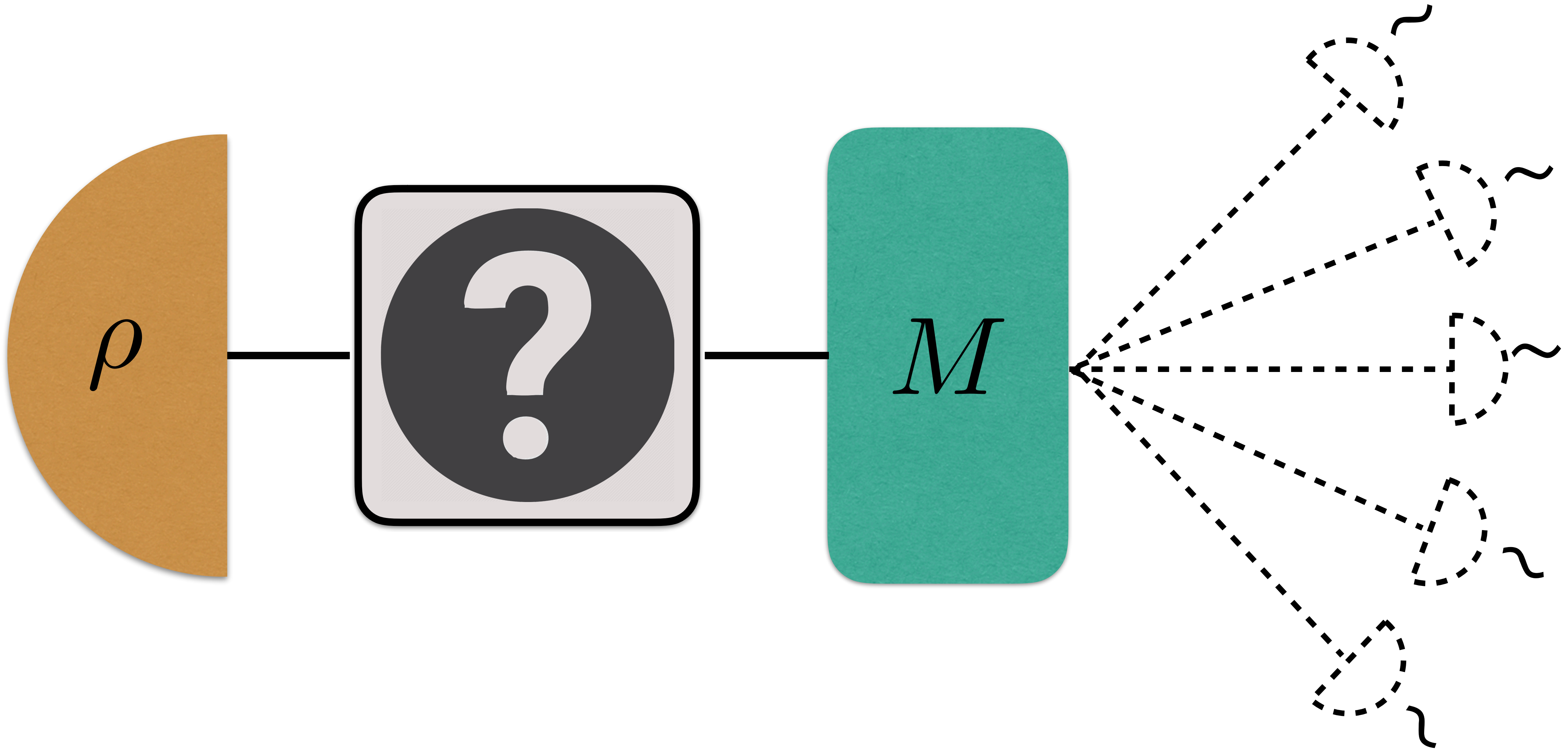
p_1 p_2 p_3

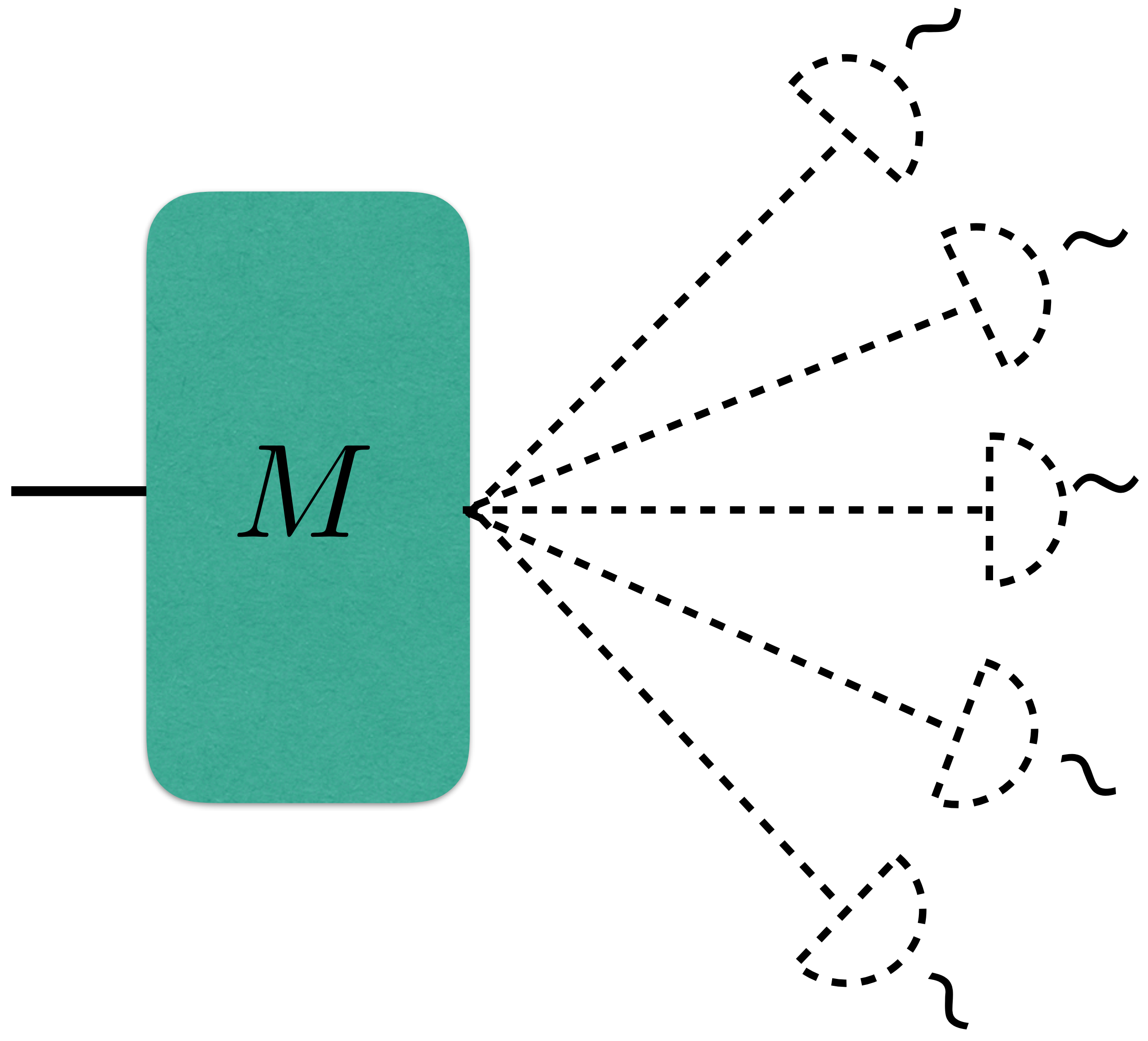
STRATEGY

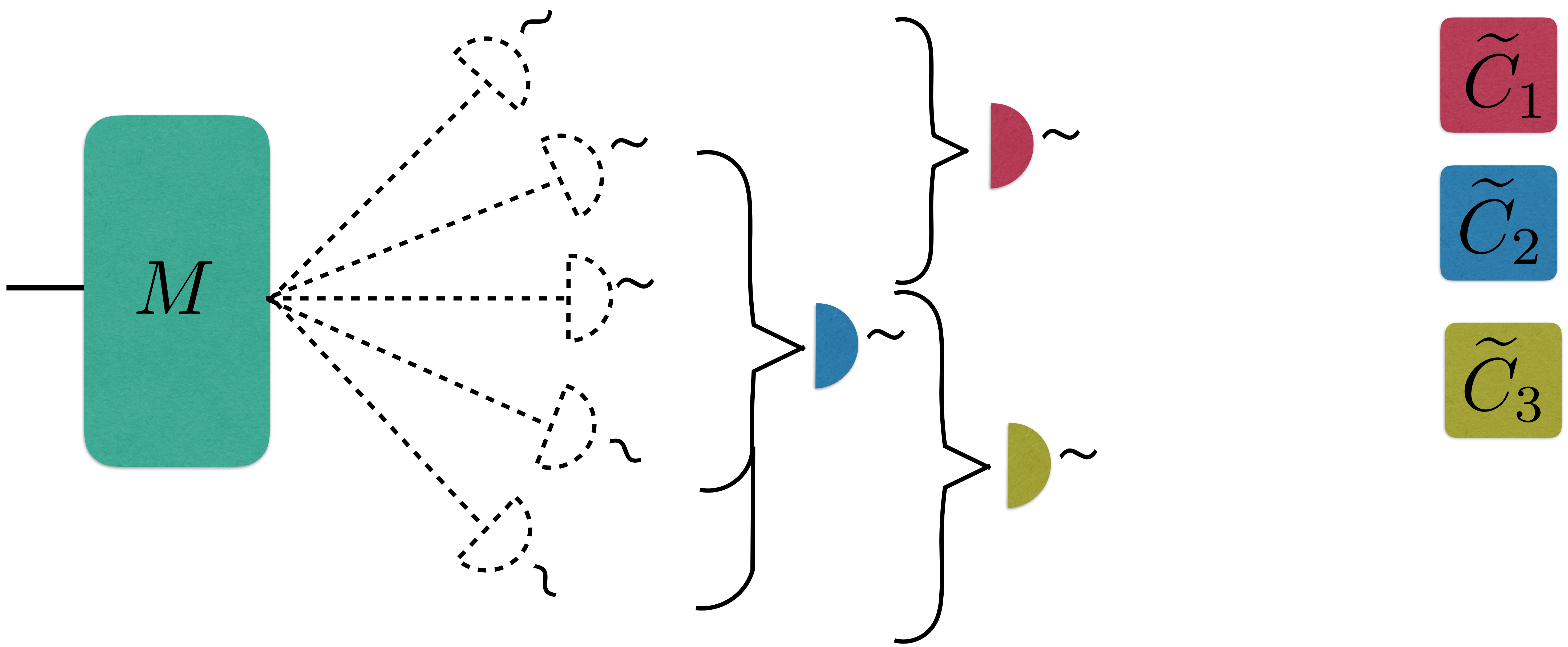
STRATEGY

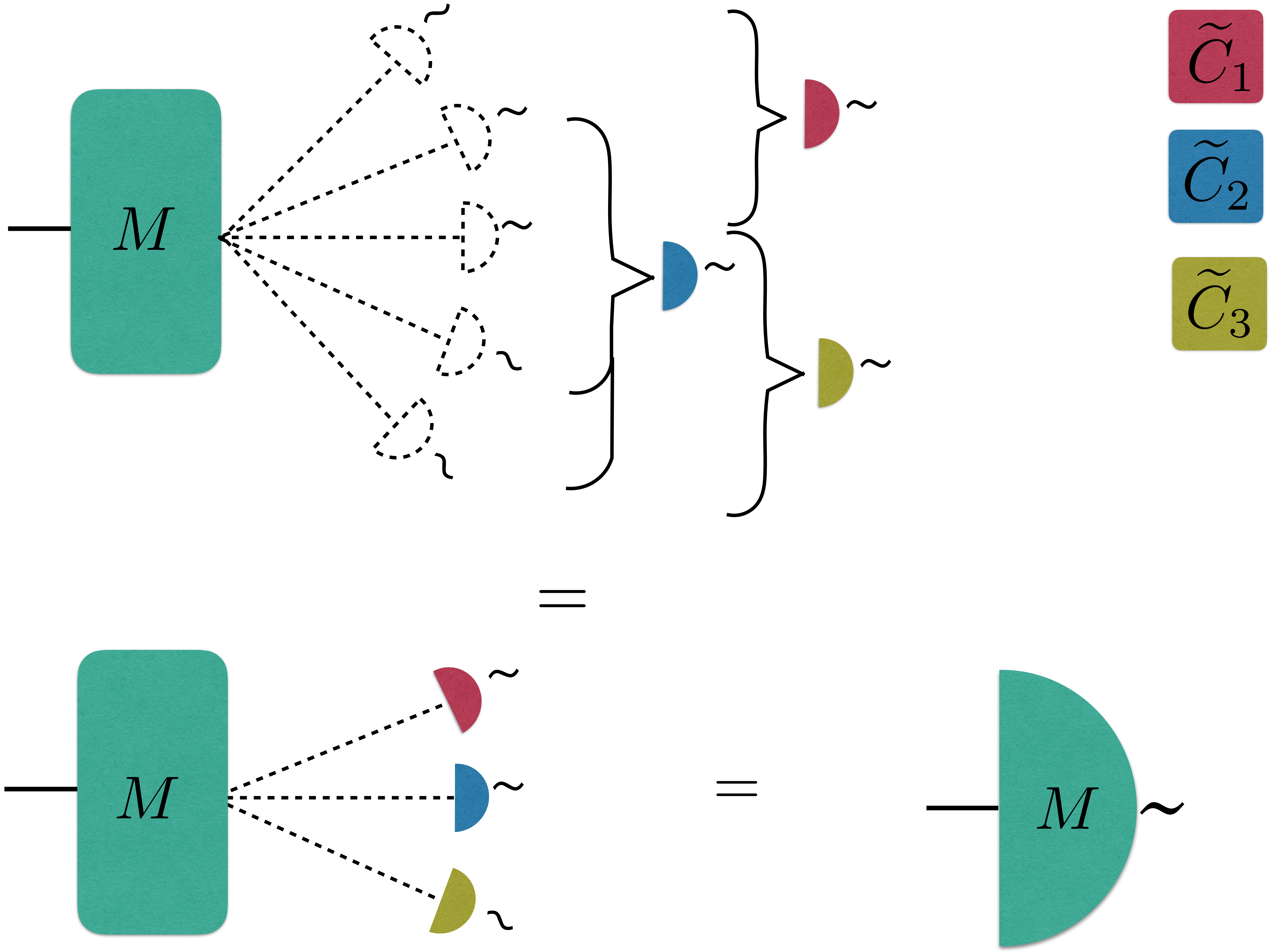


STRATEGY

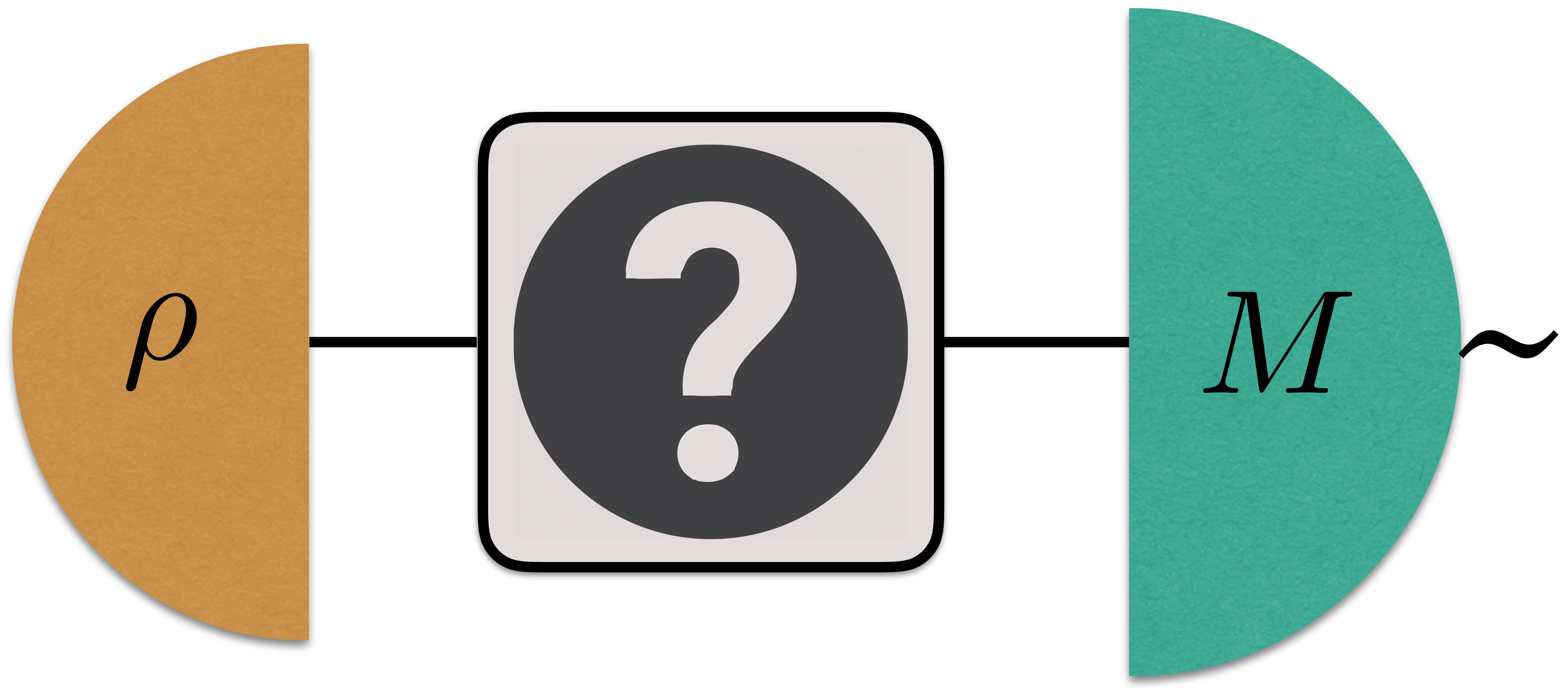



M

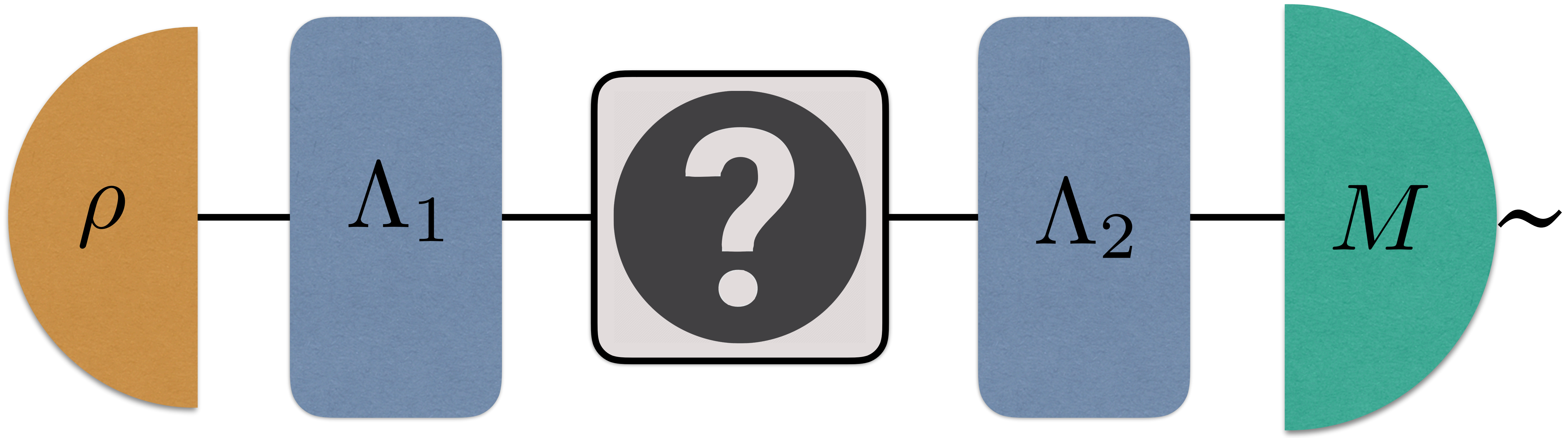

M
$\widetilde{C}_1$
$\widetilde{C}_2$
$\widetilde{C}_3$



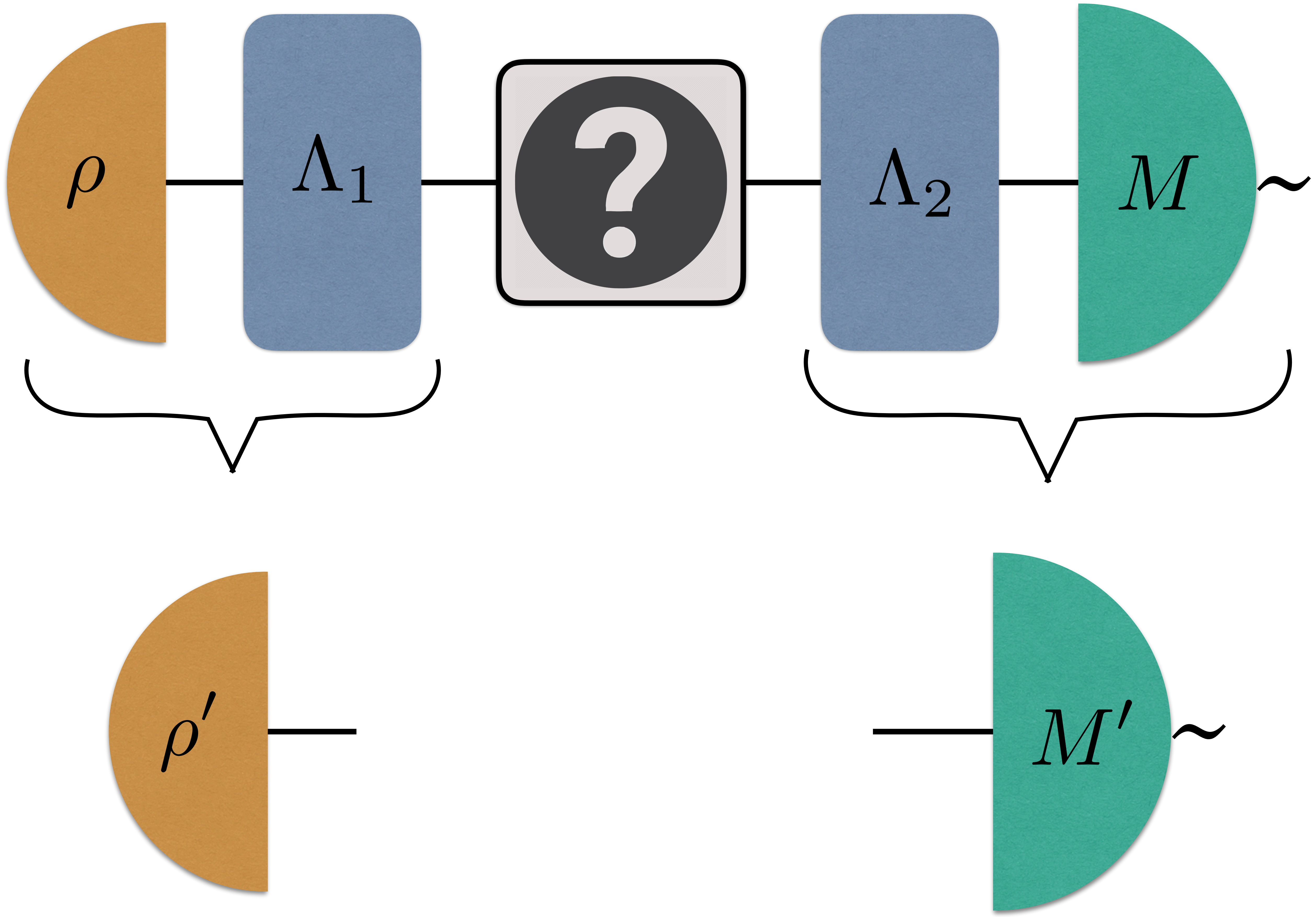
(as many outcomes as candidates)



