The Power of Catalytic Local Operations: Activation of Bell nonlocality and quantum state transformations

Jessica Bavaresco

Based on

Catalytic Activation of Bell Nonlocality

Jessica Bavaresco, Nicolas Brunner, Antoine Girardin, Pavel Sekatski, Patryk Lipka-Bartosik

Phys. Rev. Lett. 135, 220203 (2025), arXiv:2504.02042 [quant-ph]

and

The power of quantum catalytic local operations

Patryk Lipka-Bartosik, Jessica Bavaresco, Nicolas Brunner, Pavel Sekatski

arXiv:2510.08320 [quant-ph]

UNIVERSITÉ DE GENÈVE

Catalysis

Catalysis

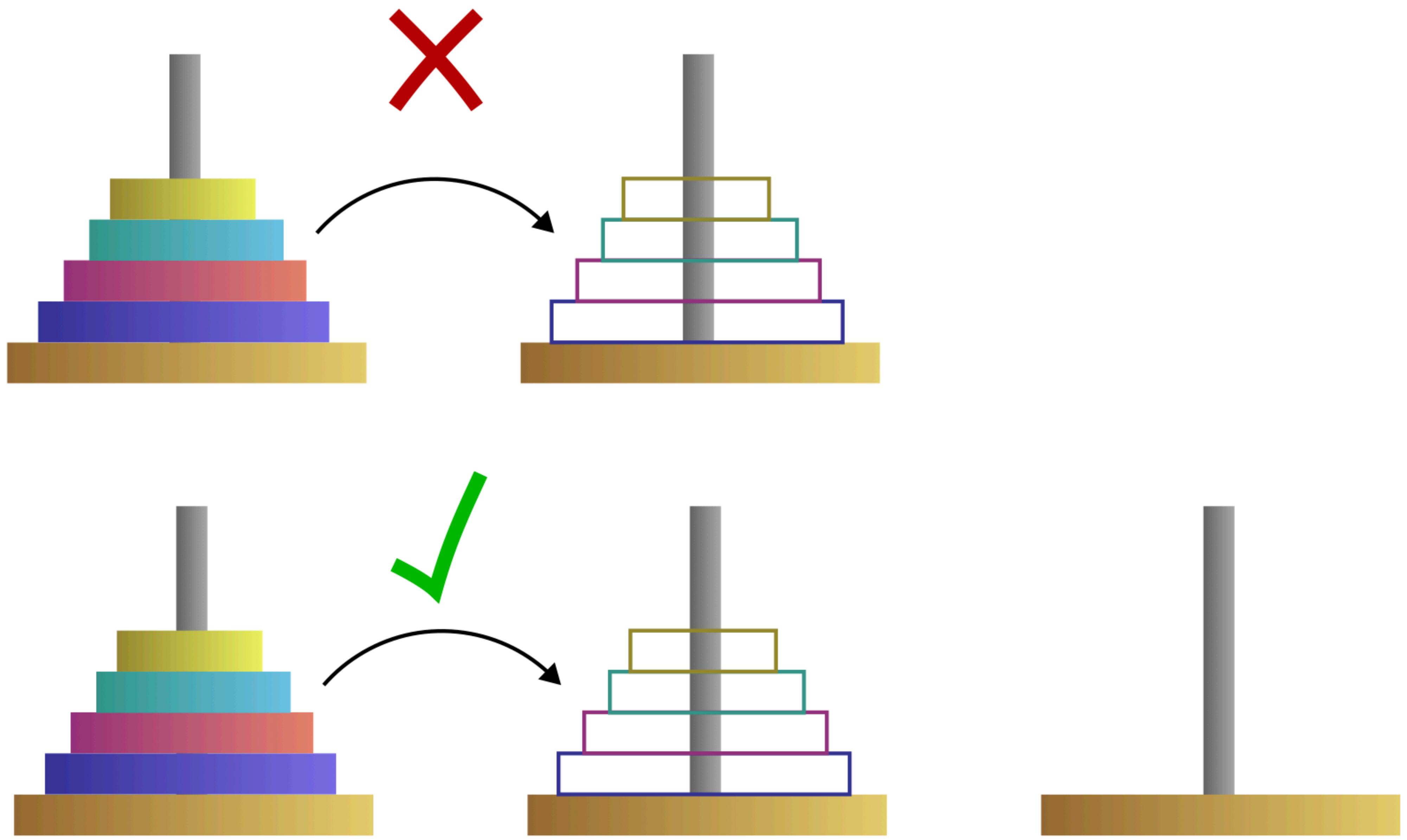

Image: P. Lipka-Bartosik, H. Wilming, N.H.Y. Ng, Rev. Mod. Phys. 96, 025005 (2024)

