Most incompatible measurements for robust steering tests

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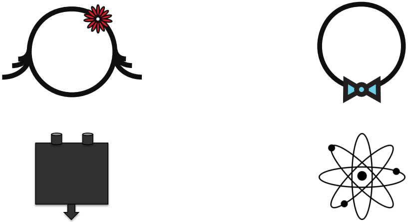
Joint work with: **M. T. Quintino, L. Guerini, T. O. Maciel, D. Cavalcanti, and M. T. Cunha.**

Quantum Steering

Approach

SEMI-DEVICE INDEPENDENT



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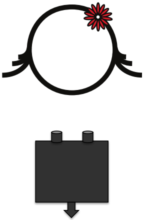
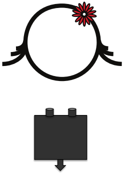


$p(a|x)$

$\rho_{a|x}$

Object of interest



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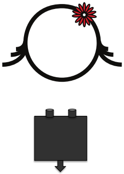


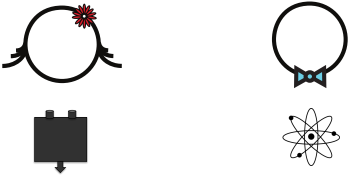
$\{p(a|x)\}$

$\{\rho_{a|x}\}$

$$\{\sigma_{a|x}\} : \quad \sigma_{a|x} = p(a|x)\rho_{a|x}$$

$\rightarrow$ assemblage

Object of interest



$\{p(a|x)\}$ $\{\rho_{a|x}\}$

$$\{\sigma_{a|x}\}: \quad \sigma_{a|x} = p(a|x)\rho_{a|x} \qquad \rightarrow \text{assemblage}$$

$$= \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{1}\, \rho_{AB})$$

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Unsteerable assemblage $\{\sigma_{a|x}^{\text{uns}}\}$:

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$$\sum_{a,x} \text{Tr}(F_{a|x}\sigma_{a|x}) \leq \beta^{\text{uns}} \rightarrow \text{Steering Inequality}$$

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Decidable by semidefinite programming (SDP)

Steerability of quantum states

But what about the steerability of quantum states?

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$$\downarrow$$

Depends on the measurements

Steerability of quantum states

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$$\sigma_{a|x} = \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{1}\, \rho_{AB})$$

$$\downarrow$$

Depends on the measurements

Not decidable by semidefinite programming (SDP)

White noise robustness

Depolarizing channel:

$$A \mapsto \Lambda^{\eta}(A) = \eta A + (1-\eta) tr(A) \frac{\mathbb{1}}{d}$$

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White noise robustness for assemblages:

$$\eta(\sigma_{a|x}) = \max \left\{ \eta \mid \{\Lambda^{\eta}(\sigma_{a|x})\}_{a,x} \in UNS \right\} \rightarrow \text{SDP!}$$

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White noise robustness for quantum states:

$$\eta^*(\rho_{AB}, N, k) = \min_{\{M_{a|x}\}} \left\{ \eta(\sigma_{a|x}) \mid \sigma_{a|x} = \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{1} \rho_{AB}) \right\}$$

White noise robustness

Maximally entangled state $\left|\Phi^+\right\rangle = \frac{1}{\sqrt{d}} \sum_i |ii\rangle$:

$$\sigma_{a|x} = \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{1} \left|\Phi^+\right\rangle\!\left\langle\Phi^+\right|) = \frac{1}{d} M_{a|x}^T$$

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$$\Lambda^\eta(\{\sigma_{a|x}\}) \equiv \Lambda^\eta(\{M_{a|x}\})$$

White noise robustness

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$$\Lambda^\eta(\{\sigma_{a|x}\}) \equiv \Lambda^\eta(\{M_{a|x}\})$$

$\eta^*(|\Phi^+\rangle\langle\Phi^+|, N, k) = \eta^*(N, k) \rightarrow$ for joint measurability!