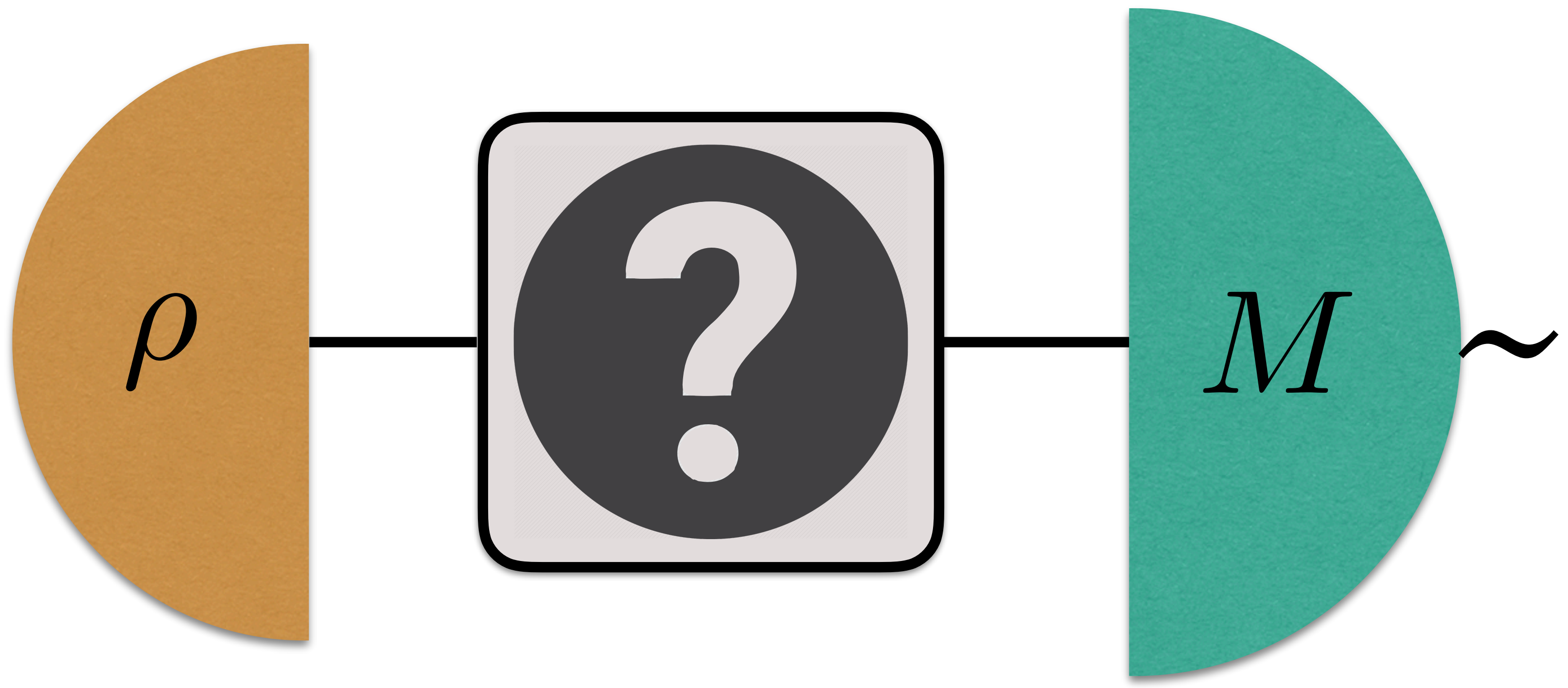
ρ
?
M

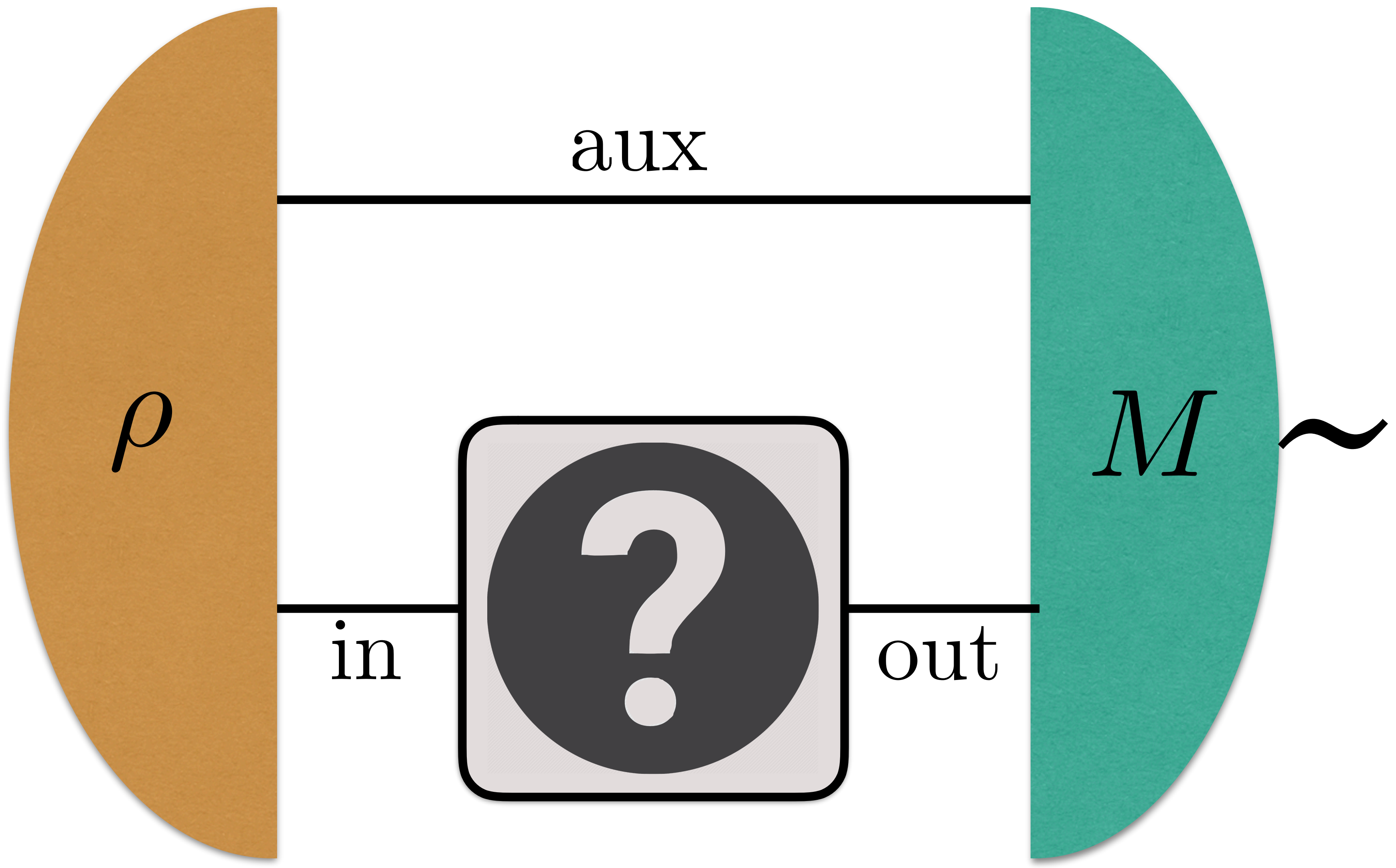


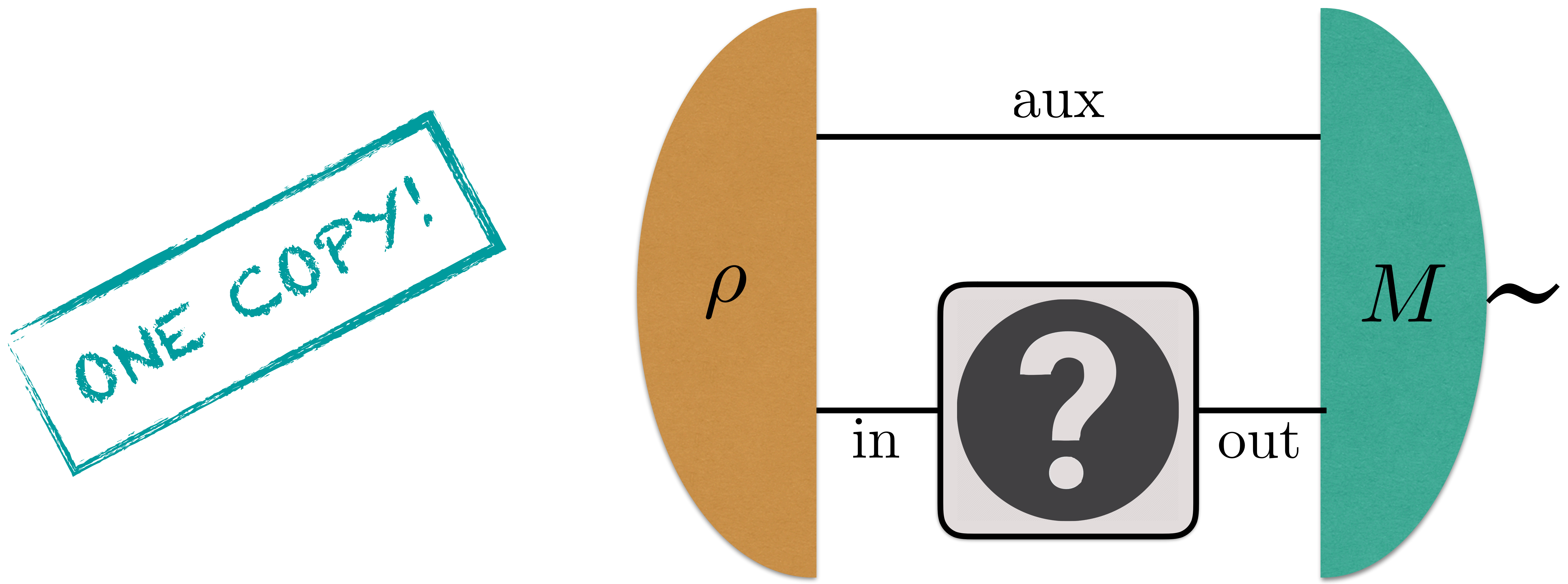
ρ
Λ_1
Λ_2
M

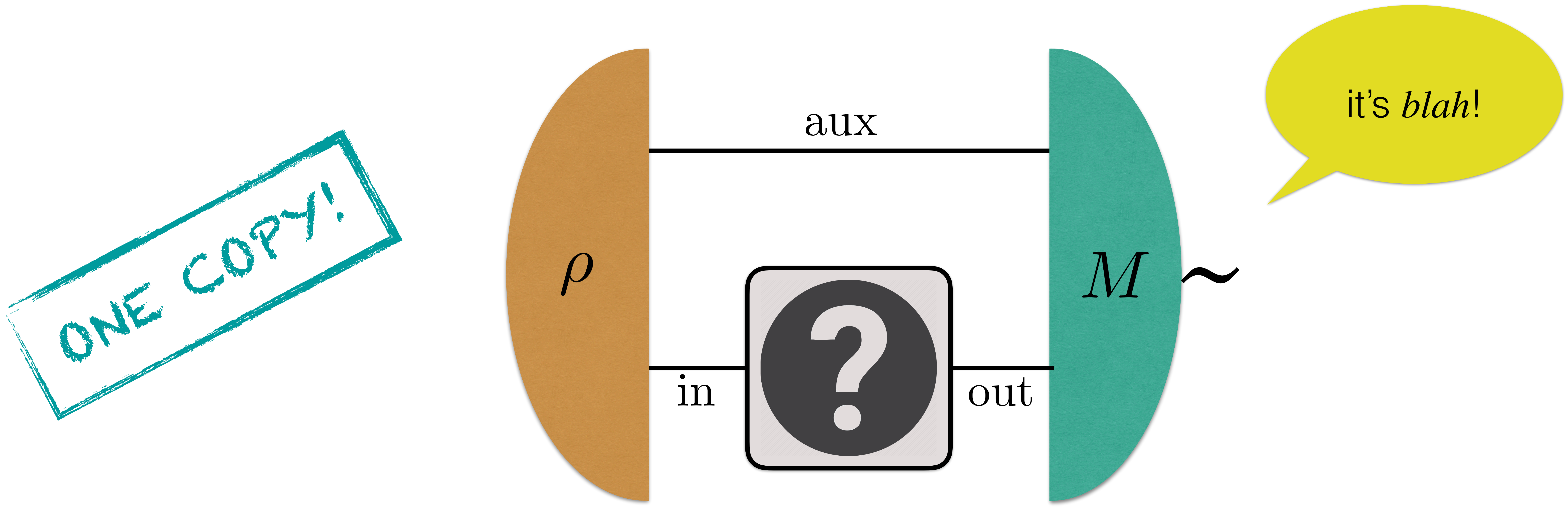


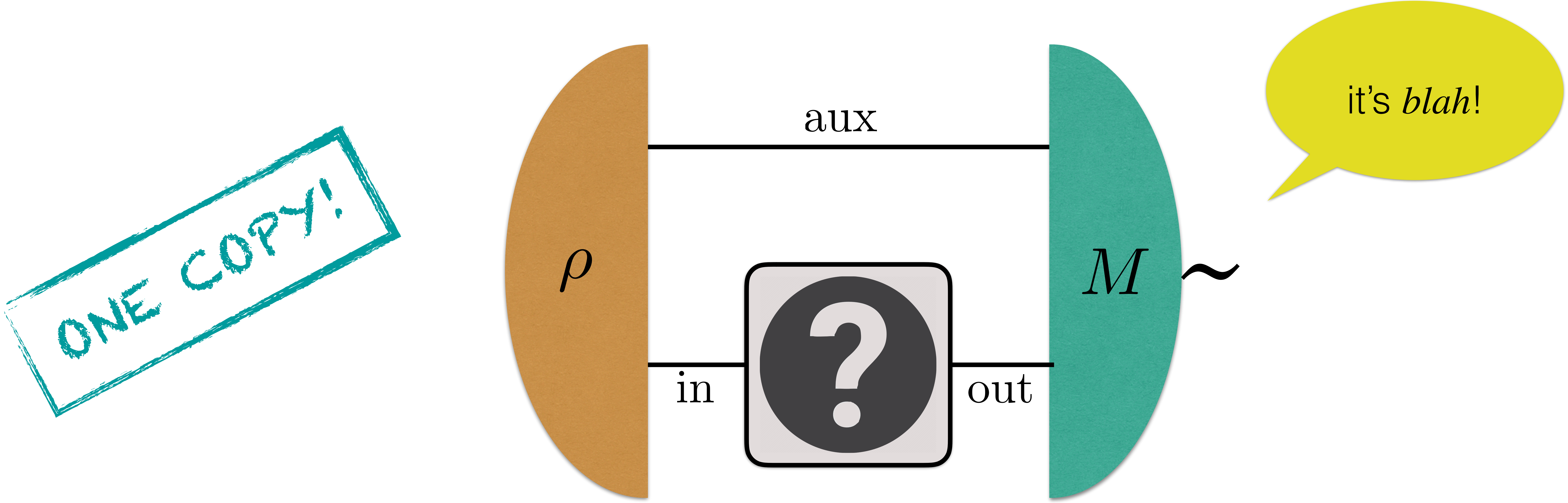
ρ
Λ_1
?
Λ_2
M
ρ'
M'


ρ
?
M


aux
ρ
in
out
M
~

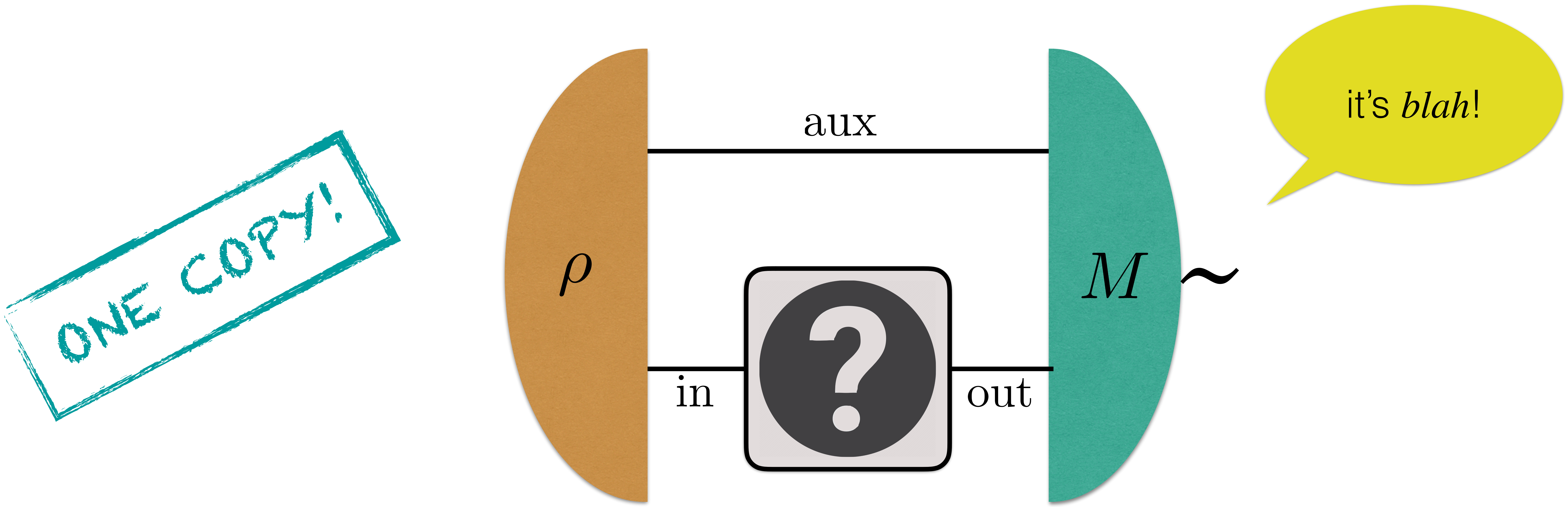

ONE COPY!
aux
ρ
M
in
out
?
~


ONE COPY!
aux
ρ
in
out
M
~
it's *blah*!



$$p_{\mathrm{succ}} = p_1\, p(1|\widetilde{C}_1, \rho, M) + p_2\, p(2|\widetilde{C}_2, \rho, M) + p_3\, p(3|\widetilde{C}_3, \rho, M)$$

$$= \sum_{i=1}^{N} p_i \,\mathrm{Tr}[(\widetilde{C}_i \otimes \widetilde{\mathbb{I}})(\rho)\, M_i]$$



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EXAMPLE

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ENSEMBLE:

$$\{p_i\} = \left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right\}$$

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EXAMPLE

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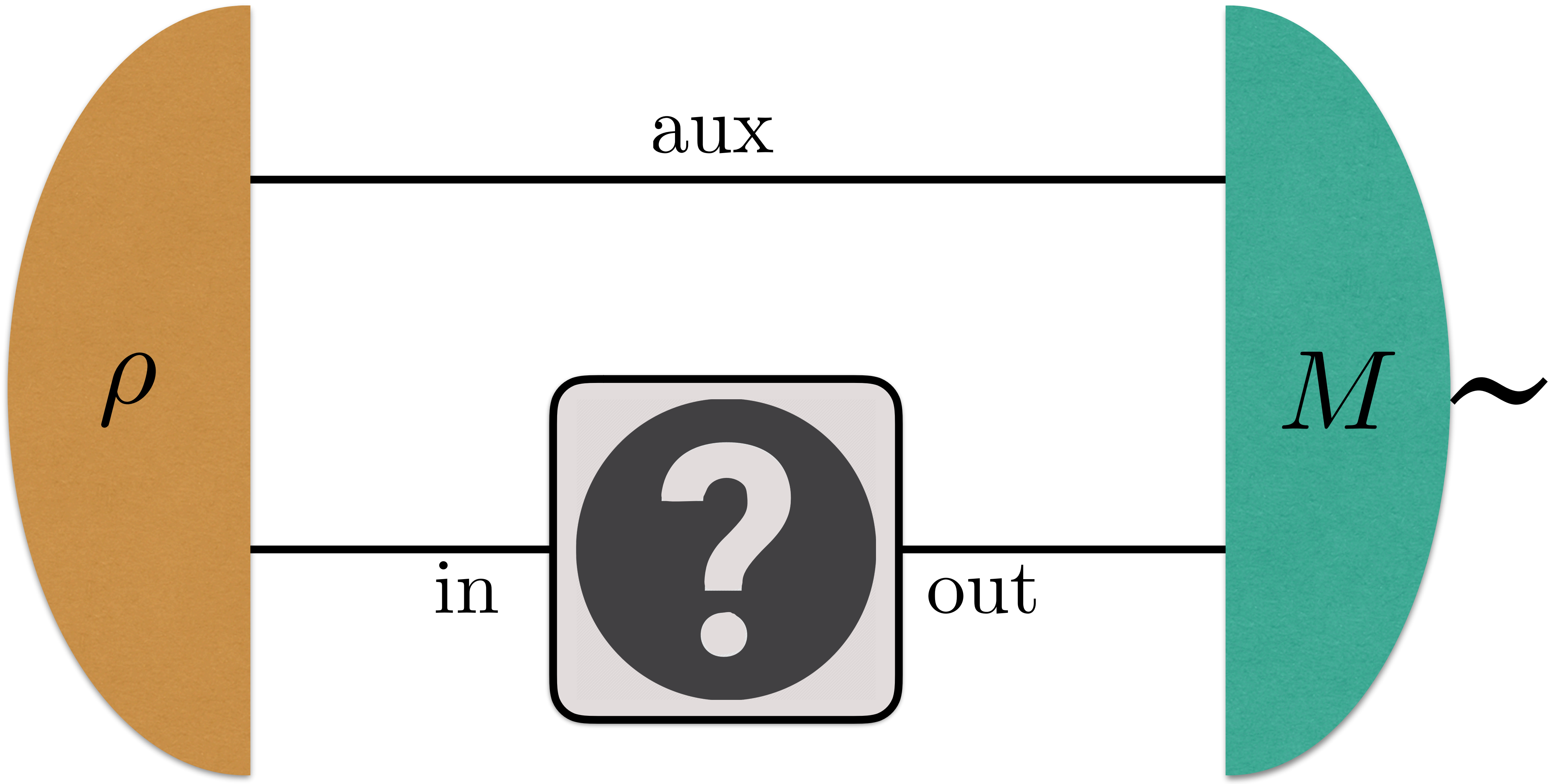