Quantum catalysis

$$|\psi_1\rangle \overset{\text{LOCC}}{\not\longrightarrow} |\psi_2\rangle$$

$$|\psi_1\rangle \otimes |\phi\rangle \xrightarrow{\text{LOCC}} |\psi_2\rangle \otimes |\phi\rangle$$

D. Jonathan and M. B. Plenio, Phys. Rev. Lett 83, 3566 (1999)

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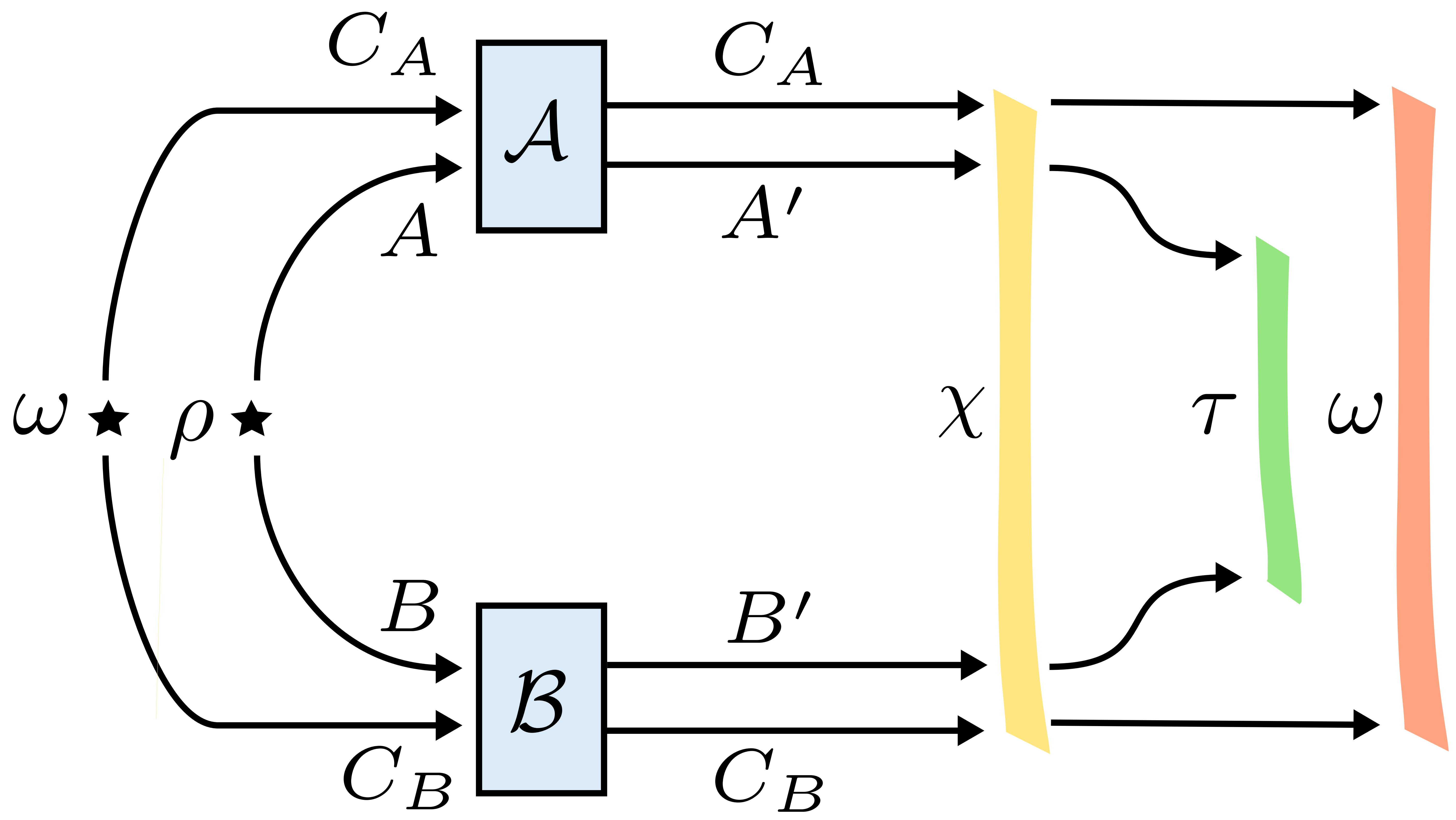
classical
communication