Contents

General states

Infinite number of measurements:

General states

Infinite number of measurements:

(i) All qubit projective measurements: $\eta \leq \frac{1}{2}$

R. Werner, *Phys. Rev. A* **40**, 4277–4281 (1989)

(ii) All qubit general POVMs: $\eta \leq \frac{5}{12}$

J. Barrett, *Phys. Rev. A* **65**, 042302 (2002)

General states

Infinite number of measurements:

(i) D. Cavalcanti, L. Guerini, R. Rabelo, and P. Skrzypczyk
"General Method for Constructing Local Hidden Variable Models for Entangled Quantum States"
Phys. Rev. Lett. **117**, 190401 (2016)

(ii) F. Hirsch, M. T. Quintino, T. Vértesi, M. F. Pusey, and N. Brunner
"Algorithmic Construction of Local Hidden Variable Models for Entangled Quantum States"
Phys. Rev. Lett. **117**, 190402 (2016)

General states

Different scenario: compatibility of a set of measurements

General states

Different scenario: compatibility of a set of measurements

(i) Finite number of measurements N

General states

Different scenario: compatibility of a set of measurements

(i) Finite number of measurements N

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General states

Different scenario: compatibility of a set of measurements

(i) Finite number of measurements N

(ii) Finite number of outcomes k

(iii) POVMs of a specific structure

Projective measurements vs. general POVMs

Are general POVMs more relevant for steering than projective measurements?

Projective measurements vs. general POVMs

Are general POVMs more relevant for steering than projective measurements?

(Can a set of N non-projective POVMs be "more incompatible" than a set of N projective measurements of the same dimension?)

Methods

Methods

$$\eta^*(\rho_{AB}, N, k)$$

Search algorithm
See-saw algorithm
(upper bounds)

Outer polytope approximation of convex sets
(lower bounds)

Methods

See-saw algorithm
(upper bounds)

(i) T. Moroder, O. Gittsovich, M. Huber, and O. Gühne
"Steering Bound Entangled States: A Counterexample to the Stronger Peres Conjecture"
Phys. Rev. Lett. **113**, 050404 (2014)

(ii) D. Cavalcanti and P. Skrzypczyk
"Quantum steering: a review with focus on semidefinite programming"
Rep. Prog. Phys. **80**, 024001 (2017)

Outer polytope approximation of convex sets
(lower bounds)

(i) M. Oszmaniec, L. Guerini, P. Wittek, and A. Acín
"Simulating positive-operator-valued measures with projective measurements"
Phys. Rev. Lett. **119**, 190501 (2017)

Search algorithm

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Measurement set parametrization:

$$\{M\} : M(x)$$

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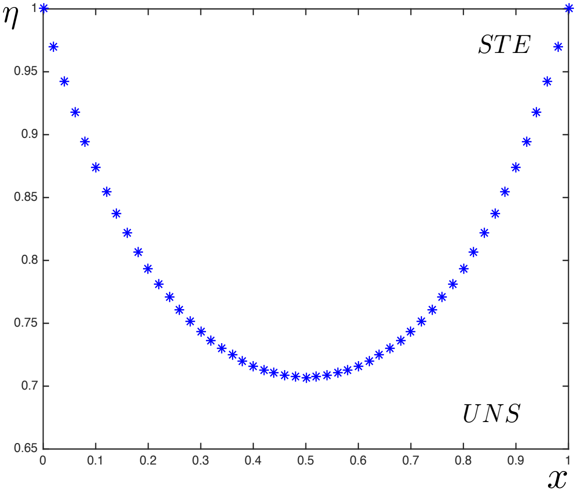
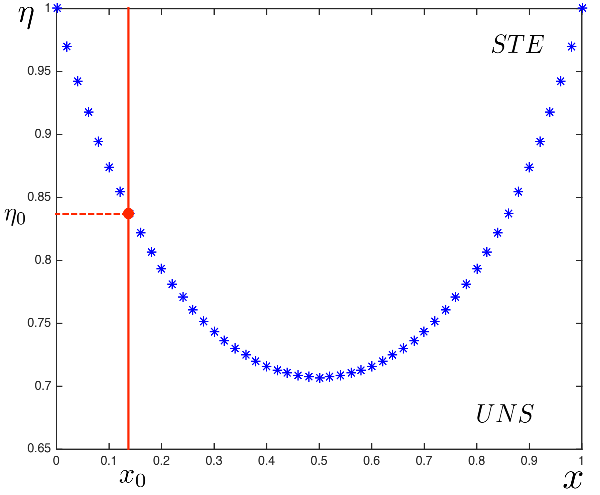
$$\{M\} : M(x)$$

For a fixed state ρ_{AB}, optimize over x.