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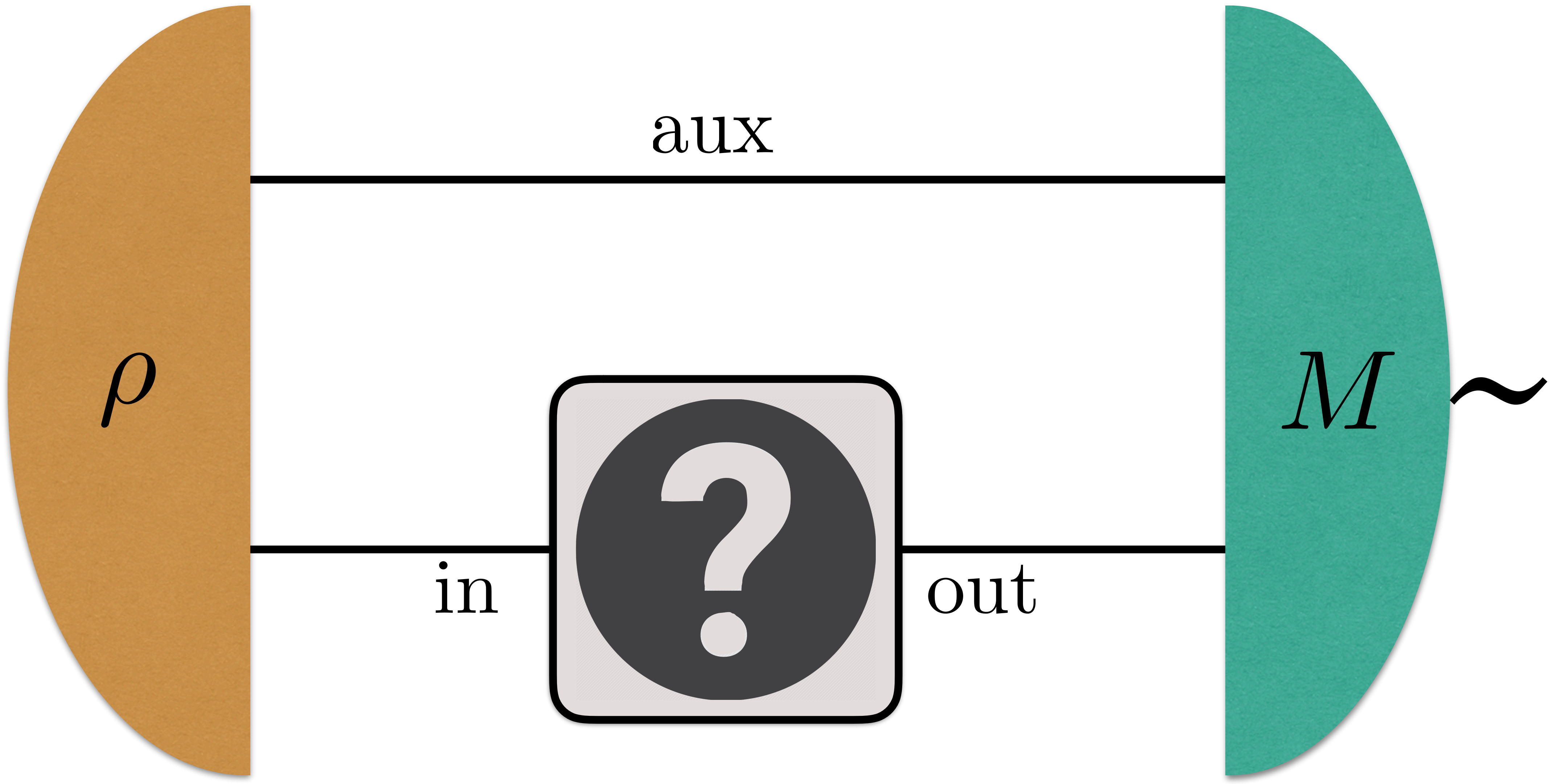
STRATEGY:

$$\rho = |\Phi^+\rangle\langle\Phi^+|$$

$$\{M_i\} = \{|\Phi^+\rangle\langle\Phi^+|, |\Phi^-\rangle\langle\Phi^-|, |\Psi^+\rangle\langle\Psi^+|, |\Psi^-\rangle\langle\Psi^-|\}$$



EXAMPLE

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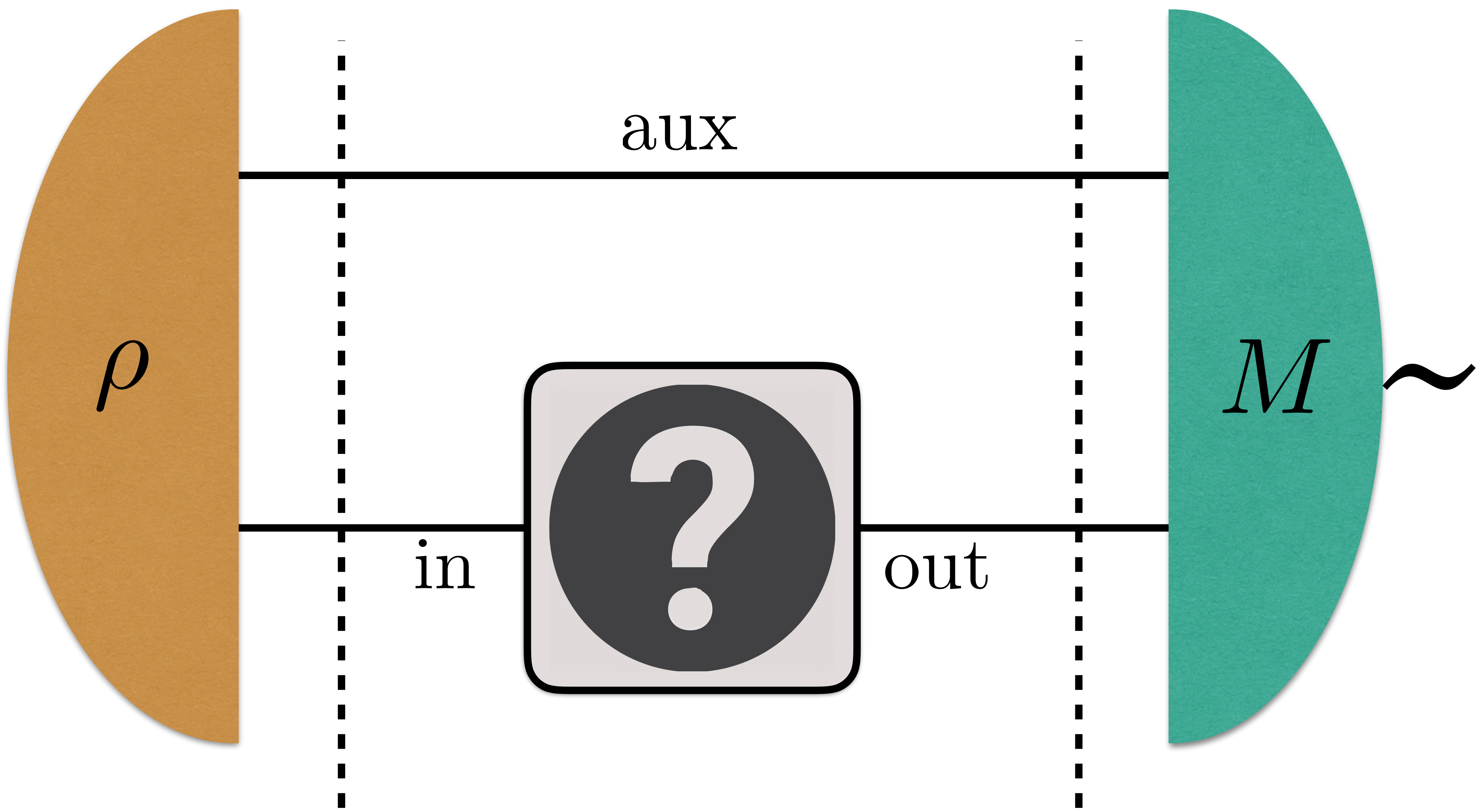

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EXAMPLE

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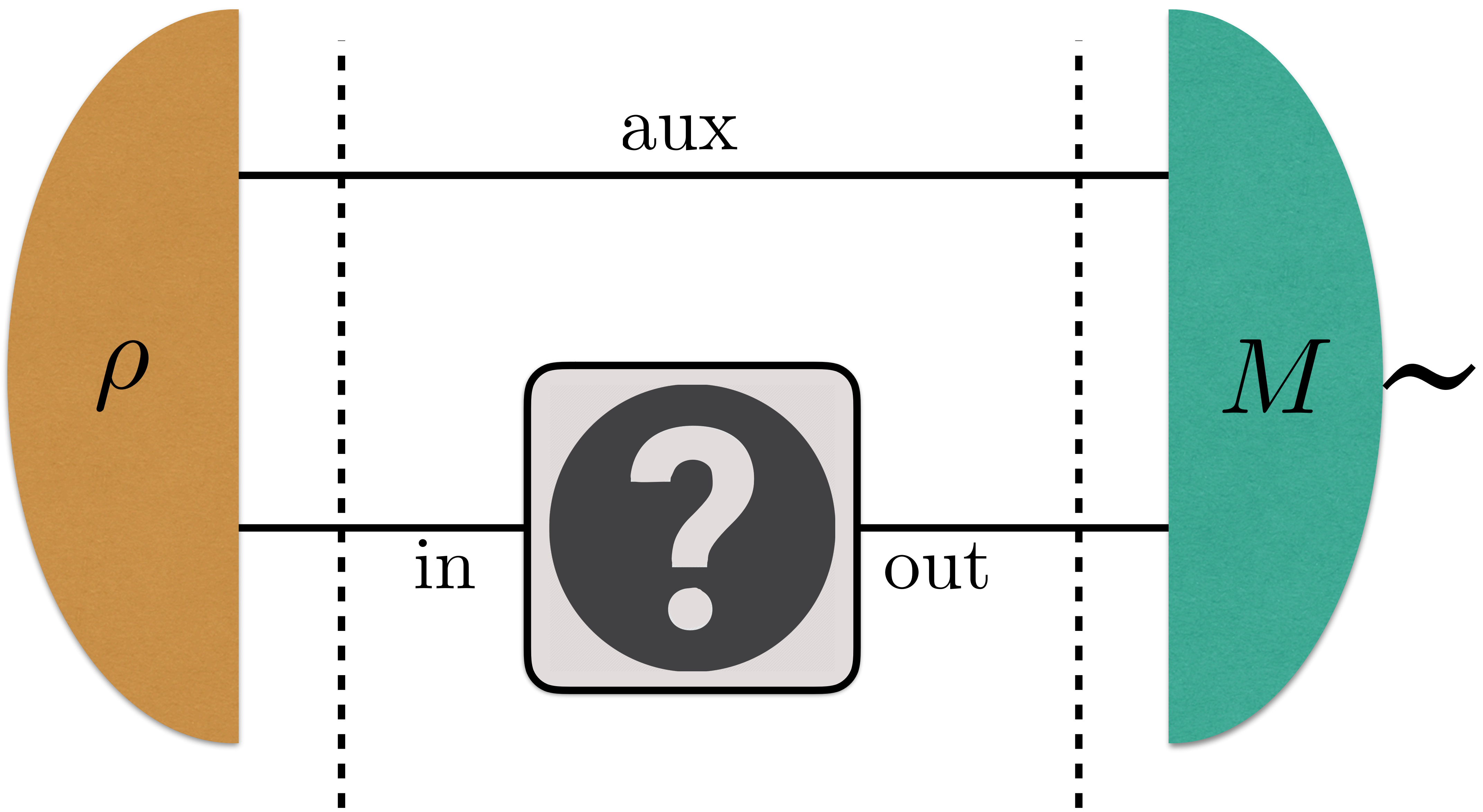

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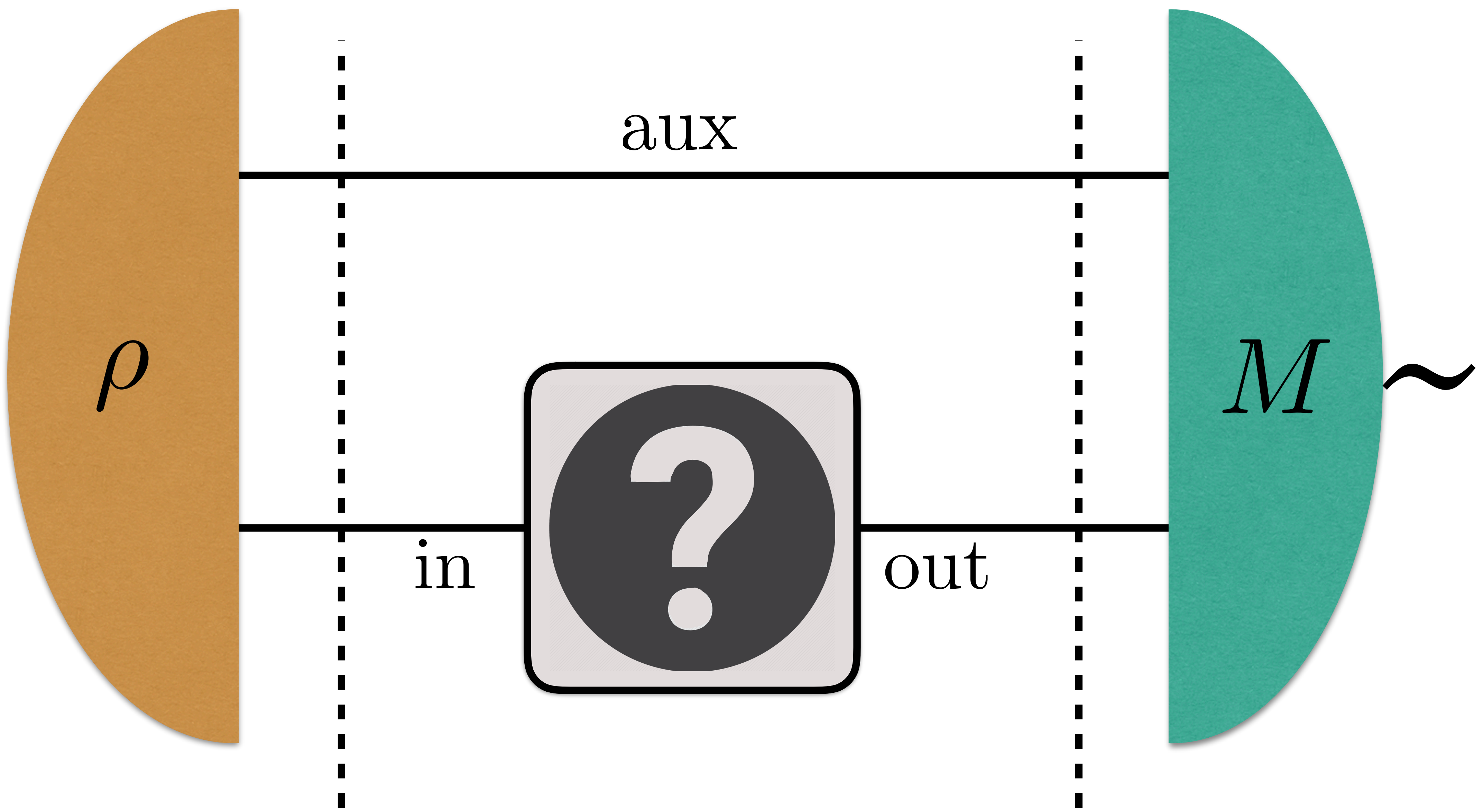

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$$p_{\text{succ}} = 1$$

$$(\mathbb{I} \otimes \mathbb{I})|\Phi^+\rangle\langle\Phi^+| = |\Phi^+\rangle\langle\Phi^+|$$

$$(\mathbb{I} \otimes \sigma_X)|\Phi^+\rangle\langle\Phi^+| = |\Psi^+\rangle\langle\Psi^+|$$

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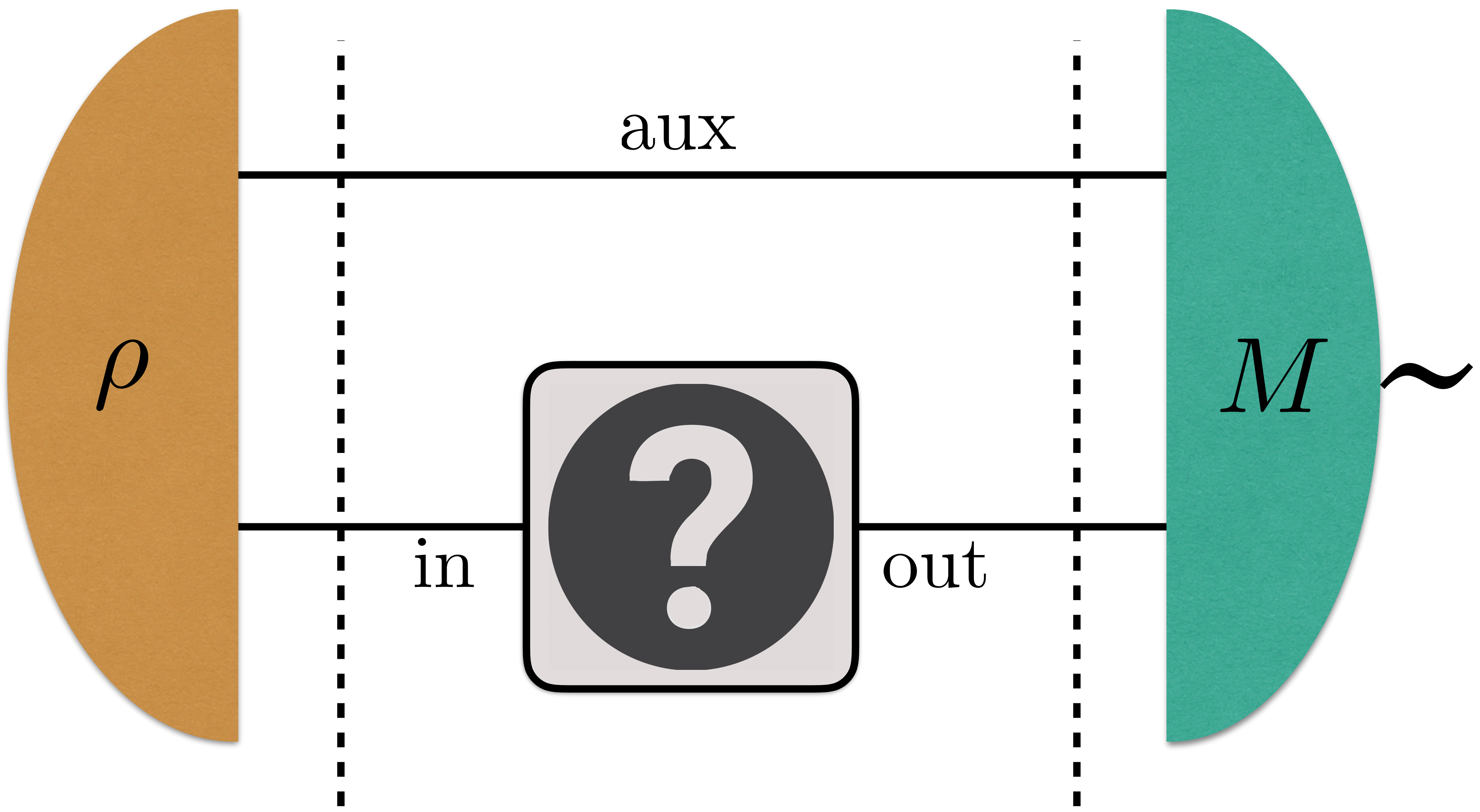
$$P = 1$$



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CHOI-JAMIOŁKOWSKI ISOMORPHISM

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$$\widetilde{C} : L(H^I) \to L(H^O) \qquad \mapsto \qquad C \in L(H^I \otimes H^O)$$

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CHOI-JAMIOŁKOWSKI ISOMORPHISM

$$\widetilde{C} : L(H^I) \rightarrow L(H^O) \qquad \mapsto \qquad C \in L(H^I \otimes H^O)$$

$$C := (\widetilde{\mathbb{I}} \otimes \widetilde{C})(|\Phi^+\rangle\langle\Phi^+|)$$

$$\widetilde{C} \text{ is a CPTP map, then} \qquad C \geq 0$$

$$\mathrm{Tr}_O C = \mathbb{I}^I$$

$$P := \max_{\rho,\{M_i\}} \sum_{i=1}^{N} p_i \, \mathrm{Tr}\left[(\widetilde{C}_i \otimes \widetilde{\mathbb{I}})(\rho)\, M_i\right]$$

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MAP

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$$P := \max_{\rho, \{M_i\}} \sum_{i=1}^{N} p_i \operatorname{Tr}\left[(\widetilde{C}_i \otimes \widetilde{\mathbb{I}})(\rho)\, M_i \right]$$

$$= \max_{\rho, \{M_i\}} \sum_{I=1}^{N} p_i \operatorname{Tr}\left[(\rho^{\mathrm{in,aux}} \otimes \mathbb{I}^{\mathrm{out}})(C_i^{\mathrm{in,out}} \otimes \mathbb{I}^{\mathrm{aux}})(M_i^{\mathrm{aux,out}} \otimes \mathbb{I}^{\mathrm{in}}) \right]$$