D. Jonathan and M. B. Plenio, Phys. Rev. Lett 83, 3566 (1999)

We are interested in detangling the role of **classical communication** from that of **quantum catalysis** in state transformations.

Catalytic Local Operations



$$\chi_{A'B'C_AC_B} \neq \tau_{A'B'} \otimes \omega_{C_AC_B}$$

may be classically correlated
or entangled

First result: A Catalytic Transformation

$$\forall \;\; \rho_{AB} \in \mathcal{L}(\mathbb{C}^d \otimes \mathbb{C}^d), \quad \forall \;\; n \in \mathbb{N}$$

$$\rho_{AB} \xrightarrow{\text{CLO}} \tau_{A'B'} = \frac{1}{n} (\rho_{AB})^{\otimes n} \otimes [00]_{R_A R_B} + \frac{n-1}{n} (\sigma_A \otimes \sigma_B)^{\otimes n} \otimes [11]_{R_A R_B}$$

R. Duan, Y. Feng, X. Li, and M. Ying, Phys. Rev. A 71, 042319 (2005)

P. Lipka-Bartosik, P. Skrzypczyk, Phys. Rev. Lett. 127, 080502 (2021)

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deterministic
no shared randomness
no classical communication

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deterministic

no shared randomness

no classical communication

catalyst state

$$\omega_{C_A C_B} = \frac{1}{n}\sum_{i=0}^{n-1} (\rho_{\widetilde{A}\widetilde{B}})^{\otimes i} \otimes (\sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}})^{\otimes (n-1-i)} \otimes [ii]_{\widetilde{R}_A \widetilde{R}_B}$$

R. Duan, Y. Feng, X. Li, and M. Ying, Phys. Rev. A 71, 042319 (2005)

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probabilistic cloning?

No!

catalyst not universal

$$\omega_{C_A C_B} = \frac{1}{n} \sum_{i=0}^{n-1} (\rho_{\widetilde{A}\widetilde{B}})^{\otimes i} \otimes (\sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}})^{\otimes (n-1-i)} \otimes [ii]_{\widetilde{R}_A \widetilde{R}_B}$$

R. Duan, Y. Feng, X. Li, and M. Ying, Phys. Rev. A 71, 042319 (2005)
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Proof: A Catalytic Transformation

$n = 2$ $$\rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$$

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1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

Proof: A Catalytic Transformation

$n = 2$ $\qquad \rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) If in branch: $[00]_{\tilde{R}_A \tilde{R}_B}$ $\qquad \rho_{AB} \otimes \sigma_{\tilde{A}\tilde{B}} \otimes [00]_{\tilde{R}_A \tilde{R}_B}$

Proof: A Catalytic Transformation

$n = 2$ $\quad \rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\tilde{A}} \otimes \sigma_{\tilde{B}} \otimes [00]_{\tilde{R}_A \tilde{R}_B} + \rho_{AB} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}\right)$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) If in branch: $[00]_{\tilde{R}_A \tilde{R}_B}$ $\quad \rho_{AB} \otimes \sigma_{\tilde{A}\tilde{B}} \otimes [00]_{\tilde{R}_A \tilde{R}_B}$

(i) Swap systems $A \leftrightarrow \tilde{A}$ and $B \leftrightarrow \tilde{B}$ $\quad \mapsto \sigma_{AB} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [00]_{\tilde{R}_A \tilde{R}_B}$

(ii) Flip classical registers $[00]_{\tilde{R}_A \tilde{R}_B} \mapsto [11]_{\tilde{R}_A \tilde{R}_B}$ $\quad \mapsto \sigma_{AB} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}$