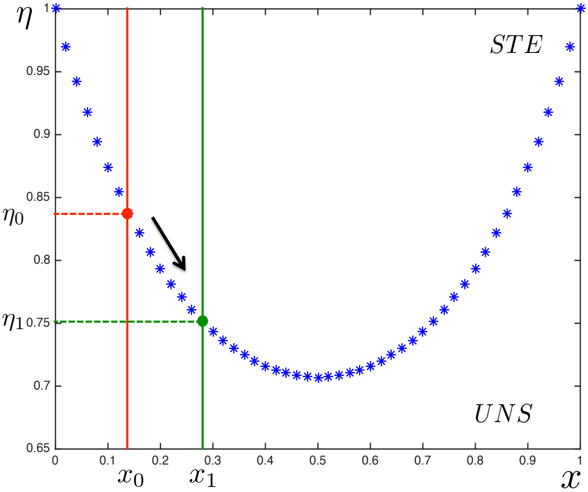
Search algorithm



Search algorithm



Search algorithm



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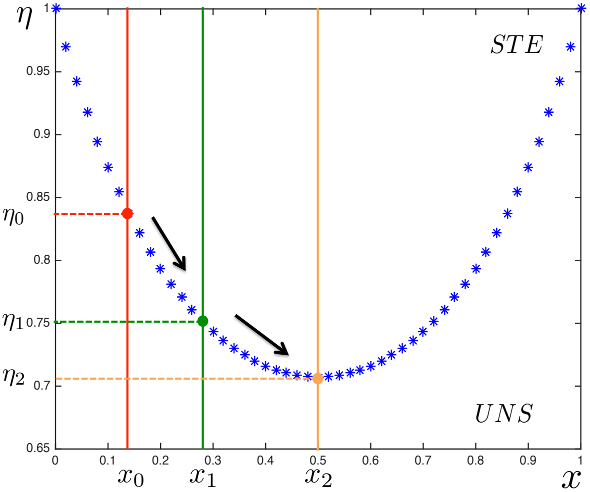

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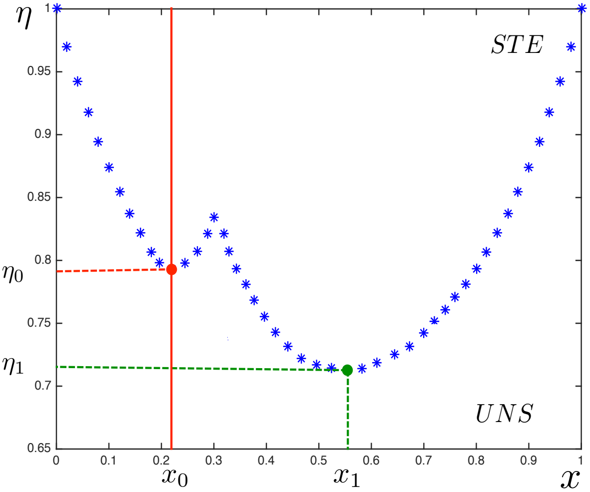

See-saw algorithm

See-saw algorithm

$x_1 = \texttt{rand}(n)$
while `<convergence condition>` **do**
    $x_2 = \texttt{SDP\_1}(x_1)$
    $x_1 = \texttt{SDP\_2}(x_2)$
end while

See-saw algorithm

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x_1 = rand(n)
while <convergence condition> do
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`SDP_1`: Fixed measurements → Optimize inequality

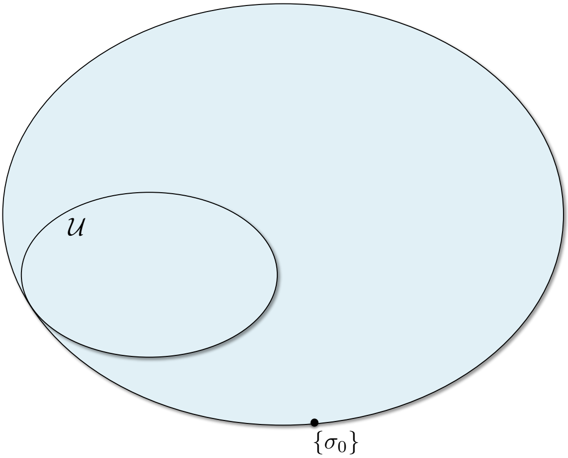
See-saw algorithm

```
1: x_1 = rand(n)
2: while <convergence condition> do
3:     x_2 = SDP_1(x_1)
4:     x_1 = SDP_2(x_2)
5: end while
```