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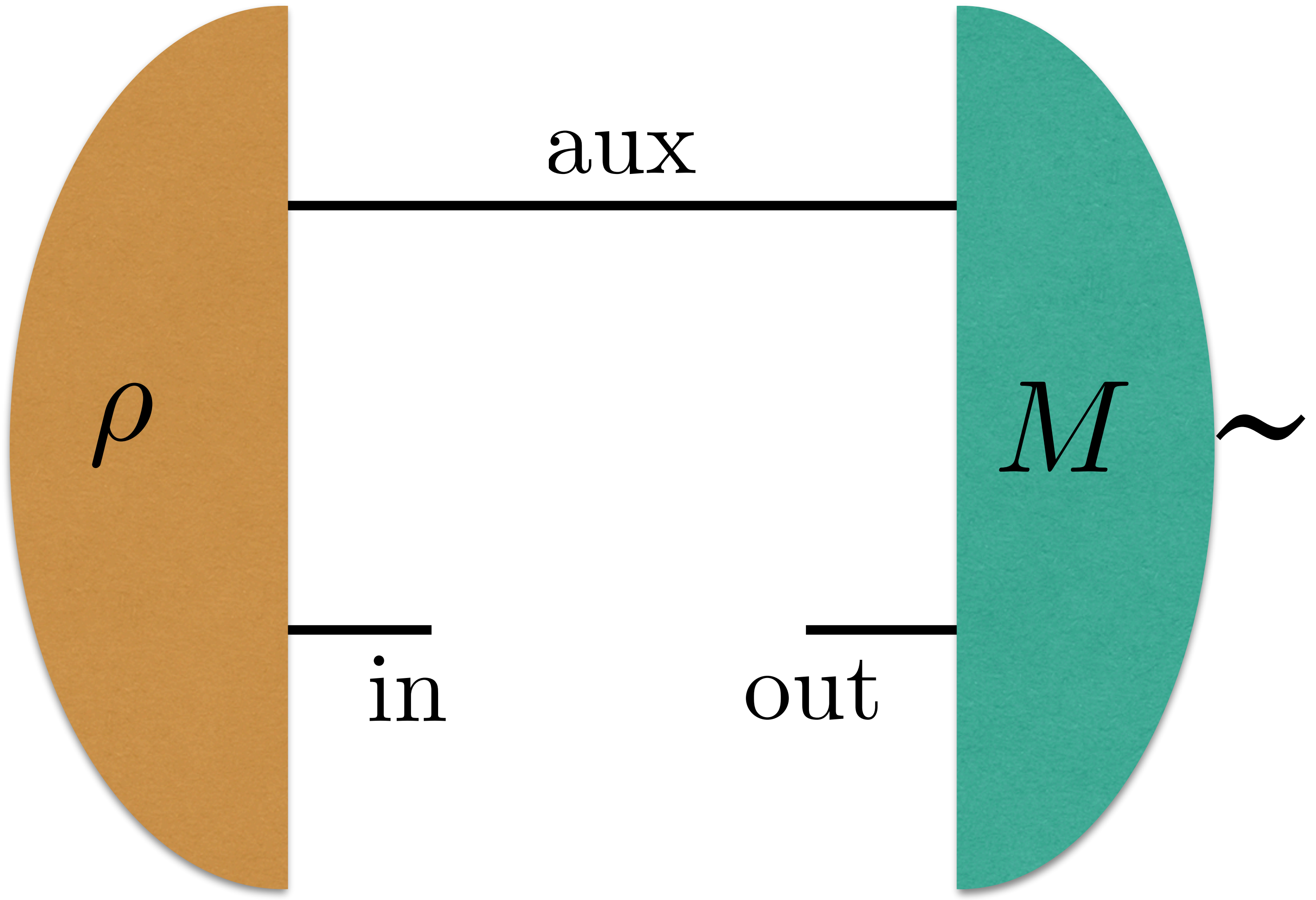
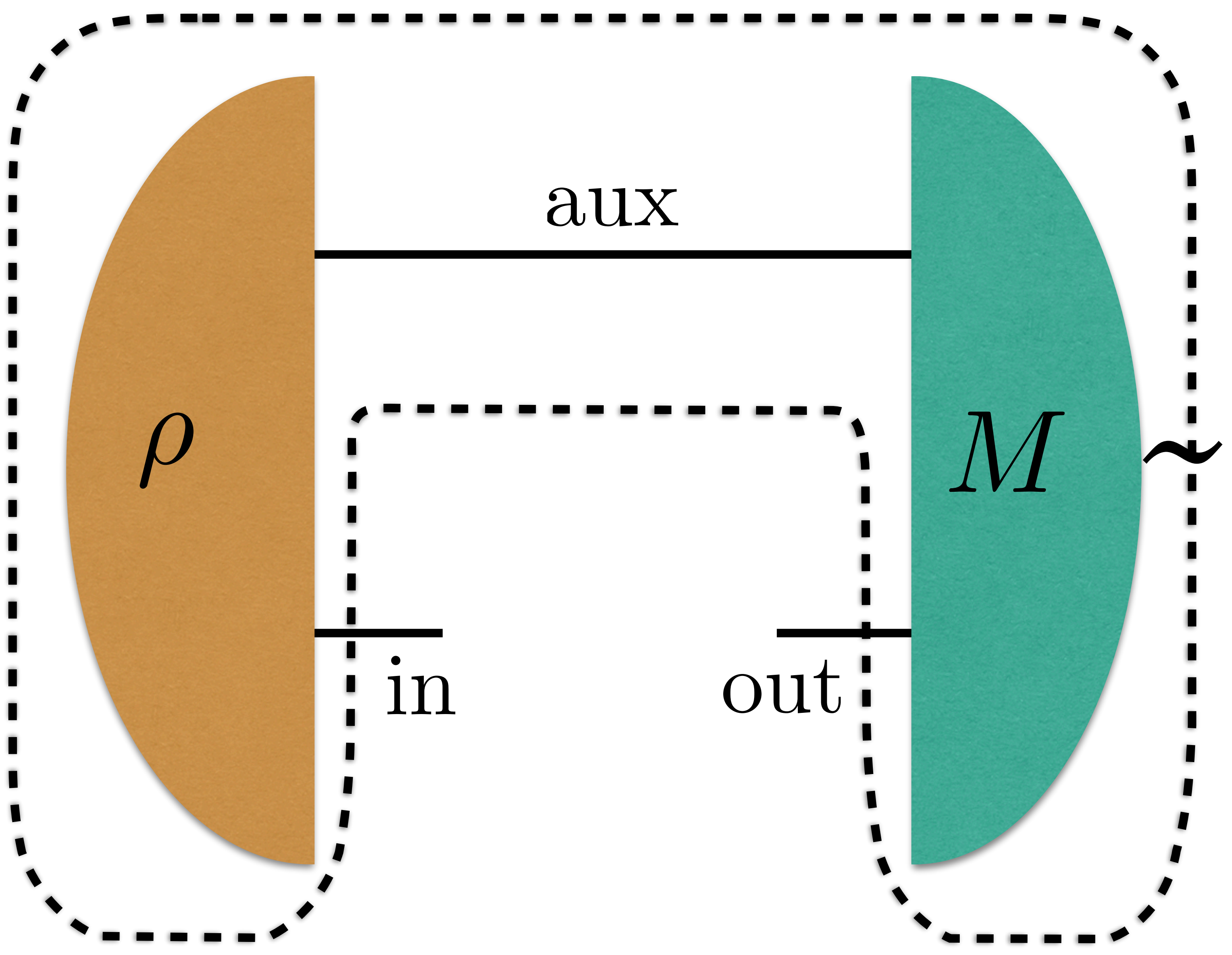
MAP

“CHOI STATE”

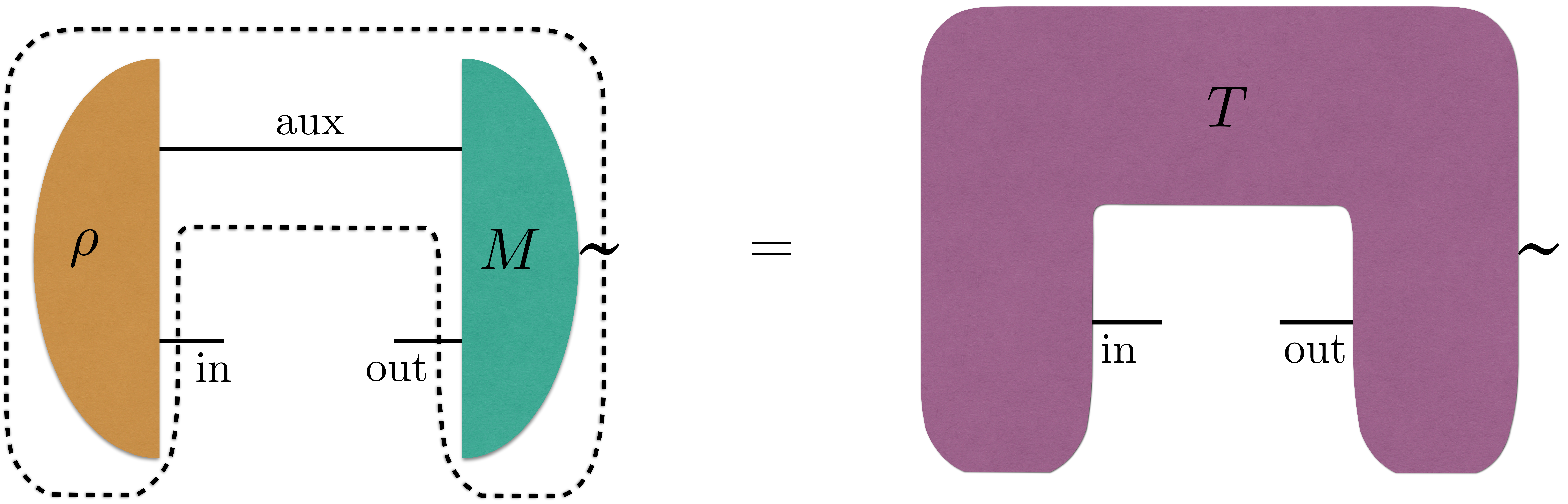
TESTERS



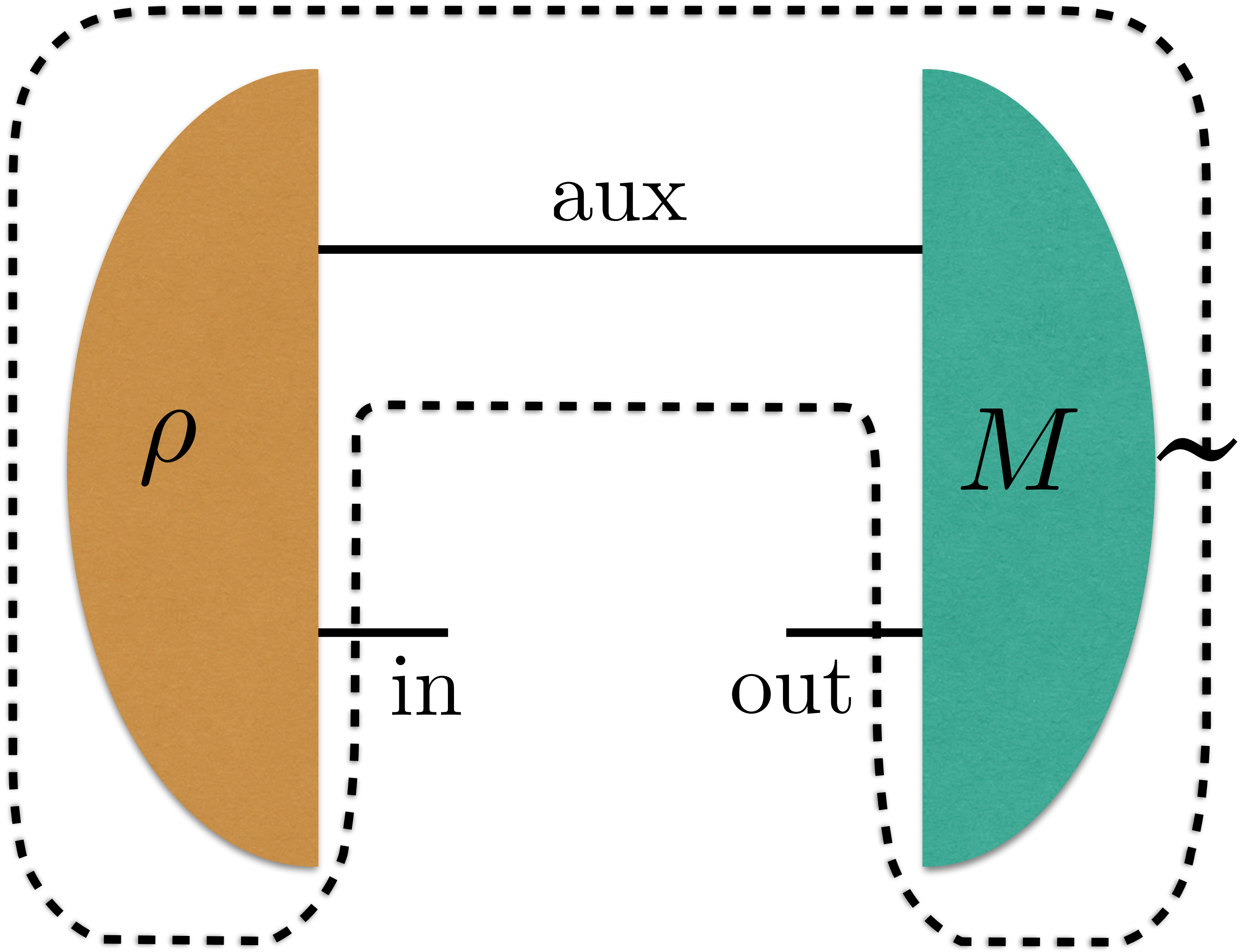
TESTERS



TESTERS



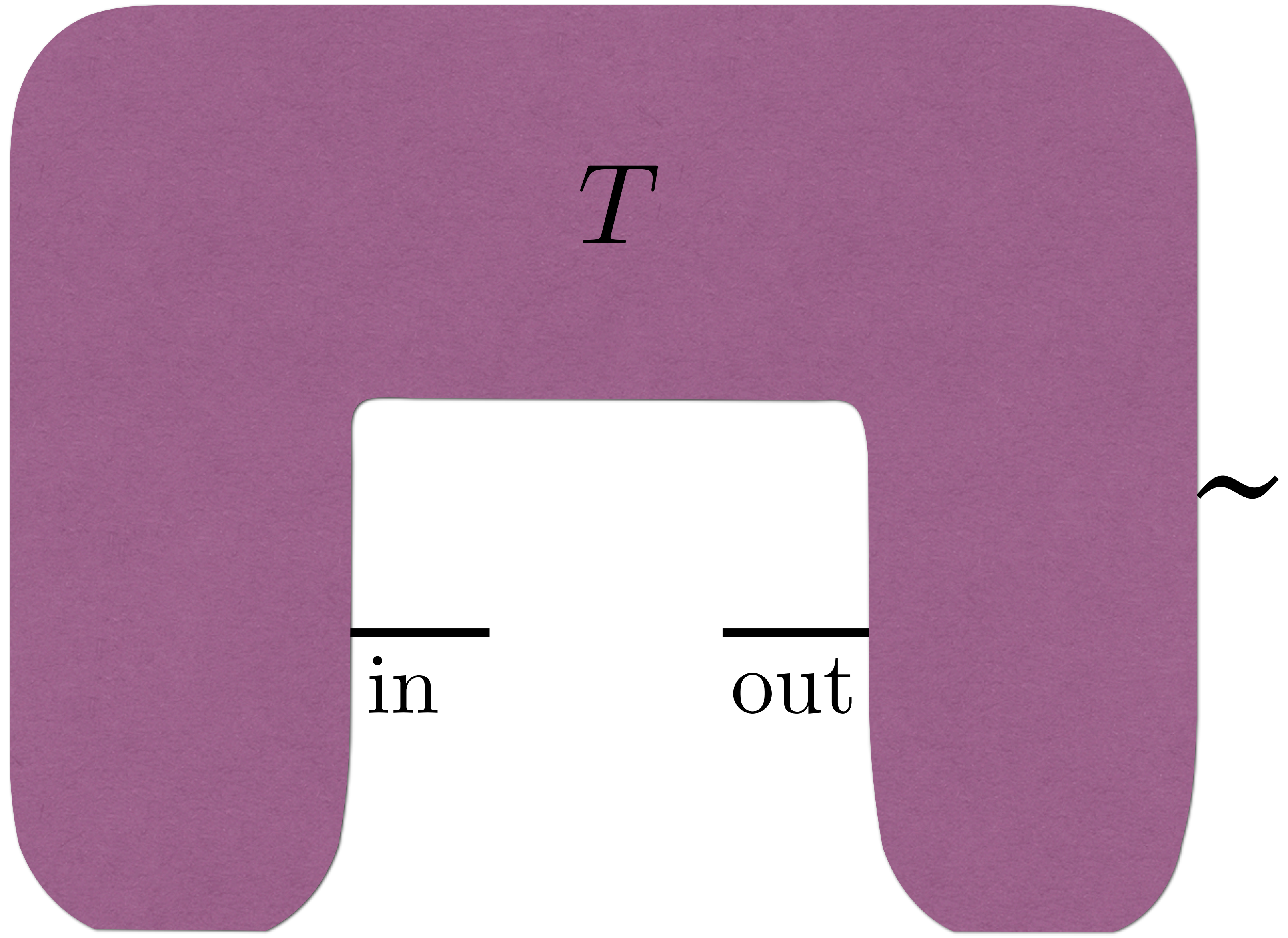
TESTERS



$$\rho \in L(H^{\text{in, aux}})$$

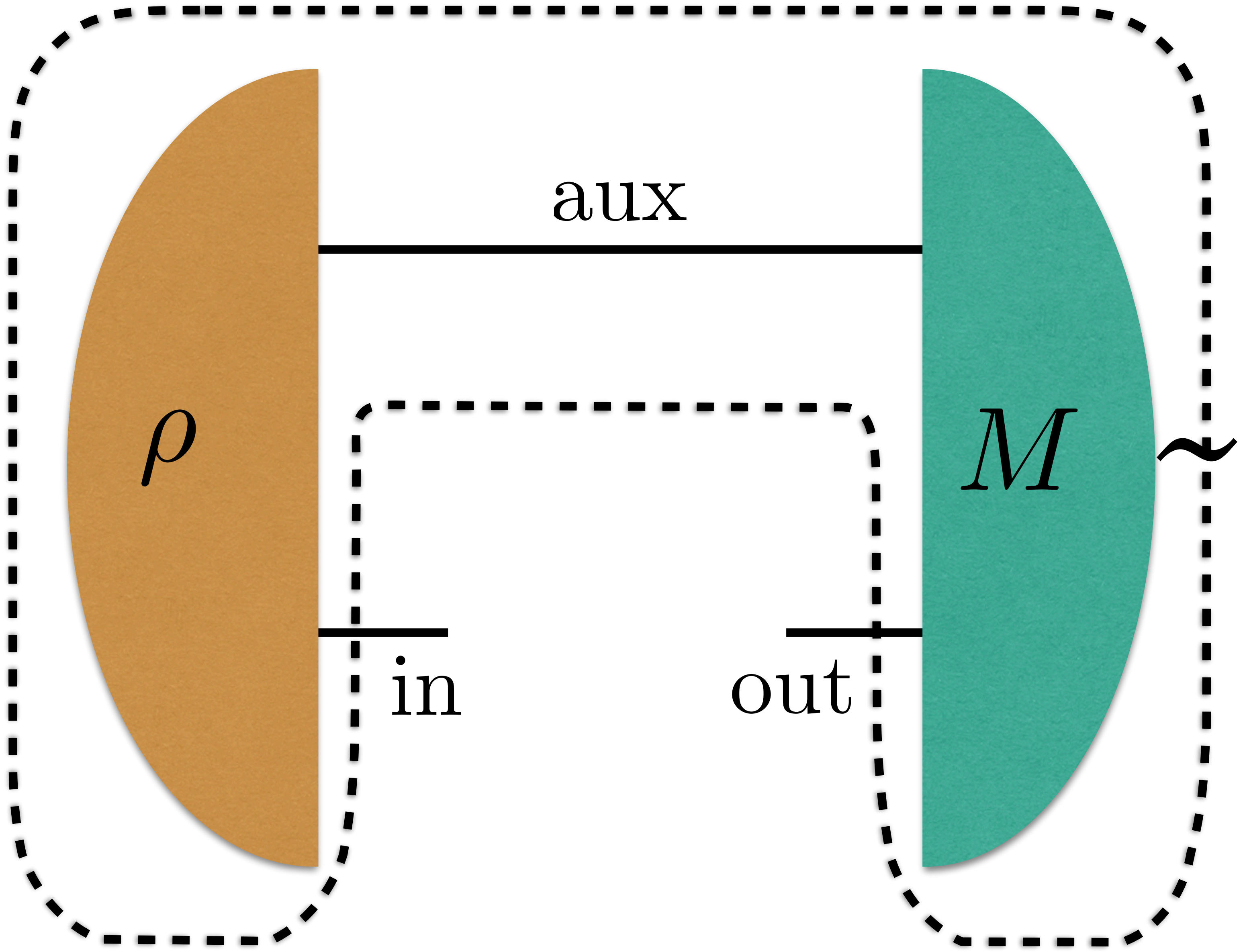
$$M_i \in L(H^{\text{out, aux}})$$

$=$



$$T_i \in L(H^{\text{in, out}})$$

TESTERS



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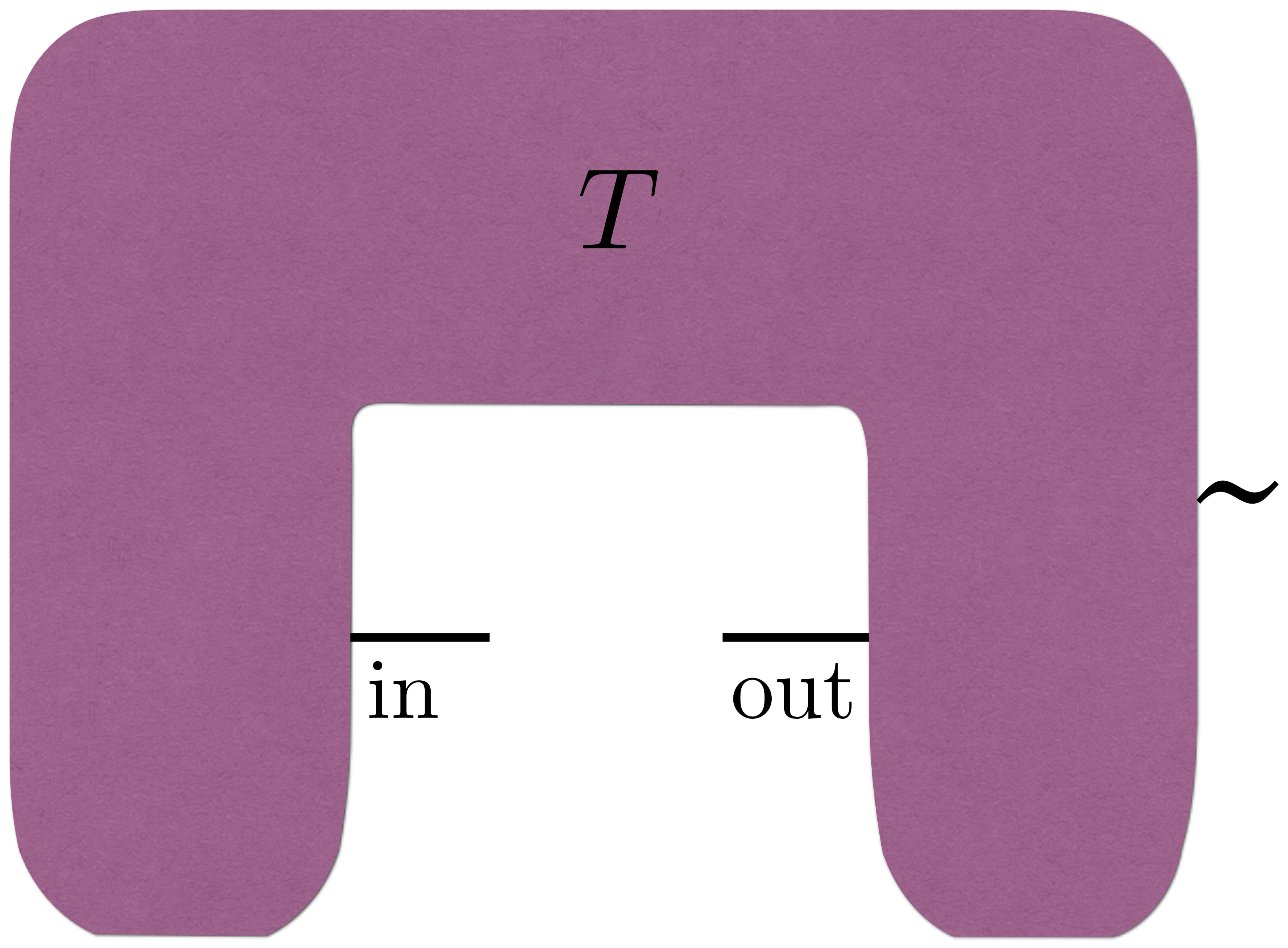


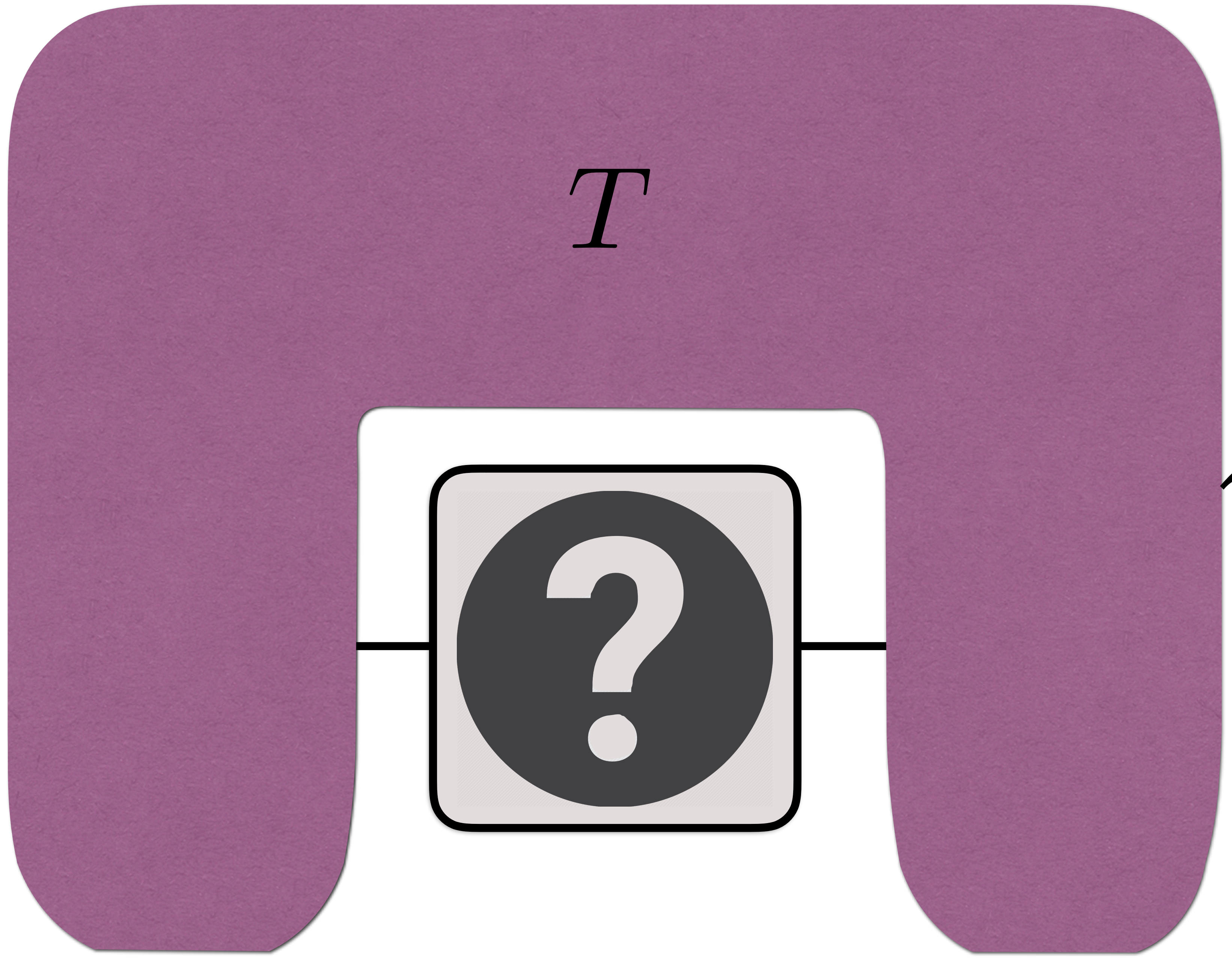
$\rho \in L(H^{\text{in, aux}})$

$M_i \in L(H^{\text{out, aux}})$

$T_i \in L(H^{\text{in, out}})$

$\rho, \{M_i\}$ $\{T_i\}$

TESTERS



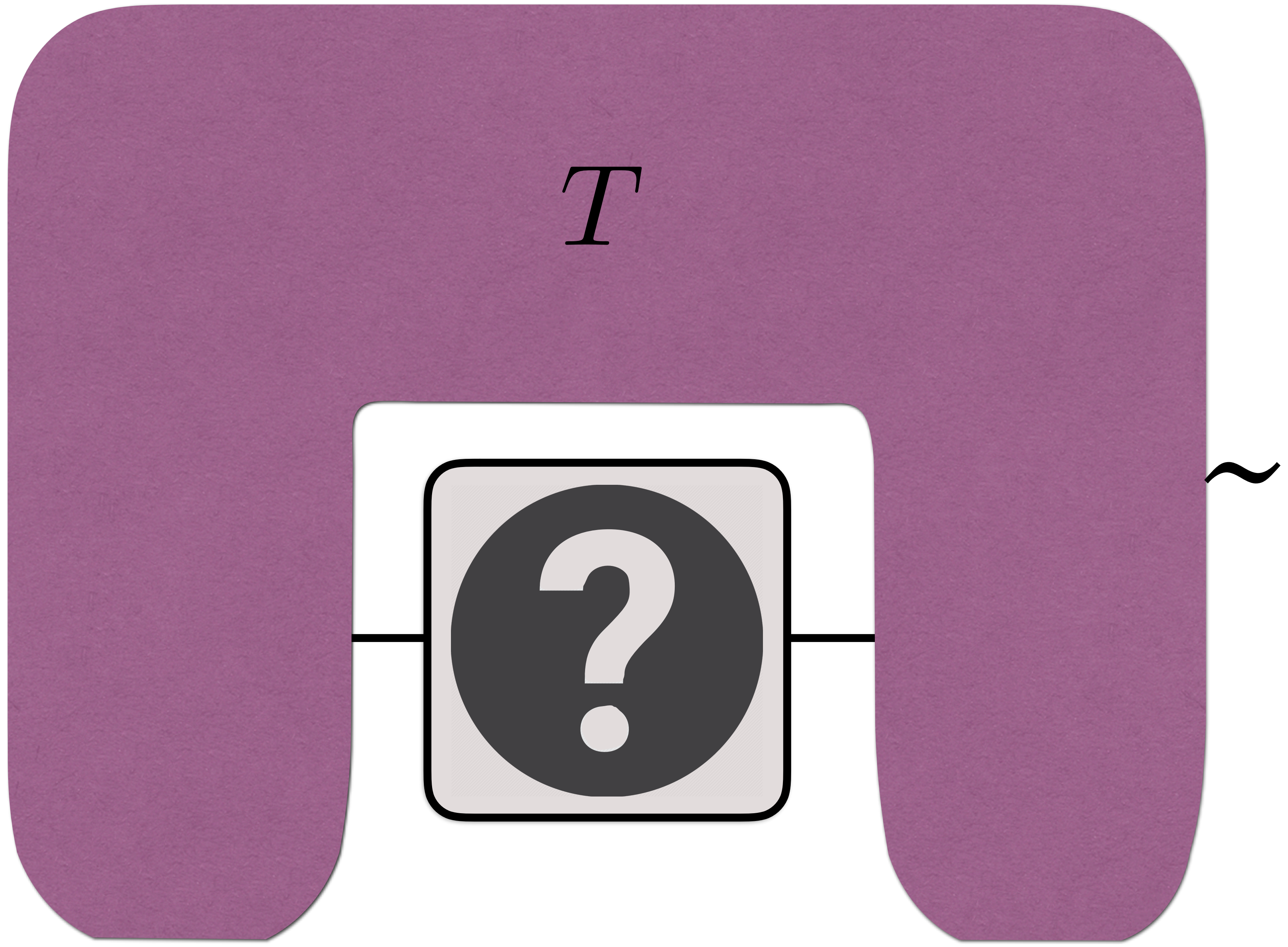
$\sim$

$$T = \{T_i\} \quad :$$

$$T_i \geq 0$$

$$\sum_i T_i = \rho_o^{\text{in}} \otimes I^{\text{out}}$$

TESTERS



$$T = \{T_i\} \quad :$$

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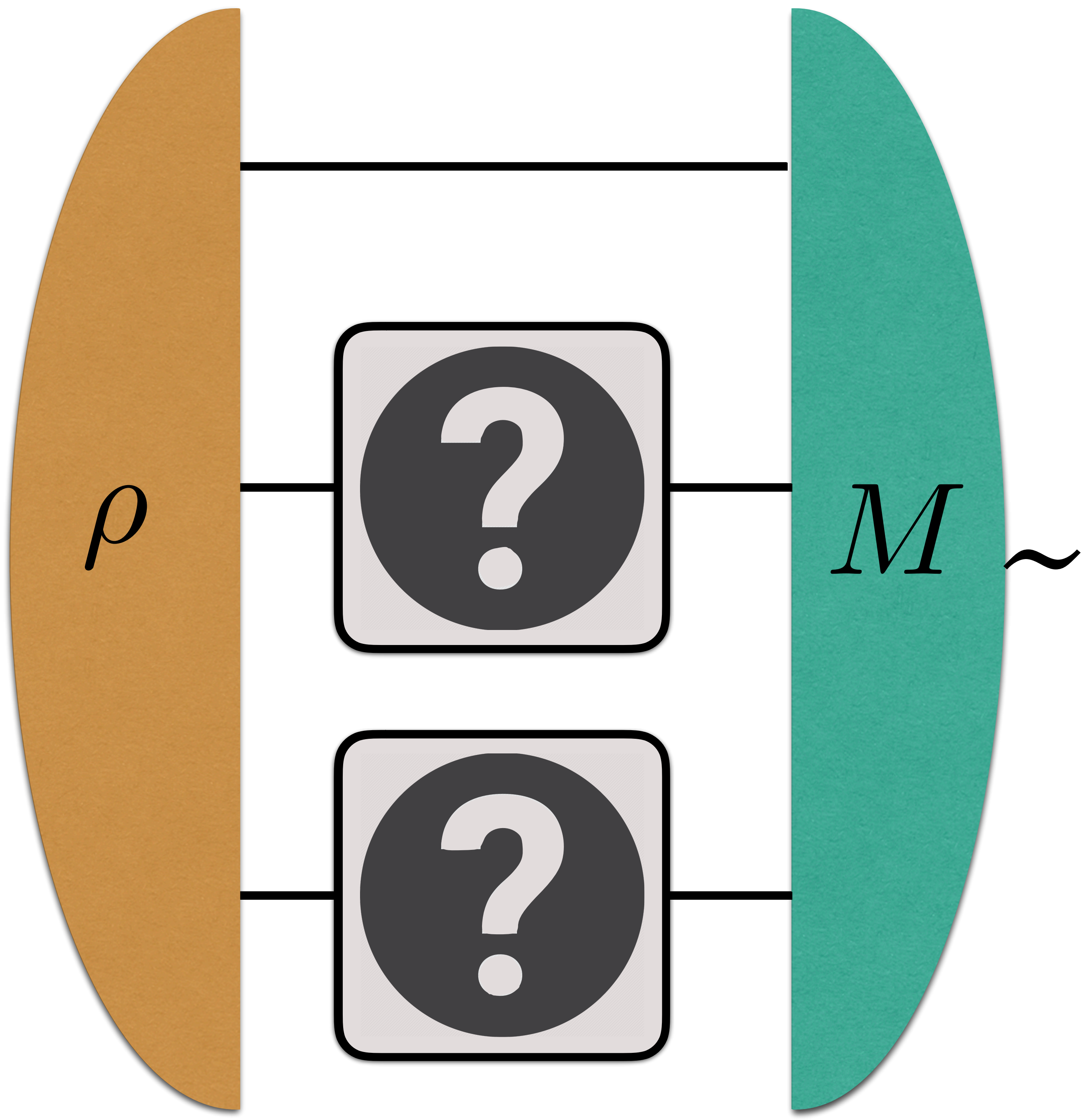
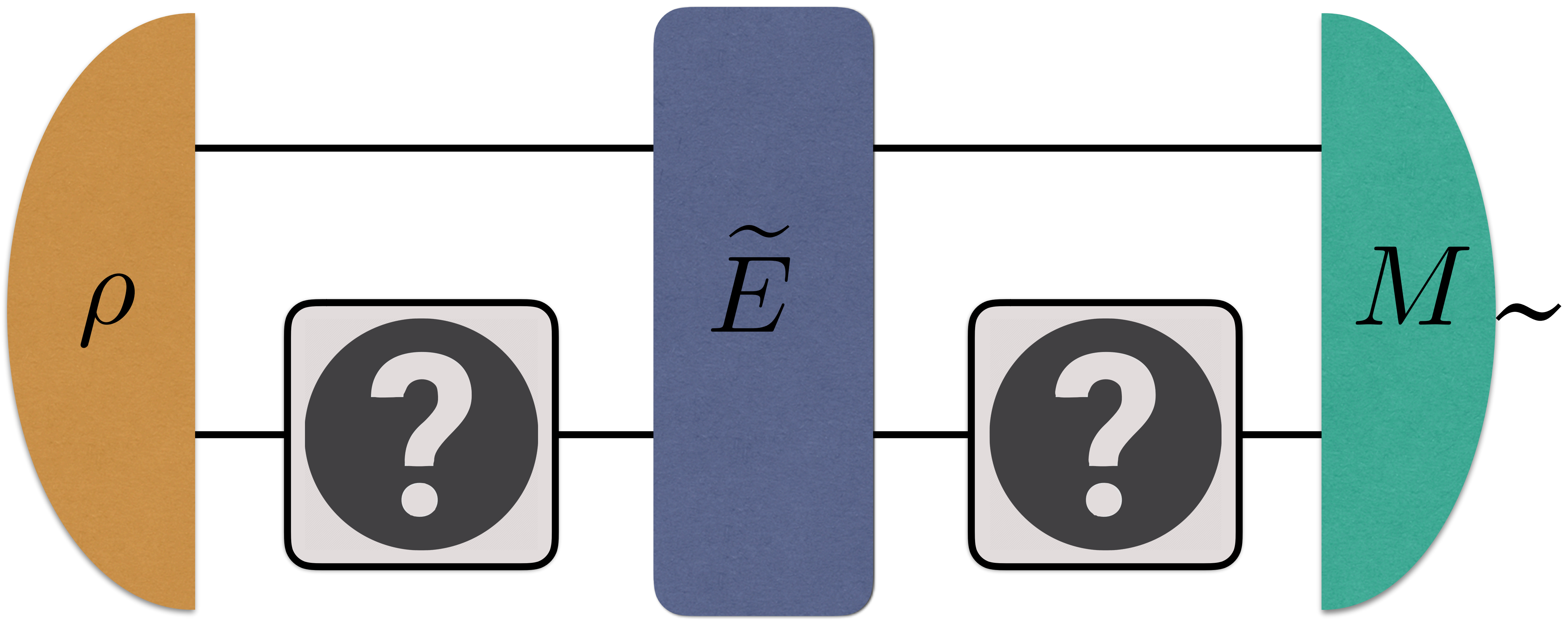
$$P = \max_{\{T_i\}} \sum_i p_i \, \mathrm{Tr}(C_i \, T_i)$$