SDP_1: Fixed measurements $\rightarrow$ Optimize inequality

SDP_2: Fixed inequality $\rightarrow$ Optimize measurements

See-saw algorithm



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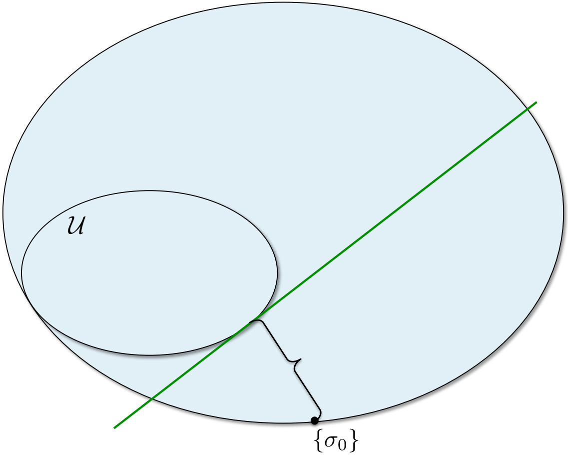

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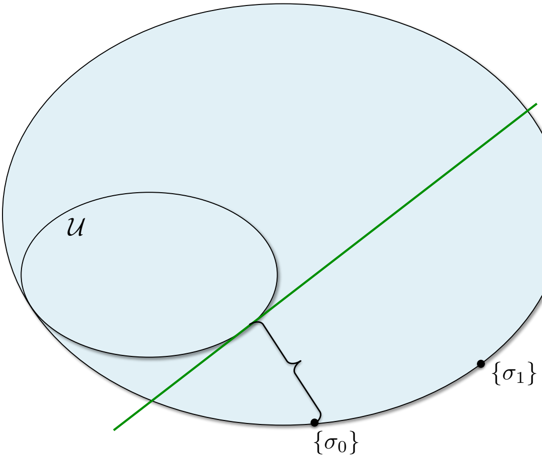

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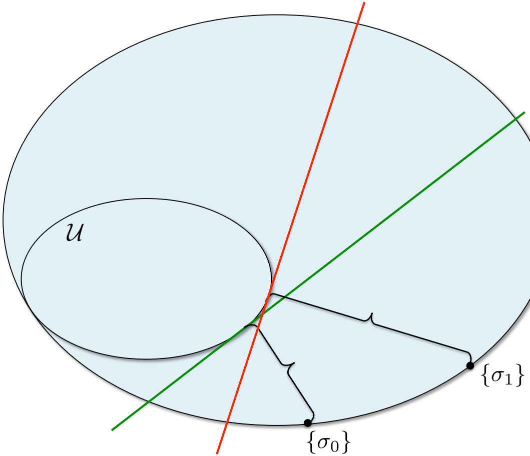

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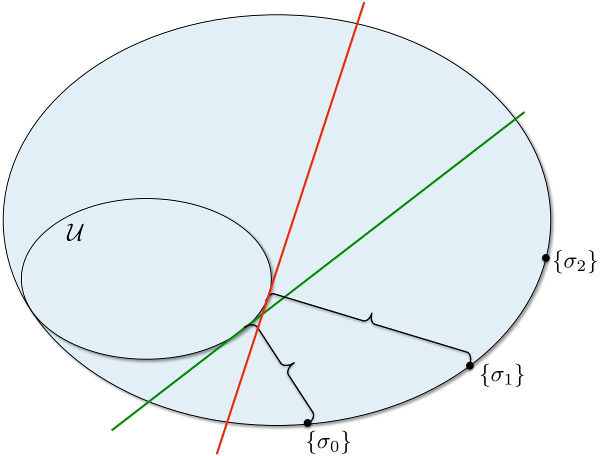

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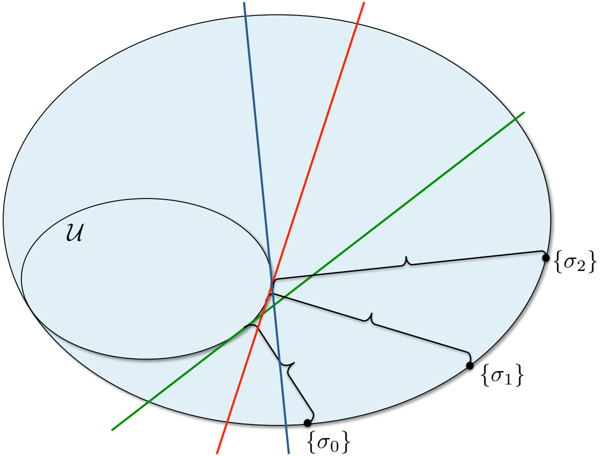
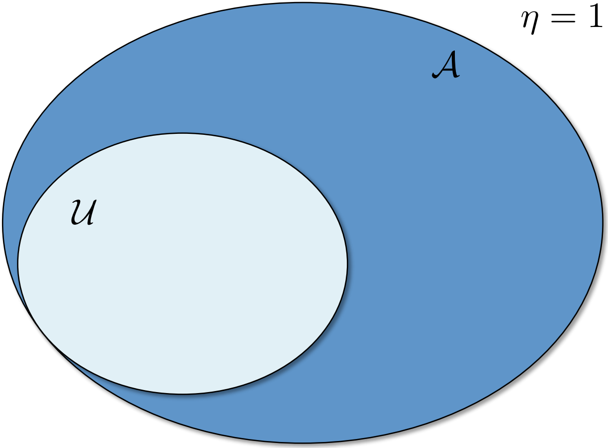