TWO COPIES


ρ
M ~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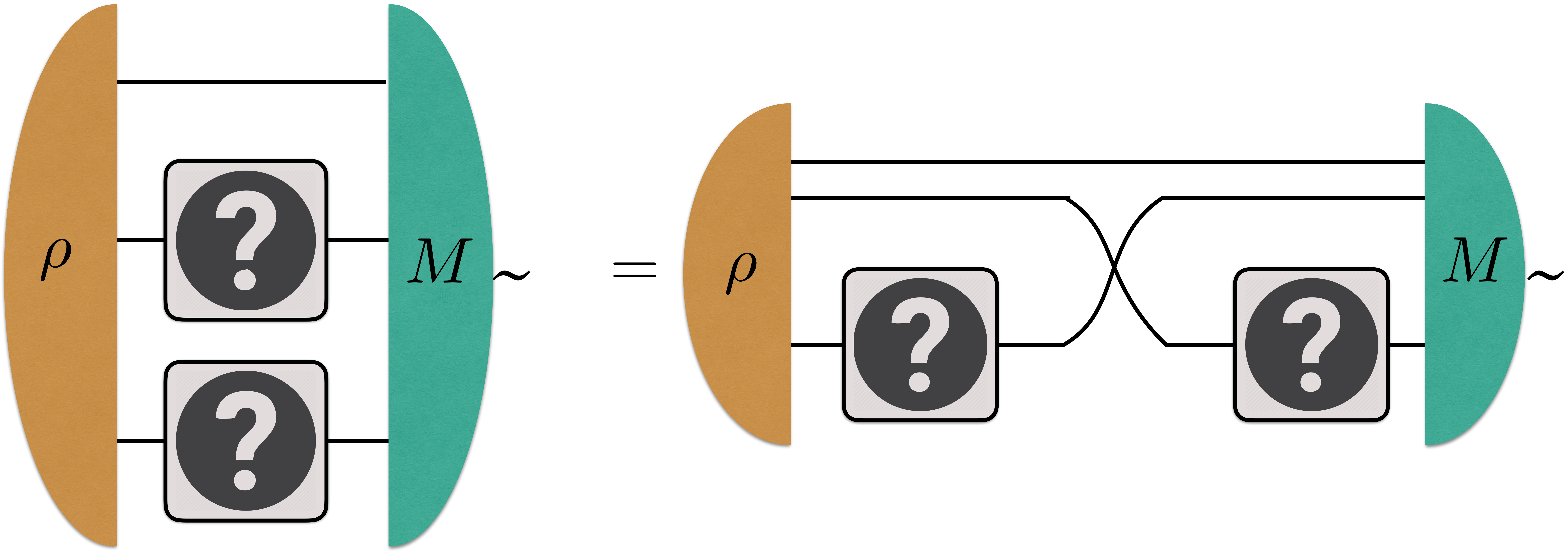
ρ
M
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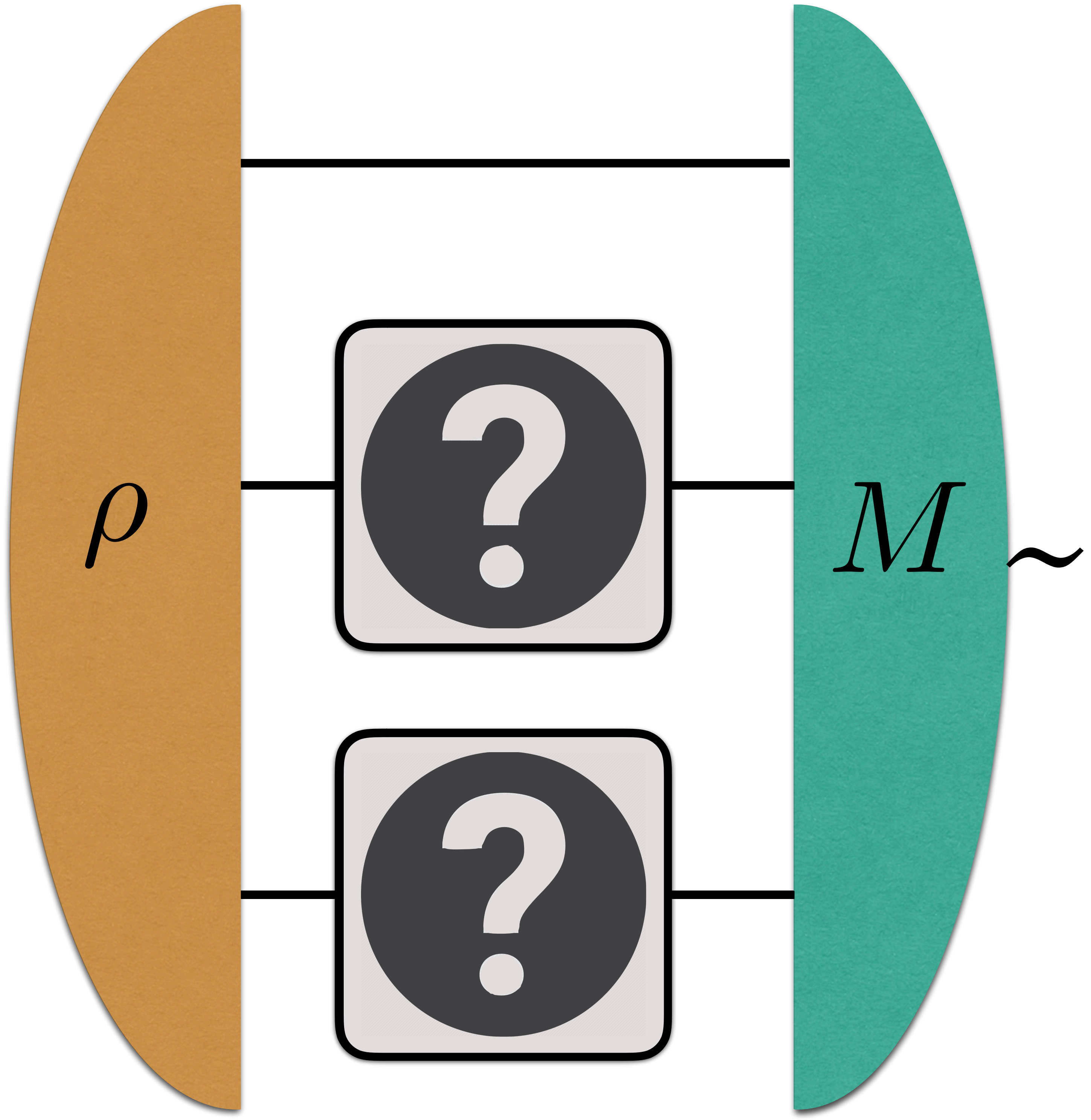
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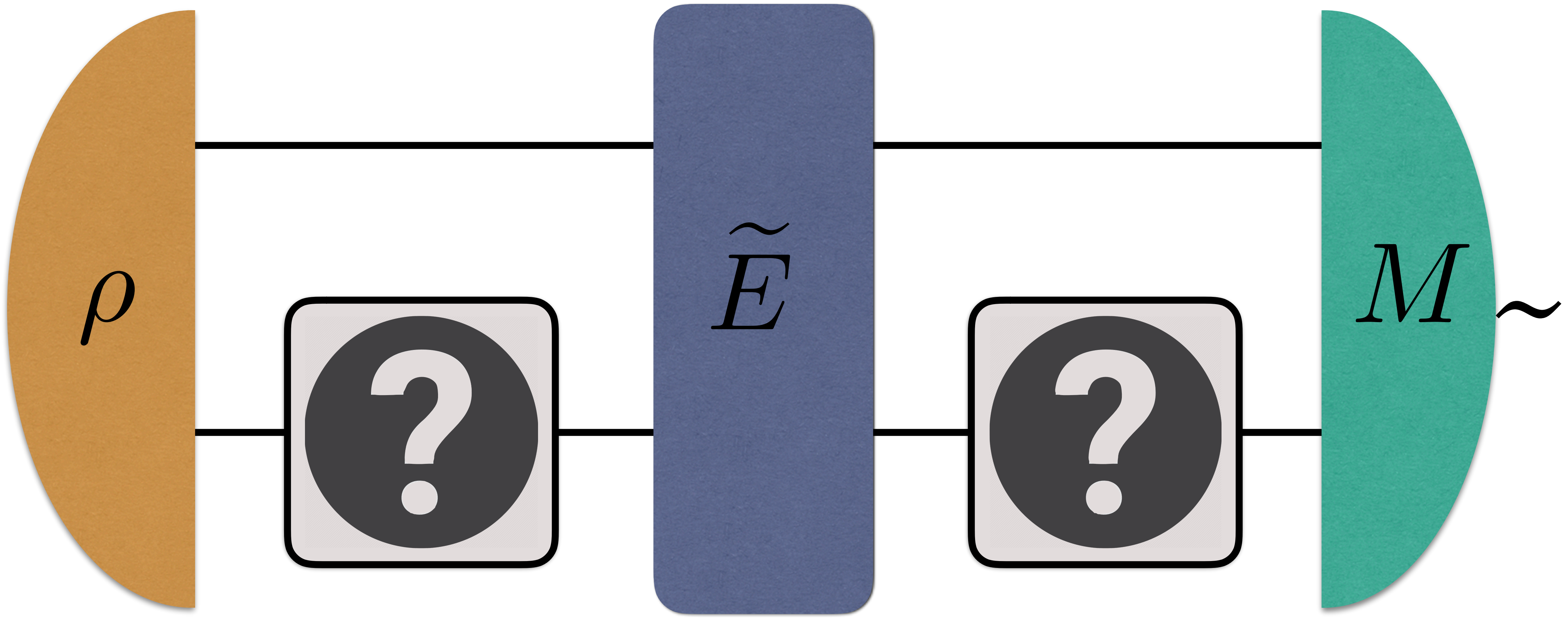
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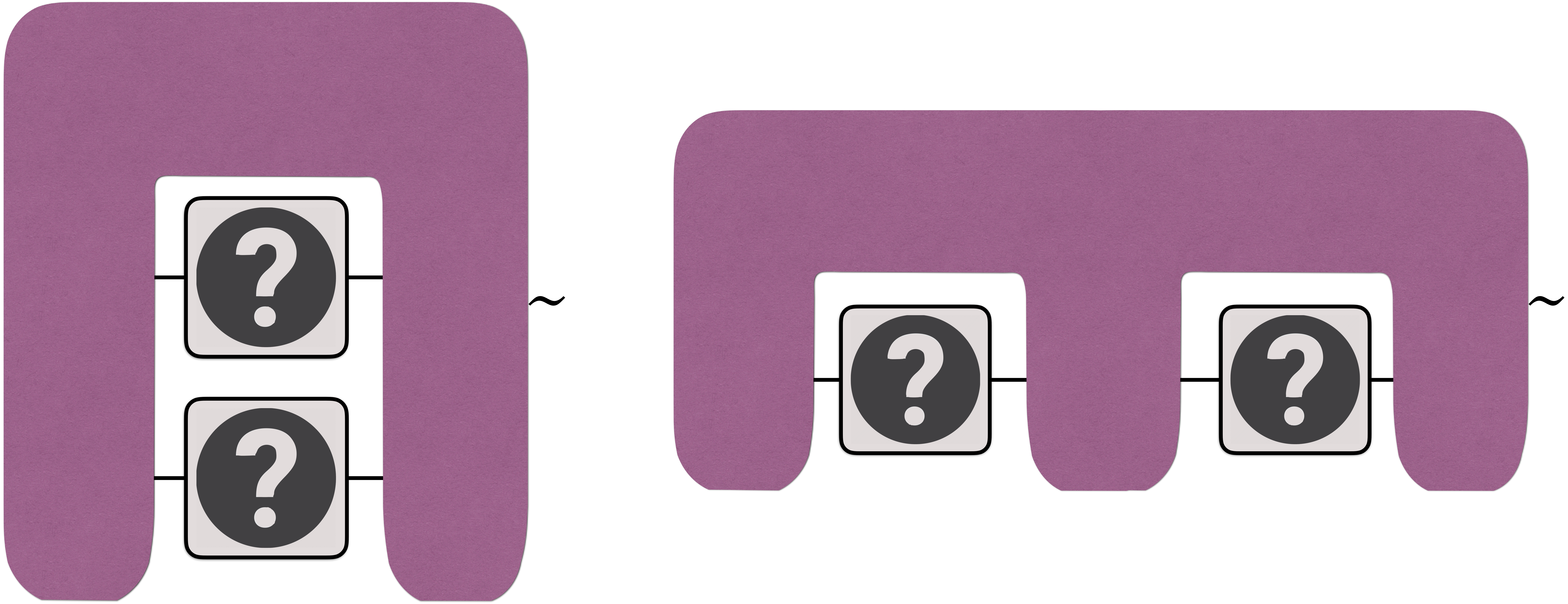

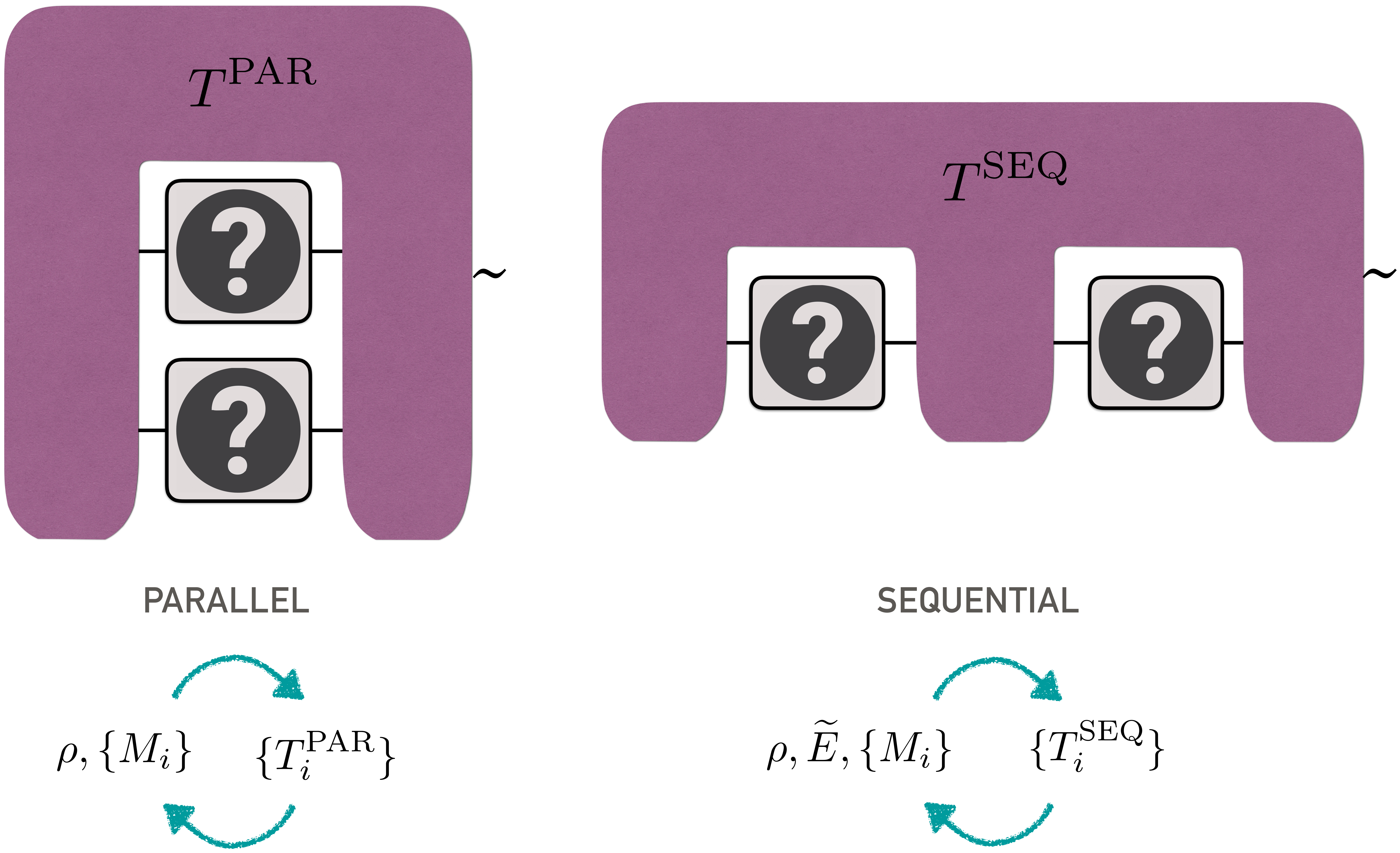
T^{PAR}
$\sim$
T^{SEQ}
$\sim$
PARALLEL
SEQUENTIAL
$\rho, \{M_i\}$
$\{T_i^{\mathrm{PAR}}\}$
$\rho, \widetilde{E}, \{M_i\}$
$\{T_i^{\mathrm{SEQ}}\}$

PARALLEL

$$T^{\mathrm{PAR}} = \{T_i^{\mathrm{PAR}}\} \ :$$

$$T_i^{\mathrm{PAR}} \geq 0$$

$$\sum_i T_i^{\mathrm{PAR}} = W^{\mathrm{PAR}}$$

SEQUENTIAL

$$T^{\mathrm{SEQ}} = \{T_i^{\mathrm{SEQ}}\} \ :$$

$$T_i^{\mathrm{SEQ}} \geq 0$$

$$\sum_i T_i^{\mathrm{SEQ}} = W^{\mathrm{SEQ}}$$

$$T = \{T_i\} \ :$$

$$T_i \geq 0$$

$$\sum_i T_i = \rho_o^{\mathrm{in}} \otimes I^{\mathrm{out}}$$

PARALLEL

$$T^{\mathrm{PAR}} = \{T_i^{\mathrm{PAR}}\} \;\; :$$

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SEQUENTIAL

$$T^{\mathrm{SEQ}} = \{T_i^{\mathrm{SEQ}}\} \;\; :$$

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PROCESS

$$T = \{T_i\} \;\; :$$

$$T_i \geq 0$$

$$\sum_i T_i = \rho_o^{\mathrm{in}} \otimes I^{\mathrm{out}}$$

PARALLEL	SEQUENTIAL
$T^{\mathrm{PAR}} = \{T_i^{\mathrm{PAR}}\} \;\;:$	$T^{\mathrm{SEQ}} = \{T_i^{\mathrm{SEQ}}\} \;\;:$
$T_i^{\mathrm{PAR}} \geq 0$	$T_i^{\mathrm{SEQ}} \geq 0$
$\sum_i T_i^{\mathrm{PAR}} = W^{\mathrm{PAR}}$	$\sum_i T_i^{\mathrm{SEQ}} = W^{\mathrm{SEQ}}$
$P^{\mathrm{PAR}} = \max_{\{T_i^{\mathrm{PAR}}\}} \sum_i p_i \operatorname{Tr}\left(C_i^{\otimes 2}\, T_i^{\mathrm{PAR}}\right)$	$P^{\mathrm{SEQ}} = \max_{\{T_i^{\mathrm{SEQ}}\}} \sum_i p_i \operatorname{Tr}\left(C_i^{\otimes 2}\, T_i^{\mathrm{SEQ}}\right)$

GENERAL TESTERS

GENERAL TESTERS

- Extracting probability distributions from channels:

GENERAL TESTERS

- Extracting probability distributions from channels:

The most general bilinear function $f:(C_1, C_2) \to \mathbb{R}$ *that extracts valid probability distributions from a pair of Choi states of quantum channels* $C_1 \in L(H^{I_1} \otimes H^{O_1})$ *and* $C_2 \in L(H^{I_2} \otimes H^{O_2})$ *is*

$$p(i|C_1, C_2) = \mathrm{Tr}[(C_1 \otimes C_2)\, T_i^{\mathrm{GEN}}],$$

where $T^{\mathrm{GEN}} = \{T_i^{\mathrm{GEN}}\},\ T_i^{\mathrm{GEN}} \in L(H^{I_1} \otimes H^{O_1} \otimes H^{I_2} \otimes H^{O_2})$

is a ***general tester.***

GENERAL TESTERS

$$T^{\mathrm{GEN}} = \{T_i^{\mathrm{GEN}}\} \quad :$$

$$T_i^{\mathrm{GEN}} \geq 0 \quad \forall\ i$$

$$\sum_i T_i^{\mathrm{GEN}} = W^{\mathrm{GEN}}$$

GENERAL TESTERS

$$T^{\mathrm{GEN}} = \{T_i^{\mathrm{GEN}}\} \ :$$

$$T_i^{\mathrm{GEN}} \geq 0 \ \ \forall \ i$$

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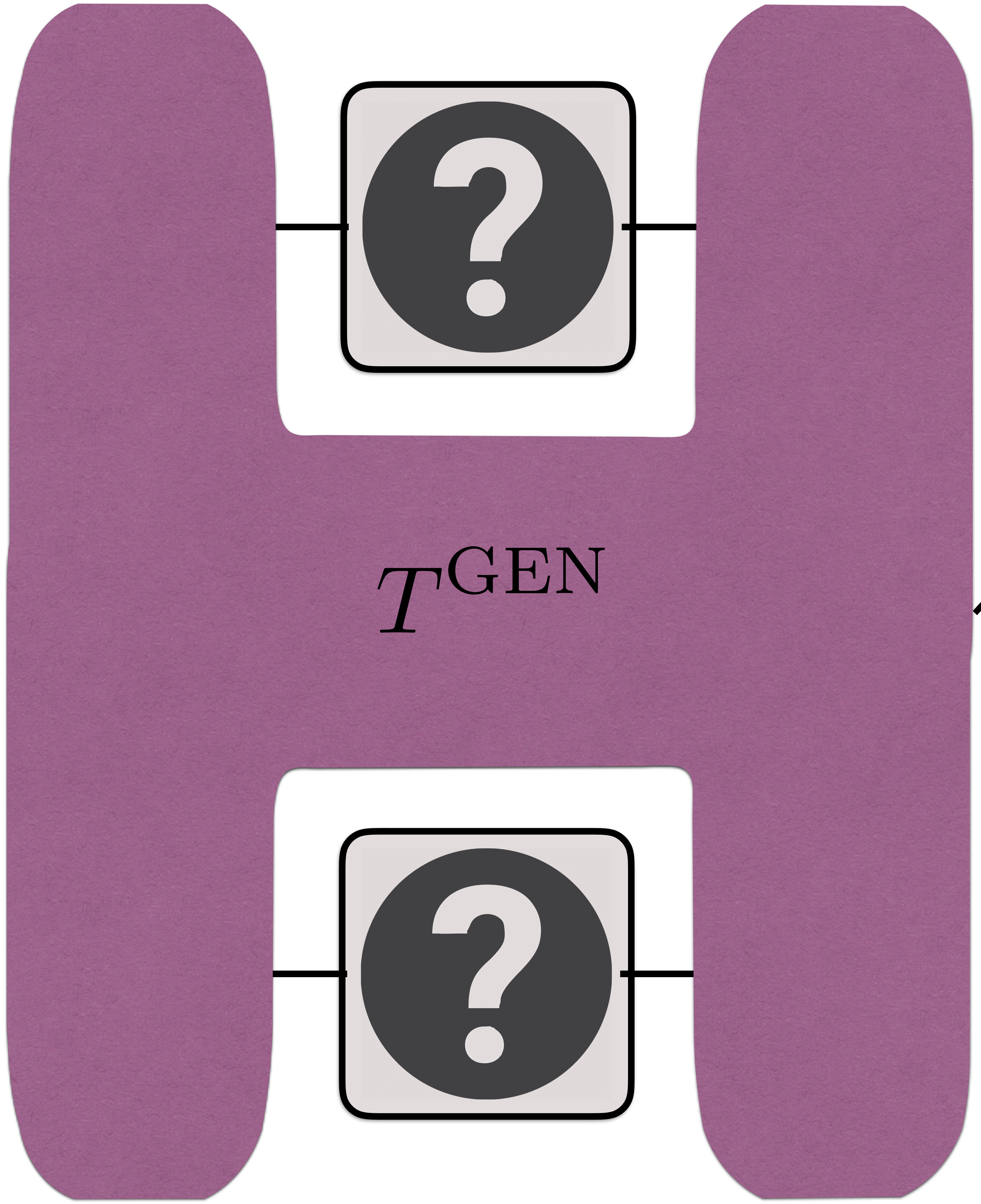
PROCESS MATRIX

GENERAL TESTERS



$\sim$

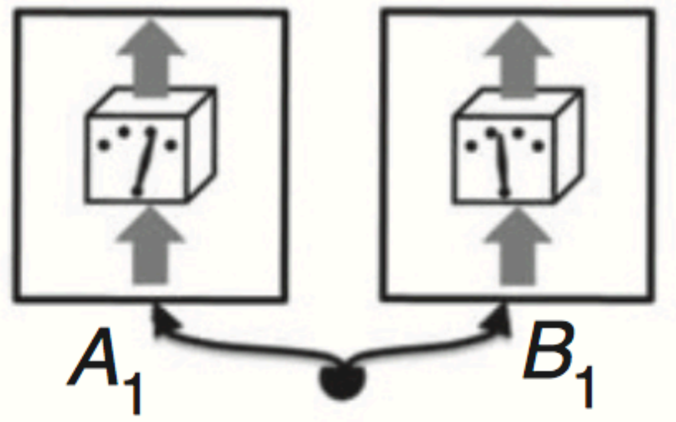
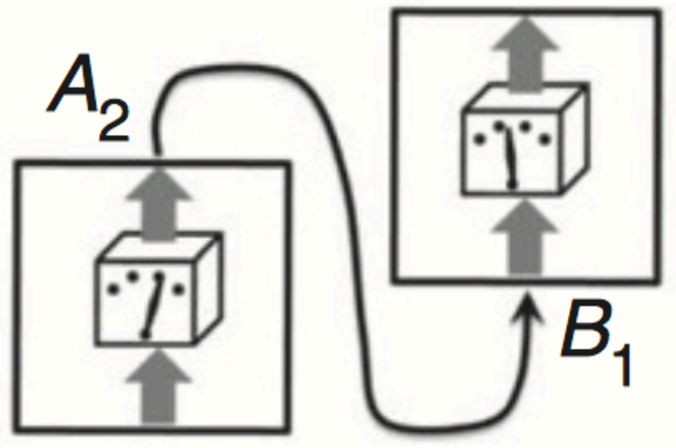
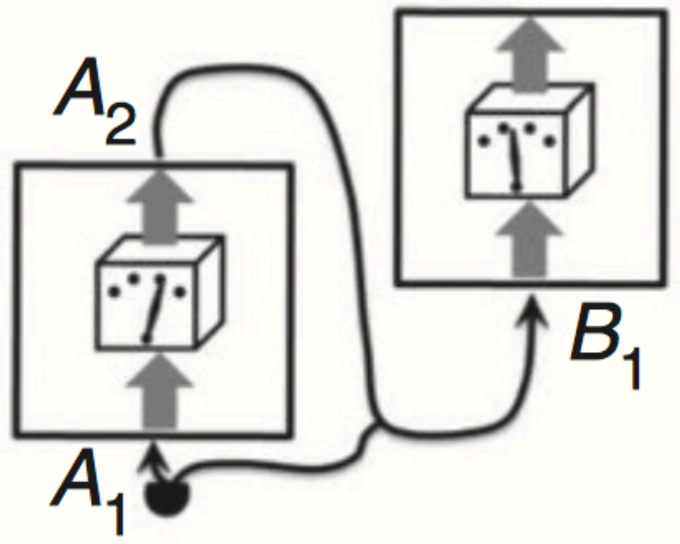
$$T^{\text{GEN}} = \{T_i^{\text{GEN}}\} \;:$$