Outer polytope approximation

Polytope approximation



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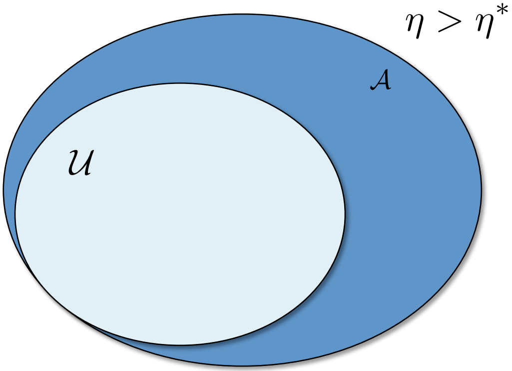
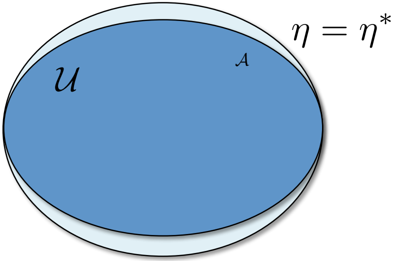


Polytope approximation



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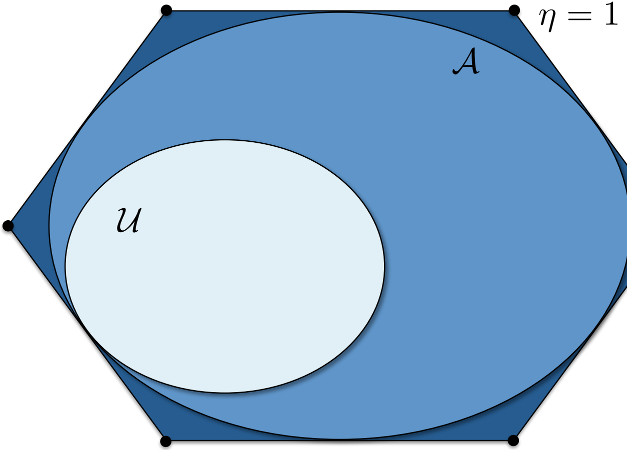
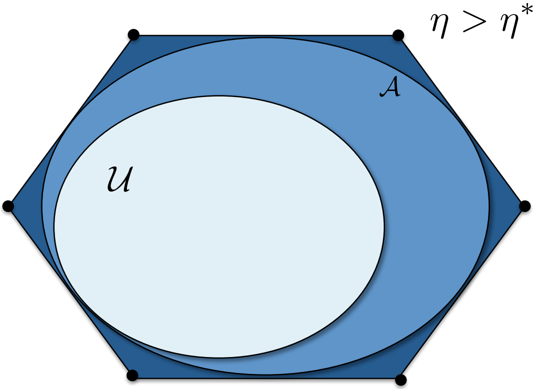


Polytope approximation



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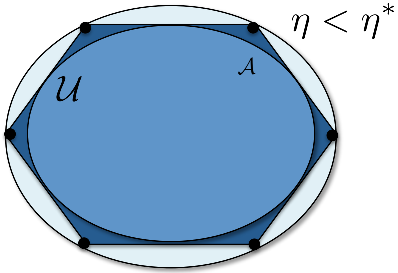