$$T_i^{\text{GEN}} \geq 0 \quad \forall\, i$$

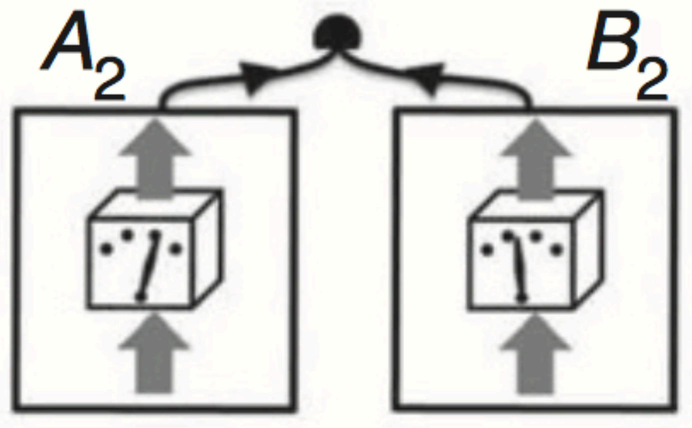
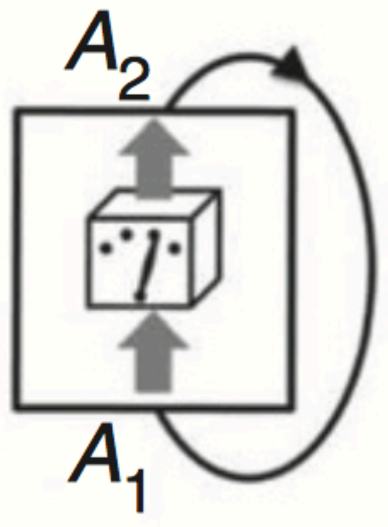
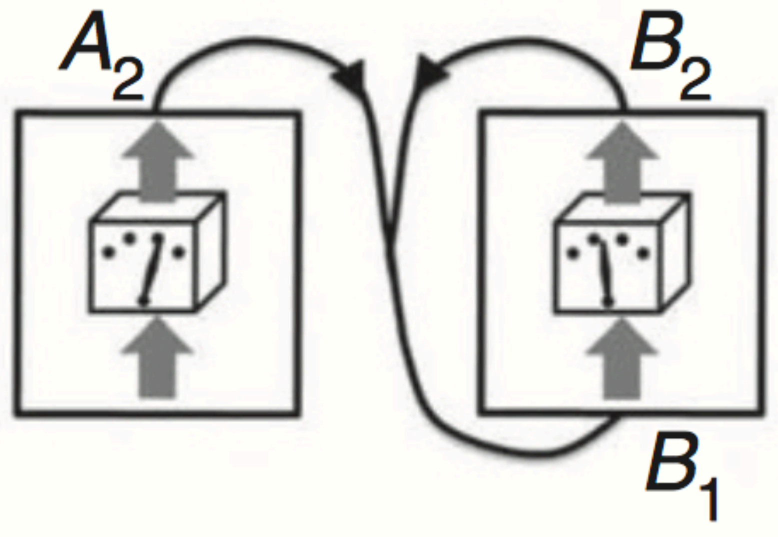
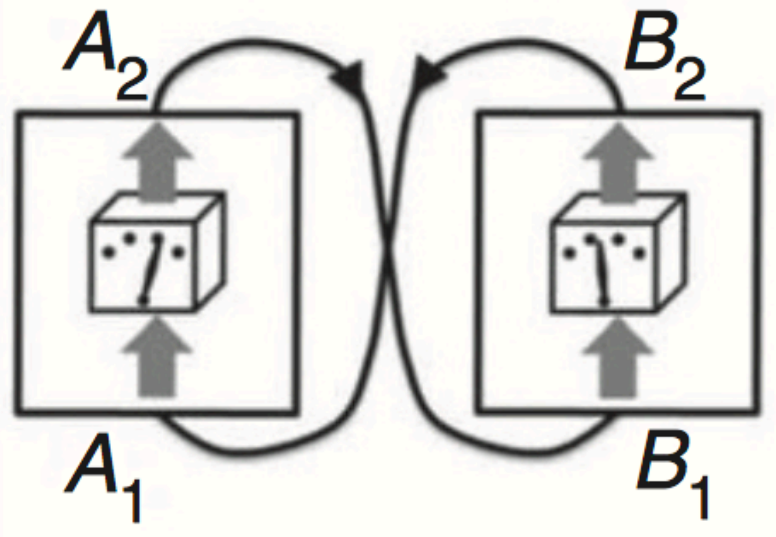
$$\sum_i T_i^{\text{GEN}} = W^{\text{GEN}}$$

PROCESS MATRIX

ALLOWED

$B \npreceq A$	$A_1, B_1, A_1 B_1$	$A_2\ B_1$	$A_1\ A_2\ B_1$
$A \npreceq B$		$A_1\ B_2$	$A_1\ B_1\ B_2$
Causal order	States	Channels	Channels with memory
			

NOT ALLOWED

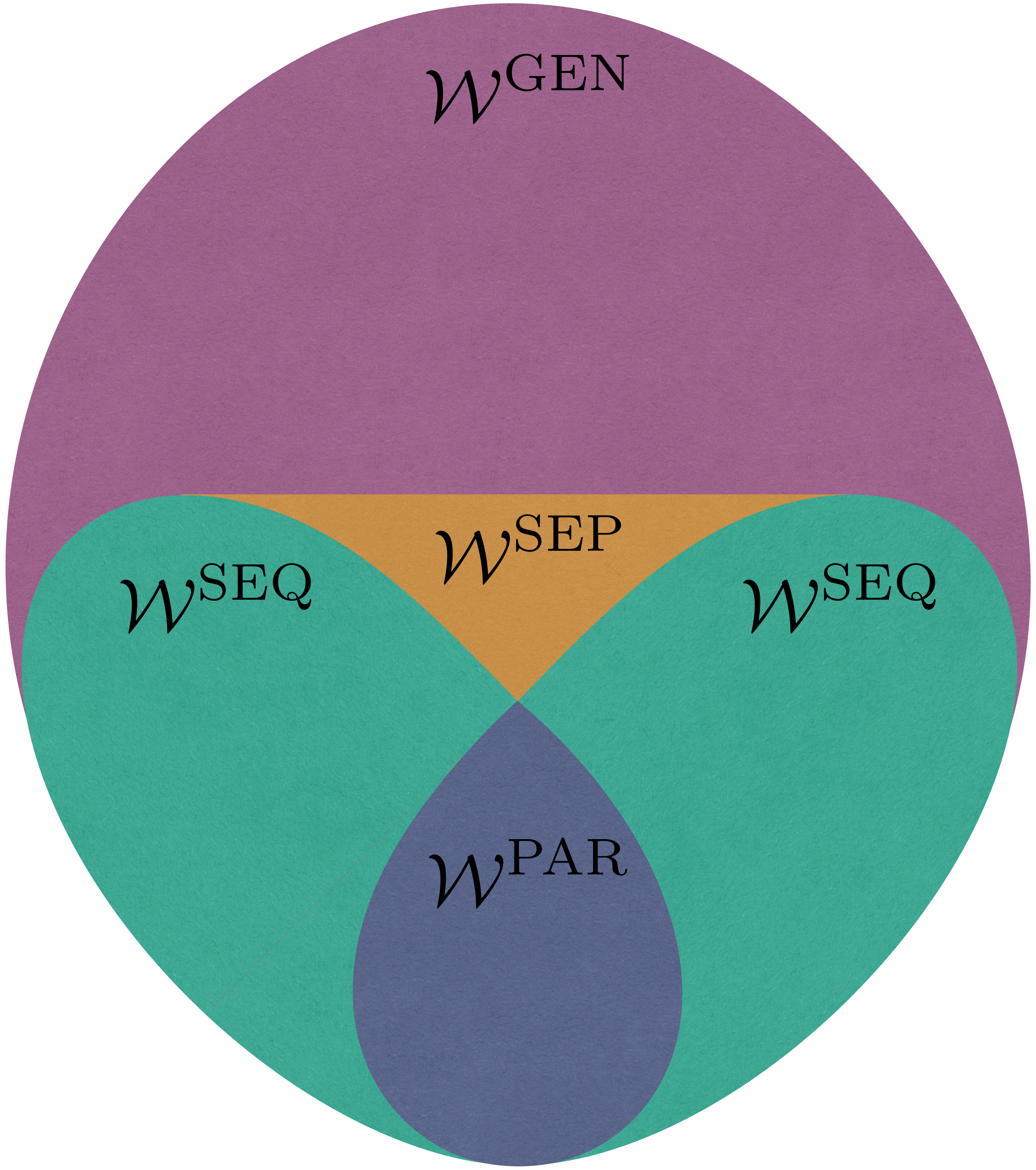
$A_2, B_2, A_2 B_2$	$A_1\ A_2, B_1 B_2$	$A_1\ A_2\ B_2, A_2 B_1 B_2$	$A_1 A_2 B_1 B_2$
Postselection	Local loops	Channels with local loops	Global loops
			

O. Oreshkov, F. Costa, and Č. Brukner, Nat. Commun. 3, 1092 (2012), arXiv:1105.4464 [quant-ph].

$$W \geq 0$$

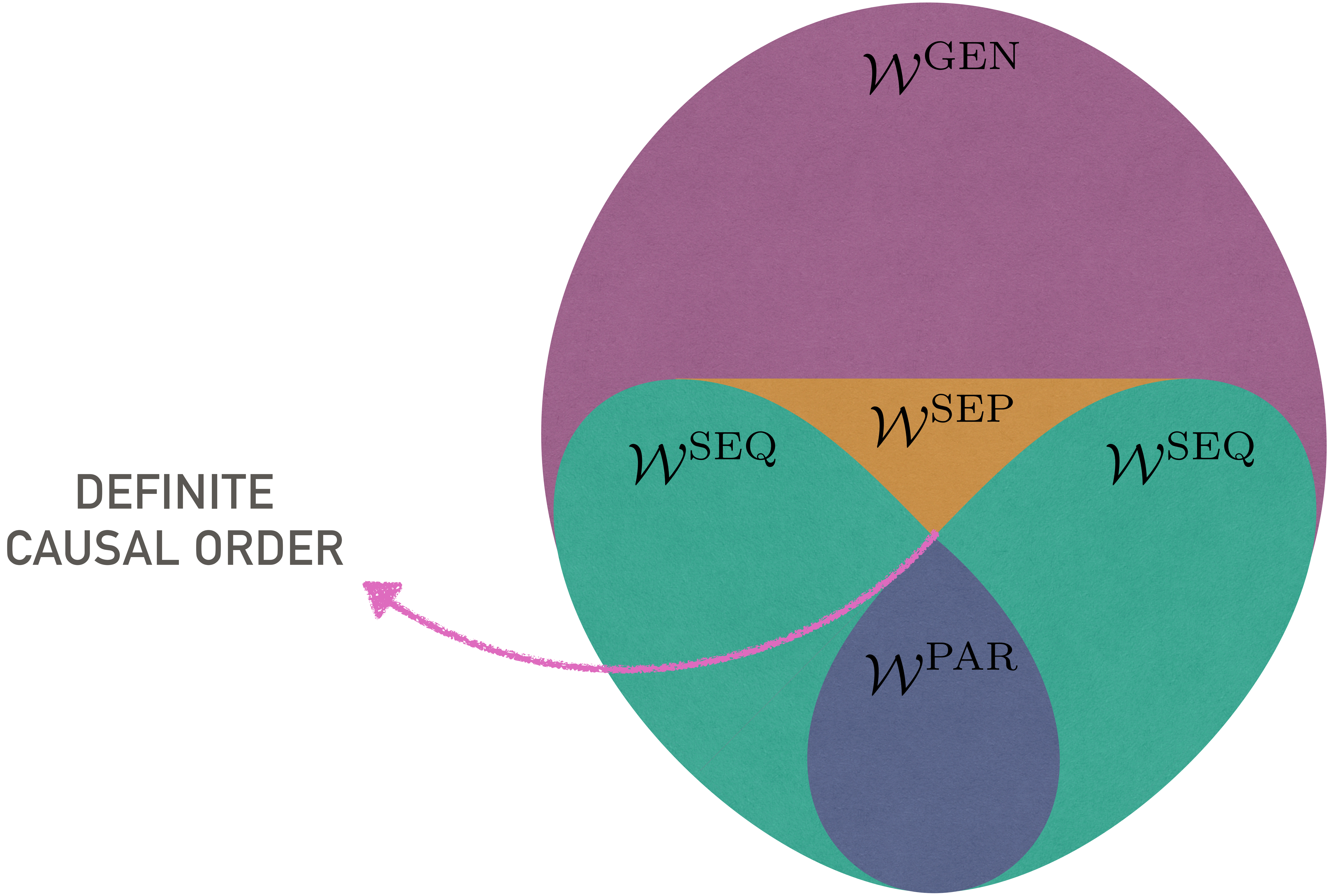
$$\mathrm{Tr}[W(C_1 \otimes C_2)] = 1$$

$$\forall \;\; C_1, C_2$$



Set of all processes $\mathcal{W}^{\mathcal{S}}$

O. Oreshkov, F. Costa, and Č. Brukner, Nat. Commun. 3, 1092 (2012), arXiv:1105.4464 [quant-ph].



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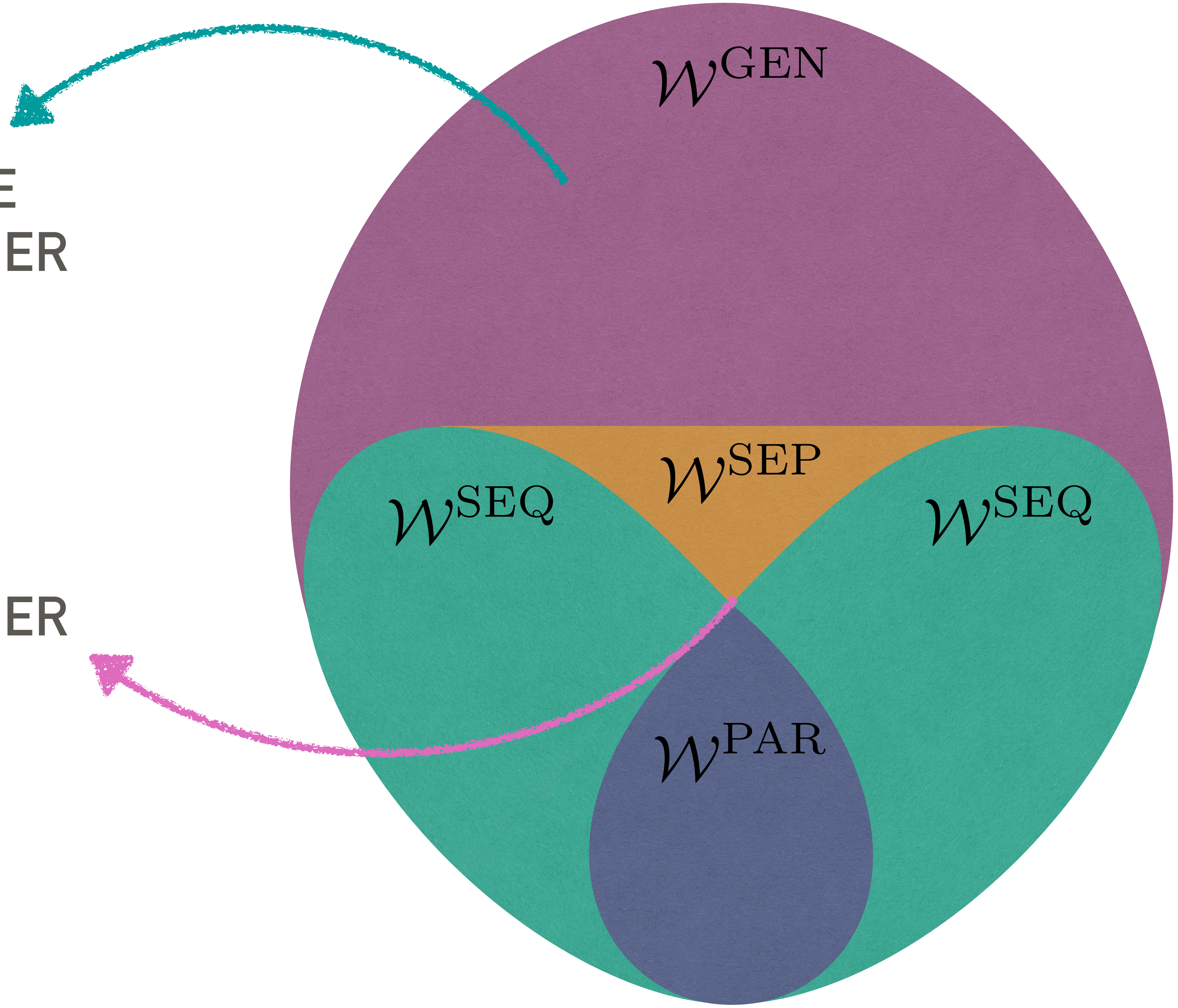
INDEFINITE CAUSAL ORDER

DEFINITE CAUSAL ORDER



$$W \geq 0$$
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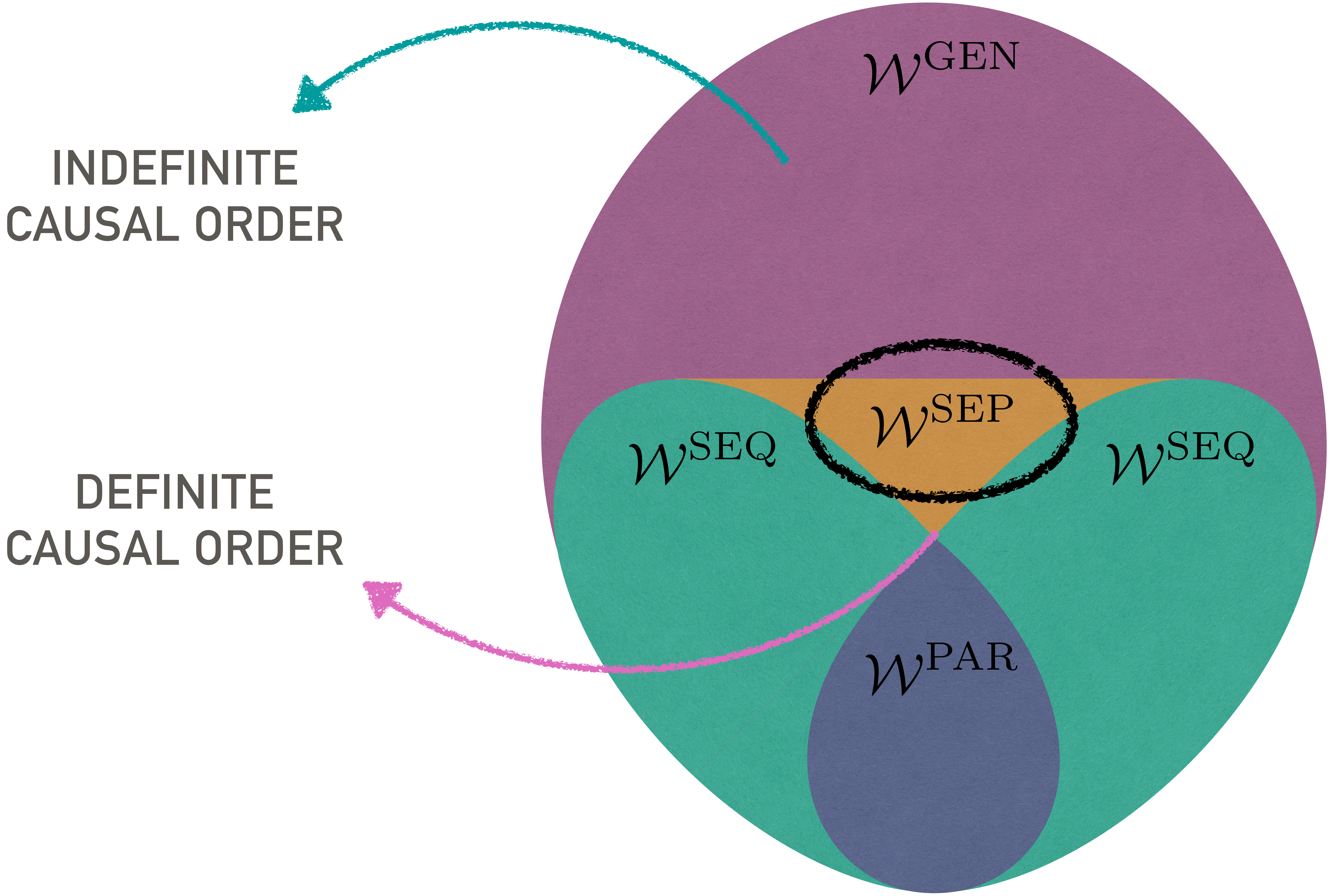

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O. Oreshkov, F. Costa, and Č. Brukner, Nat. Commun. 3, 1092 (2012), arXiv:1105.4464 [quant-ph].