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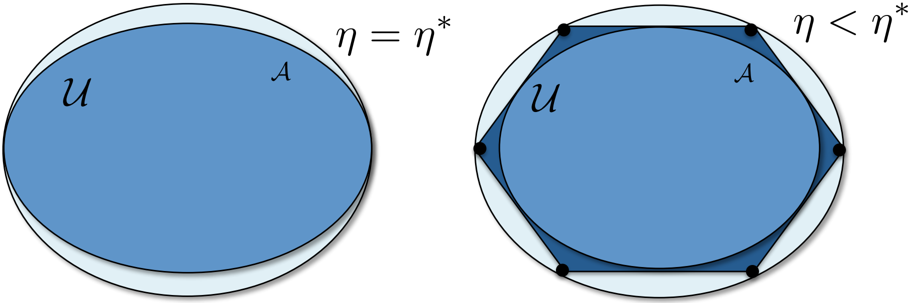
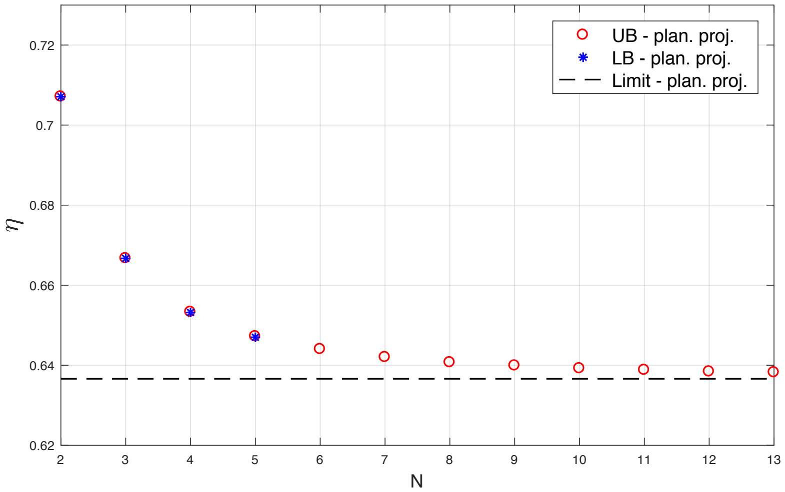


Results

Planar projective qubit measurements

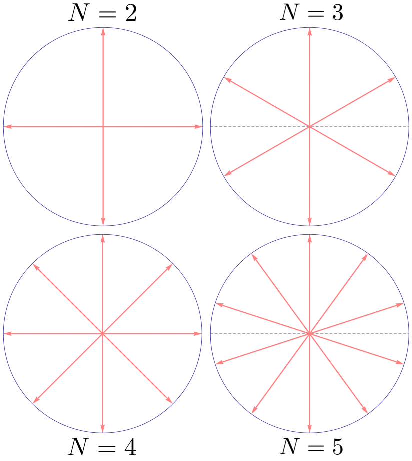
Simple case: planar projective qubit measurements.

Planar projective qubit measurements



Planar projective qubit measurements

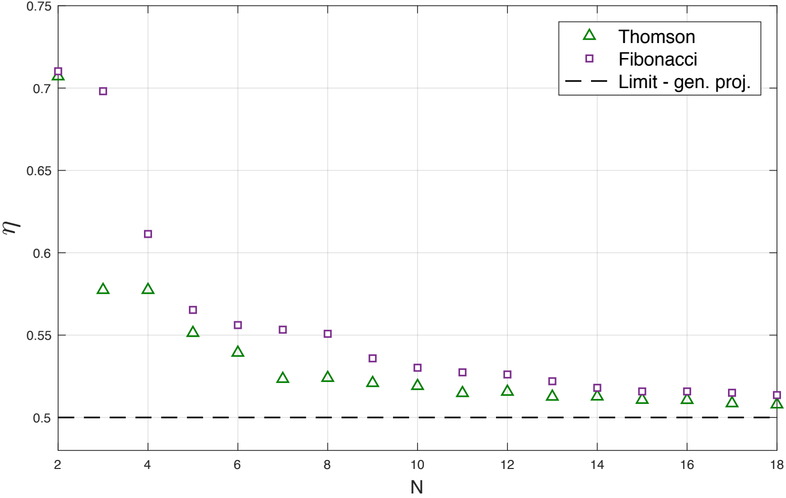
Optimal measurement set seems to be equally spaced.



General projective qubit measurements

The distribution of equally spaced points on a sphere is not trivial - specially for small N.