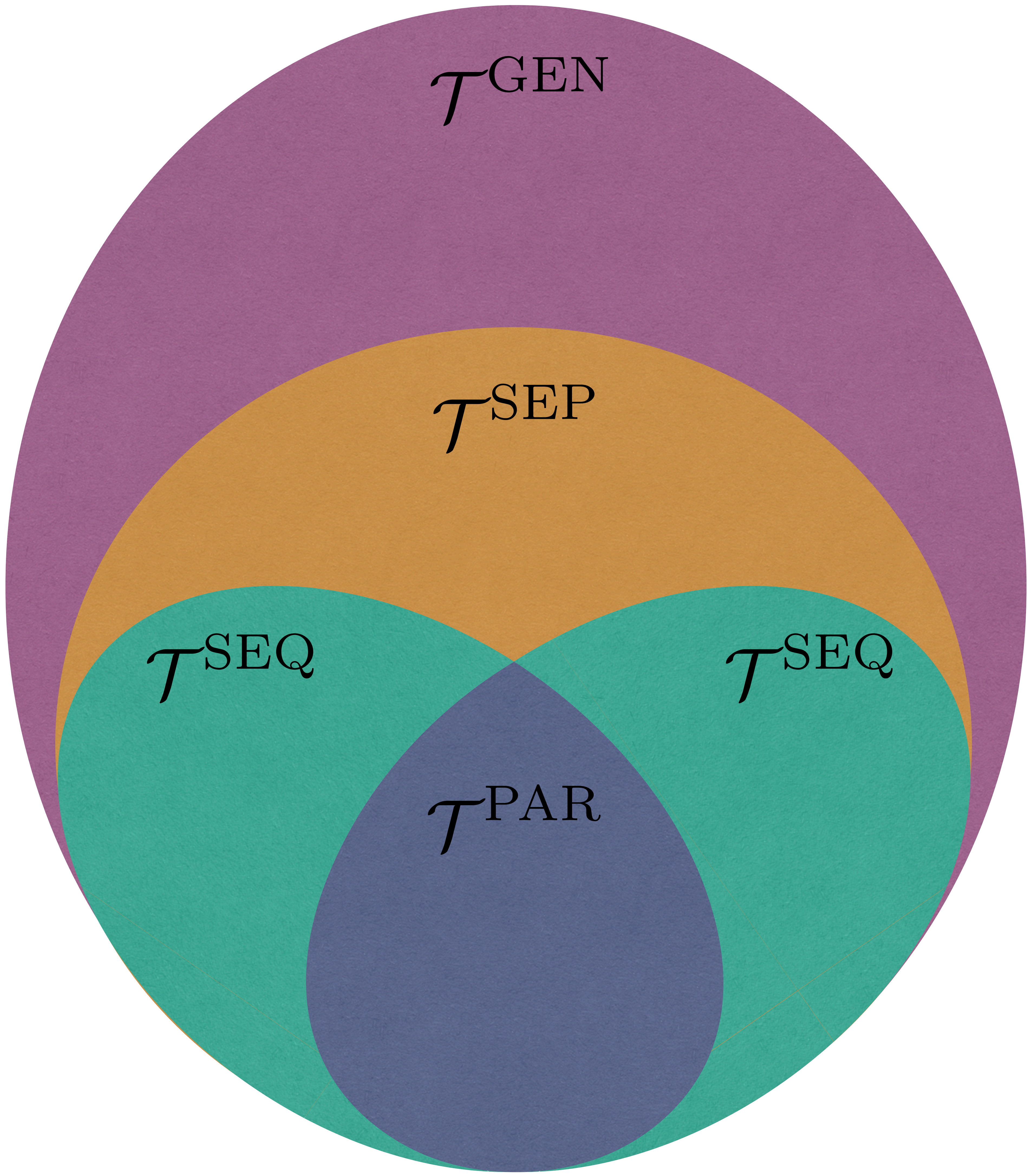
SEPARABLE TESTERS

SEPARABLE TESTERS

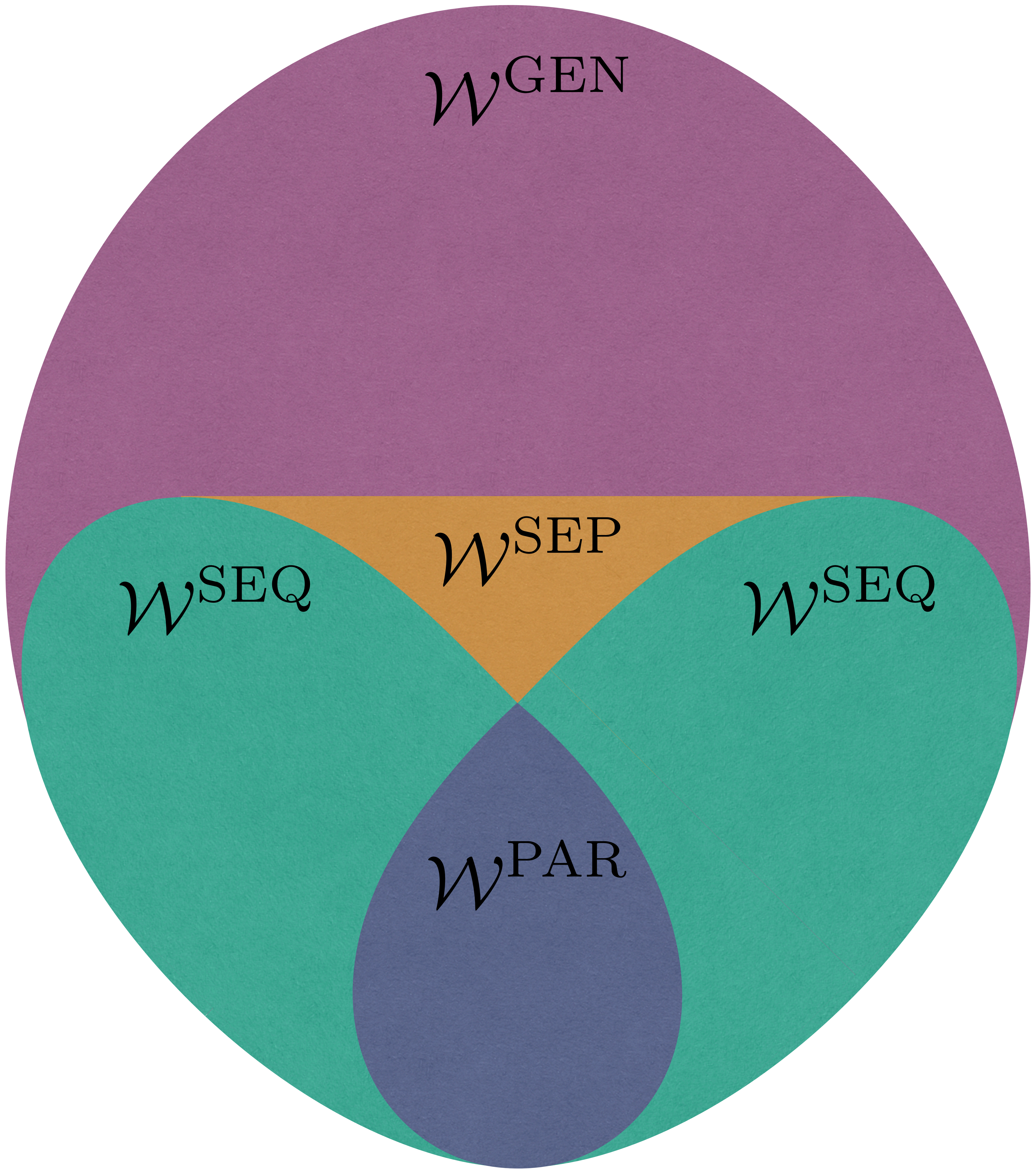
$$T^{\mathrm{SEP}} = \{T_i^{\mathrm{SEP}}\} \;\; :$$

$$T_i^{\mathrm{SEP}} \geq 0 \quad \forall\, i$$

$$\sum_i T_i^{\mathrm{SEP}} = q\, W_{1\prec 2}^{\mathrm{SEQ}} + (1-q) W_{2\prec 1}^{\mathrm{SEQ}} =: W^{\mathrm{SEP}}$$



Set of all testers $\mathcal{T}^{\mathcal{S}}$



Set of all processes $\mathcal{W}^{\mathcal{S}}$

SEMIDEFINITE PROGRAMMING (SDP)

$$P^{\mathcal{S}} = \max_{\{T_i^{\mathcal{S}}\}} \sum_i p_i \operatorname{Tr}\left(C_i^{\otimes 2}\, T_i^{\mathcal{S}}\right)$$

SEMIDEFINITE PROGRAMMING (SDP)

$$P^{\mathcal{S}} = \max_{\{T_i^{\mathcal{S}}\}} \sum_i p_i \, \mathrm{Tr}\left(C_i^{\otimes 2} \, T_i^{\mathcal{S}}\right)$$

PRIMAL

given $\{p_i, C_i\}$

maximize $\sum_i p_i \mathrm{Tr}(T_i^{\mathcal{S}} \, C_i^{\otimes 2})$

subject to $\{T_i^{\mathcal{S}}\}$ is a tester with strategy $\mathcal{S}$.

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$$P^{\mathcal{S}} = \max_{\{T_i^{\mathcal{S}}\}} \sum_i p_i \,\mathrm{Tr}\left(C_i^{\otimes 2}\, T_i^{\mathcal{S}}\right)$$

PRIMAL

given $\{p_i, C_i\}$

maximize $\sum_i p_i \mathrm{Tr}(T_i^{\mathcal{S}}\, C_i^{\otimes 2})$

subject to $\{T_i^{\mathcal{S}}\}$ is a tester with strategy $\mathcal{S}$.

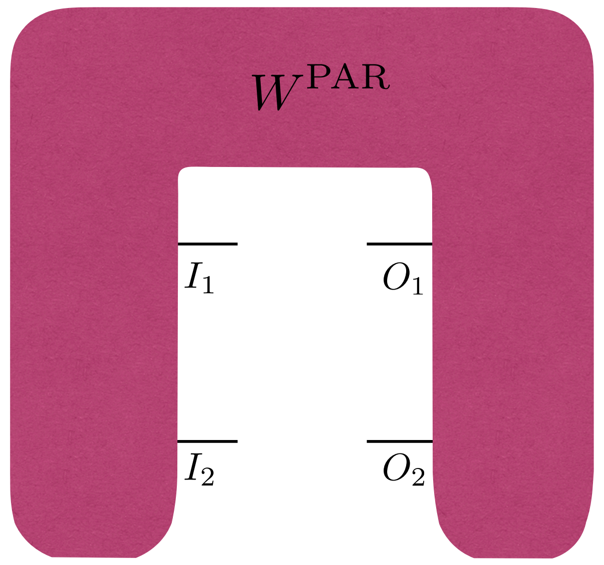
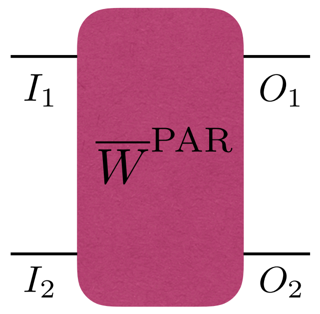
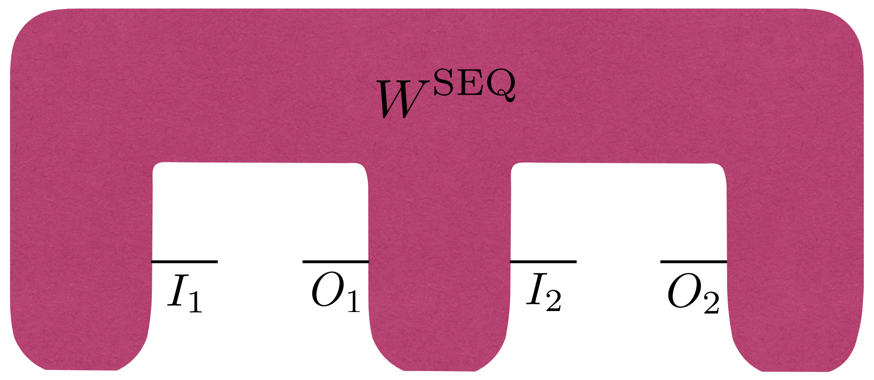
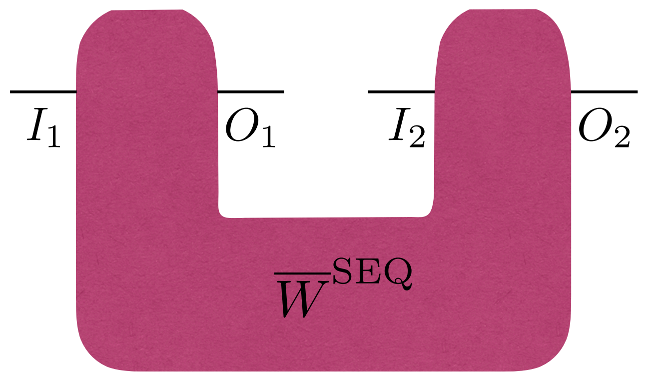
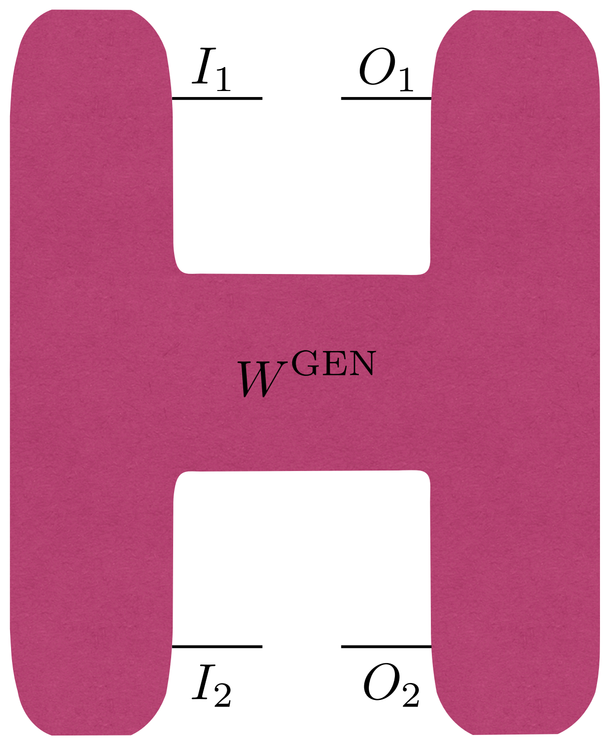
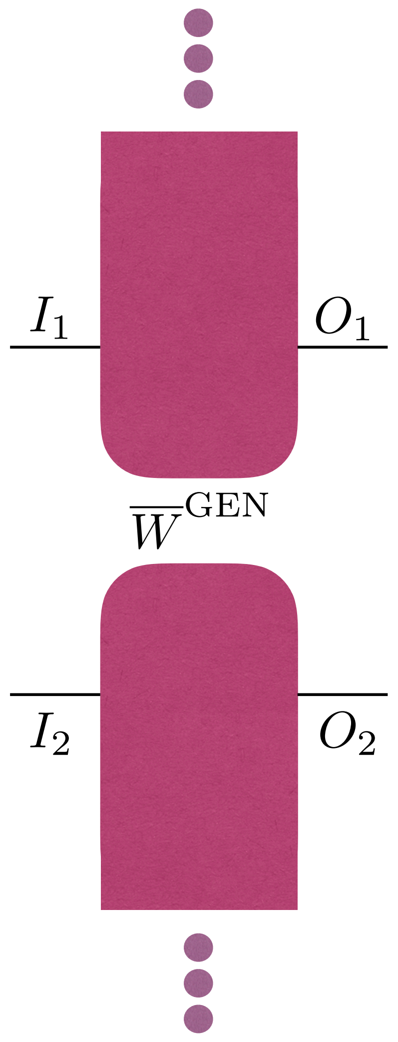
DUAL

given $\{p_i, C_i\}$

minimize λ

subject to $p_i\, C_i^{\otimes 2} \leq \lambda\, \overline{W}^{\mathcal{S}} \quad \forall\, i$

$$\mathrm{Tr}(W\,\overline{W}) = 1 \quad \forall\ \ W \in \mathcal{W},\ \ \overline{W} \in \overline{\mathcal{W}}$$

	PROCESS		DUAL AFFINE (CHANNEL)	
PARALLEL		$\mathrm{Tr}(W^{\mathrm{PAR}}) = d_{O_1}d_{O_2}$ $W^{\mathrm{PAR}} = {}_{O_1O_2}W^{\mathrm{PAR}}$		$\mathrm{Tr}(\overline{W}^{\mathrm{PAR}}) = d_{I_1}d_{I_2}$ ${}_{O_1O_2}\overline{W}^{\mathrm{PAR}} = {}_{I_1O_1I_2O_2}\overline{W}^{\mathrm{PAR}}$
SEQUENTIAL		$\mathrm{Tr}(W^{\mathrm{SEQ}}) = d_{O_1}d_{O_2}$ $W^{\mathrm{SEQ}} = {}_{O_2}W^{\mathrm{SEQ}}$ ${}_{I_2O_2}W^{\mathrm{SEQ}} = {}_{O_1I_2O_2}W^{\mathrm{SEQ}}$		$\mathrm{Tr}(\overline{W}^{\mathrm{SEQ}}) = d_{I_1}d_{I_2}$ ${}_{O_2}\overline{W}^{\mathrm{SEQ}} = {}_{I_2O_2}\overline{W}^{\mathrm{SEQ}}$ ${}_{O_1I_2O_2}\overline{W}^{\mathrm{SEQ}} = {}_{I_1O_1I_2O_2}\overline{W}^{\mathrm{SEQ}}$
GENERAL		$\mathrm{Tr}(W^{\mathrm{GEN}}) = d_{O_1}d_{O_2}$ ${}_{I_1O_1}W^{\mathrm{GEN}} = {}_{I_1O_1O_2}W^{\mathrm{GEN}}$ ${}_{I_2O_2}W^{\mathrm{GEN}} = {}_{O_1I_2O_2}W^{\mathrm{GEN}}$ $W^{\mathrm{GEN}} = {}_{O_1}W^{\mathrm{GEN}} + {}_{O_2}W^{\mathrm{GEN}} - {}_{O_1O_2}W^{\mathrm{GEN}}$		$\mathrm{Tr}(\overline{W}^{\mathrm{GEN}}) = d_{I_1}d_{I_2}$ ${}_{O_1}\overline{W}^{\mathrm{GEN}} = {}_{I_1O_1}\overline{W}^{\mathrm{GEN}}$ ${}_{O_2}\overline{W}^{\mathrm{GEN}} = {}_{I_2O_2}\overline{W}^{\mathrm{GEN}}$

$$P^{\mathcal{S}} = \max_{\{T_i^{\mathcal{S}}\}} \sum_i p_i \operatorname{Tr}\left(C_i^{\otimes 2}\, T_i^{\mathcal{S}}\right)$$

$$P^{\mathrm{PAR}} \leq P^{\mathrm{SEQ}} \leq P^{\mathrm{SEP}} \leq P^{\mathrm{GEN}}$$

$$P^{\mathrm{PAR}} < P^{\mathrm{SEQ}} < P^{\mathrm{SEP}} < P^{\mathrm{GEN}}$$

STRICT HIERARCHY BETWEEN DISCRIMINATION STRATEGIES

MAIN EXAMPLE

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$$k = 2 \text{ copies}, \; N = 2 \text{ candidates},$$
$$p_1 = p_2 = \frac{1}{2}$$

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$$k = 2 \text{ copies},\ N = 2 \text{ candidates},$$

$$p_1 = p_2 = \frac{1}{2}$$

$$\widetilde{C}_1 = \widetilde{C}_{\mathrm{AD}},\ \widetilde{C}_2 = \widetilde{C}_{\mathrm{BF}}$$

AMPLITUDE DAMPING

$$\widetilde{C}_{\mathrm{AD}}(\rho) = K_0 \rho K_0^\dagger + K_1 \rho K_1^\dagger$$

$$K_0 = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$$

$$K_1 = \sqrt{\gamma}|0\rangle\langle 1|$$

BIT FLIP

$$\widetilde{C}_{\mathrm{BF}}(\rho) = \eta\,\rho + (1-\eta)X\rho X$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$P^{\mathrm{PAR}} = 0.8346 < P^{\mathrm{SEQ}} = 0.8447 < P^{\mathrm{SEP}} = 0.8487 < P^{\mathrm{GEN}} = 0.8514$$

COMPUTER-ASSISTED PROOF

COMPUTER-ASSISTED PROOF

Example: how to create a “valid” channel

- Take numerically imprecise matrix C from the solution of an SDP
- Truncate C and define $C \mapsto \dfrac{C + C^\dagger}{2}$
- Project C onto the subspace of valid channels, $C \mapsto L(C)$
- Find coefficient η such that $C \mapsto \eta C + (1-\eta)\mathbb{I} \geq 0$
- Output $C \mapsto d_I \dfrac{C}{\mathrm{Tr}(C)}$

$$\frac{8346}{10000} < P^{\mathrm{PAR}} < \frac{8347}{10000}$$

$$\frac{8446}{10000} < P^{\mathrm{SEQ}} < \frac{8447}{10000}$$

$$\frac{8486}{10000} < P^{\mathrm{SEP}} < \frac{8487}{10000}$$

$$\frac{8514}{10000} < P^{\mathrm{GEN}} < \frac{8515}{10000}$$

QUANTUM REALISATION OF TESTERS

PARALLEL

SEQUENTIAL

SEPARABLE

GENERAL

QUANTUM REALISATION OF TESTERS

PARALLEL

states, measurements

SEQUENTIAL

SEPARABLE

GENERAL

QUANTUM REALISATION OF TESTERS

PARALLEL	states, measurements
SEQUENTIAL	states, channels, measurements
SEPARABLE	
GENERAL	

QUANTUM REALISATION OF TESTERS

PARALLEL	states, measurements
SEQUENTIAL	states, channels, measurements
SEPARABLE	coherent quantum control of causal orders[1]
GENERAL	

[1] J. Wechs, H. Dourdent, A. A. Abbott, C. Branciard, arXiv: 2101.08796 [quant-ph] (2021)

QUANTUM REALISATION OF TESTERS

PARALLEL	states, measurements
SEQUENTIAL	states, channels, measurements
SEPARABLE	coherent quantum control of causal orders[1]
GENERAL	process matrices, measurements

[1] J. Wechs, H. Dourdent, A. A. Abbott, C. Branciard, arXiv: 2101.08796 [quant-ph] (2021)

CONCLUSIONS

CONCLUSIONS

- Unified tester formalism that includes indefinite-causal-order strategies.
- Strict hierarchy between discrimination strategies in the simplest scenario (2 copies, 2 candidates, qubit channels).
- Method of computer-assisted proofs readily applicable to quantum information problems.

THANK YOU!

EXTRA

PARALLEL PROCESSES

$$W^{\mathrm{PAR}} \in L(H^{I_1} \otimes H^{O_1} \otimes H^{I_2} \otimes H^{O_2})$$

$$W^{\mathrm{PAR}} \geq 0$$

$$\mathrm{Tr}(W^{\mathrm{PAR}}) = d_{O_1} d_{O_2}$$

$$W^{\mathrm{PAR}} =_{O_1 O_2} W^{\mathrm{PAR}}$$

SEQUENTIAL PROCESSES

$$W_{1\prec 2}^{\mathrm{SEQ}} \in L(H^{I_1} \otimes H^{O_1} \otimes H^{I_2} \otimes H^{O_2})$$

$$W_{1\prec 2}^{\mathrm{SEQ}} \geq 0$$

$$\mathrm{Tr}(W_{1\prec 2}^{\mathrm{SEQ}}) = d_{O_1} d_{O_2}$$

$$W_{1\prec 2}^{\mathrm{SEQ}} =_{O_2} W_{1\prec 2}^{\mathrm{SEQ}}$$

$${}_{I_2 O_2}W_{1\prec 2}^{\mathrm{SEQ}} =_{O_1 I_2 O_2} W_{1\prec 2}^{\mathrm{SEQ}}$$

GENERAL PROCESSES

$$W^{\text{GEN}} \in L(H^{I_1} \otimes H^{O_1} \otimes H^{I_2} \otimes H^{O_2})$$

$$W^{\text{GEN}} \geq 0$$

$$\text{Tr}(W^{\text{GEN}}) = d_{O_1} d_{O_2}$$

$${}_{I_1 O_1} W^{\text{GEN}} =_{I_1 O_1 O_2} W^{\text{GEN}}$$

$${}_{I_2 O_2} W^{\text{GEN}} =_{O_1 I_2 O_2} W^{\text{GEN}}$$

$$W^{\text{GEN}} +_{O_1 O_2} W^{\text{GEN}} =_{O_1} W^{\text{GEN}} +_{O_2} W^{\text{GEN}}$$