General projective qubit measurements



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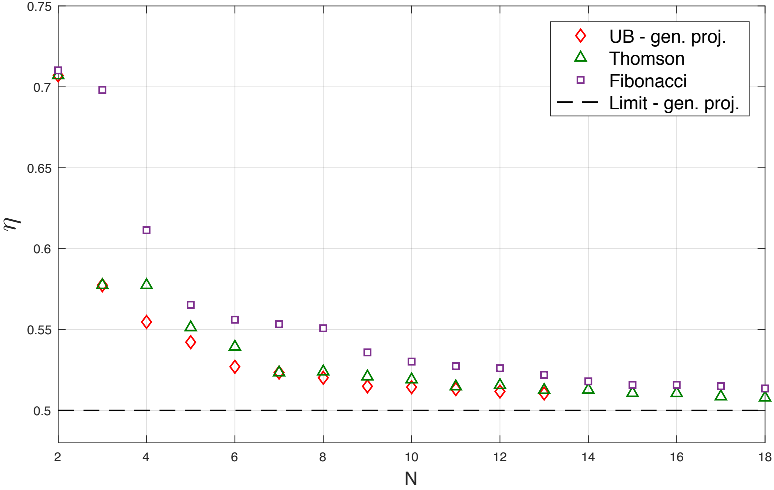
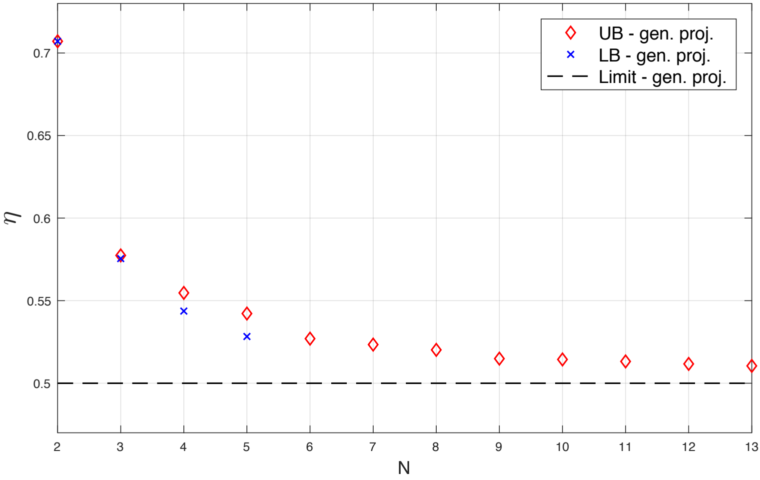
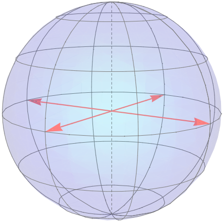


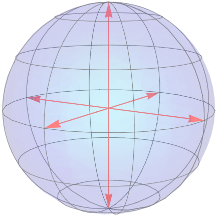
General projective qubit measurements



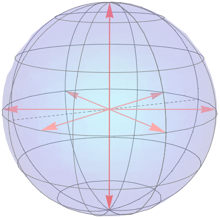
General projective qubit measurements



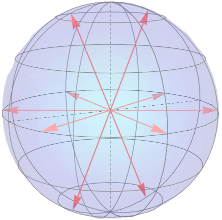
$N = 2$



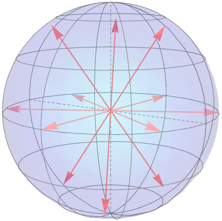
$N = 3$



$N = 4$



$N = 5$

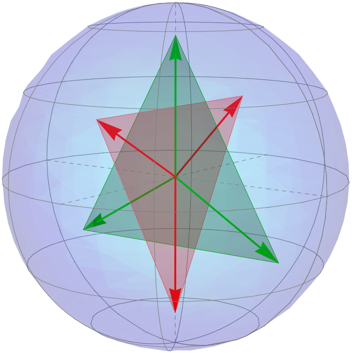


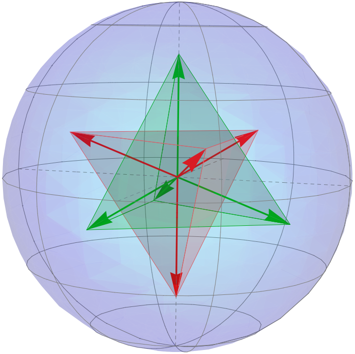
$N = 6$

General qubit POVMs

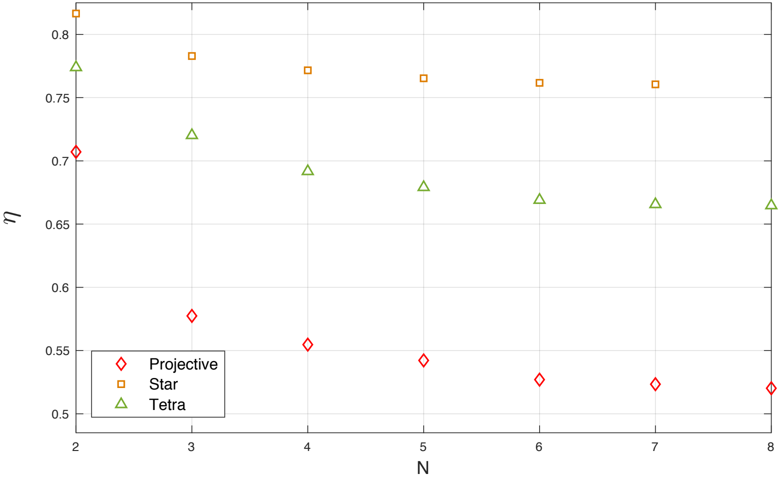
What about POVMs with more outcomes?

Symmetric qubit POVMs





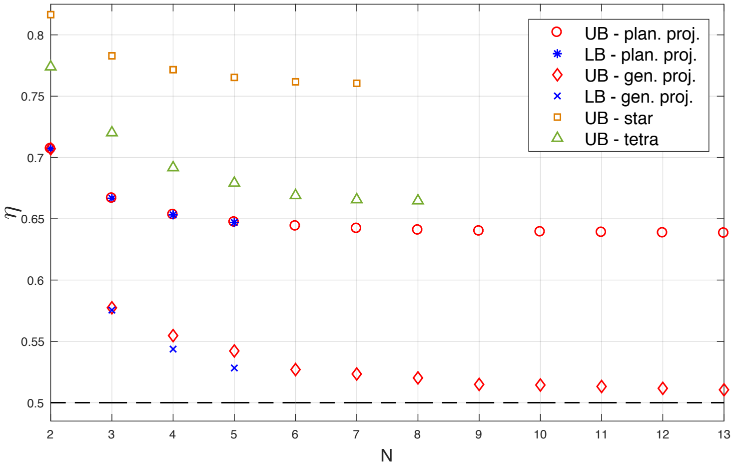
Symmetric qubit POVMs



General qubit POVMs

What about general POVMs?
(no restrictions on the structure)

General qubit POVMs



General qubit POVMs

Projective measurements seem to be optimal for steering the two-qubit Werner states.

Isotropic states

$N = 2$

m	$d = 2$	3	4	5	6
2	0.7071	0.7000	0.6901	0.6812	0.6736
3	0.7071	0.6794	0.6722	0.6621	0.6527
4		0.6794	0.6665	0.6544	0.6448
5			0.6665	0.6483	0.6429
6				0.6483	0.6390
7					0.6390

All outputed measurements are projective.

Isotropic states

In higher dimension, non-projective POVMs
also do not seem to be relevant for steering the isotropic states.

Mutually unbiased measurements

A set of mutually unbiased basis (MUBs) is a set of 2 or more orthonormal basis $\{|i_k\rangle\}_i$ in a d-dimensional Hilbert space that satisfy

$$|\langle i_k|j_l\rangle|^2 = \frac{1}{d}, \quad \forall\, i,j \in \{1,\ldots,d\},\, k \neq l, \tag{1}$$

for all basis k, l.

Mutually unbiased measurements

MUBs

N	$d=2$	3	4	5	6
2	0.7071	0.6830	0.6667	0.6545	0.6449
3	0.5774	0.5686	0.5469	0.5393	0.5204
4		0.4818	0.5000	0.4615	
5			0.4309	0.4179	
6				0.3863	

General d-outcome POVMs

N	$d=2$	3	4	5	6
2	0.7071	0.6794	0.6665	0.6483	0.6395
3	0.5774	0.5572	0.5412	0.5266	0.5139
4		0.4818	0.4797	0.4615	
5			0.4309	–	
6				–	

Isotropic states

Mutually unbiased measurements also
do not seem to be the most interesting measurements
for steering in $d > 2$.

Conclusions

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(i) General methods for certifying steering and joint measurability under restrictive scenarios.

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(ii) Candidates for the most incompatible sets of qubit measurements.

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(i) General methods for certifying steering and joint measurability under restrictive scenarios.

(ii) Candidates for the most incompatible sets of qubit measurements.

(iii) Evidence that projective measurements are optimal for steering.

JB, M. T. Quintino, L. Guerini, T. O. Maciel, D. Cavalcanti, and M. Terra Cunha
"Most incompatible measurements for robust steering tests"

Phys. Rev. A **96**, 022110 (2017)

arXiv:1704.02994 [quant-ph]

https://github.com/jessicabavaresco/most-incompatible-measurements

Thank you!