(iii) Locally prepare system $A'B' = A_1 B_1 A_2 B_2$ in product state $\sigma_{A_1 B_1} \otimes \sigma_{A_2 B_2}$ $\quad \mapsto \sigma_{A_1 B_1} \otimes \sigma_{A_2 B_2} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}$

(iv) Prepare new classical registers $[11]_{R_A R_B}$ $\quad \mapsto \sigma_{A_1 B_1} \otimes \sigma_{A_2 B_2} \otimes [11]_{R_A R_B} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}$

Proof: A Catalytic Transformation

$n = 2$ $$\rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) If in branch: $[11]_{\tilde{R}_A \tilde{R}_B}$ $\rho_{AB} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}$

Proof: A Catalytic Transformation

$n = 2$ $$\rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) If in branch: $[11]_{\tilde{R}_A \tilde{R}_B}$ $\rho_{AB} \otimes \rho_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}$

(i) Locally prepare system $A'B' = A_1B_1A_2B_2$ in state $\rho_{A_1B_1} \otimes \rho_{A_2B_2}$ and system $\tilde{A}\tilde{B}$ in product state $\sigma_{\tilde{A}\tilde{B}}$ $\mapsto \rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes \sigma_{\tilde{A}\tilde{B}} \otimes [11]_{\tilde{R}_A \tilde{R}_B}$

(ii) Flip classical registers $[11]_{\tilde{R}_A \tilde{R}_B} \mapsto [00]_{\tilde{R}_A \tilde{R}_B}$ $\mapsto \rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes \sigma_{\tilde{A}\tilde{B}} \otimes [00]_{\tilde{R}_A \tilde{R}_B}$

(iii) Prepare new classical registers $[00]_{R_A R_B}$ $\mapsto \rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes [00]_{R_A R_B} \otimes \sigma_{\tilde{A}\tilde{B}} \otimes [00]_{\tilde{R}_A \tilde{R}_B}$

Proof: A Catalytic Transformation

$n = 2$ $$\rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) Perform local operations

Proof: A Catalytic Transformation

$n = 2$ $\qquad \rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) Perform local operations

3) Resulting state: $\chi_{A'B'C_A C_B} = \frac{1}{2}(\rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes [00]_{R_A R_B} \otimes \sigma_{\widetilde{A}\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} +$

$\sigma_{A_1B_1} \otimes \sigma_{A_2B_2} \otimes [11]_{R_A R_B} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B})$

Proof: A Catalytic Transformation

$n = 2$ $$\rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) Perform local operations

3) Resulting state: $$\chi_{A'B'C_AC_B} = \frac{1}{2}(\rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes [00]_{R_AR_B} \otimes \sigma_{\widetilde{A}\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \sigma_{A_1B_1} \otimes \sigma_{A_2B_2} \otimes [11]_{R_AR_B} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B})$$

Marginals:

$$\mathrm{tr}_{A'B'}(\chi_{A'B'C_AC_B}) = \frac{1}{2}(\sigma_{\widetilde{A}\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}) = \omega_{C_AC_B} \checkmark$$

Catalyst preserved!

Proof: A Catalytic Transformation

$n = 2$ $\quad \rho_{AB} \otimes \omega_{C_A C_B} = \frac{1}{2}\left(\rho_{AB} \otimes \sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} + \rho_{AB} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B}\right)$

1) Alice and Bob measure their local classical register $\tilde{R}_A \tilde{R}_B$

2) Perform local operations

3) Resulting state: $\chi_{A'B'C_A C_B} = \frac{1}{2}(\rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes [00]_{R_A R_B} \otimes \sigma_{\widetilde{A}\widetilde{B}} \otimes [00]_{\widetilde{R}_A \widetilde{R}_B} +$

$\sigma_{A_1B_1} \otimes \sigma_{A_2B_2} \otimes [11]_{R_A R_B} \otimes \rho_{\widetilde{A}\widetilde{B}} \otimes [11]_{\widetilde{R}_A \widetilde{R}_B})$

Final state:

$$\mathrm{tr}_{C_A C_B}(\chi_{A'B'C_A C_B}) = \frac{1}{2}(\rho_{AB}^{\otimes 2} \otimes [00]_{R_A R_B} + (\sigma_A \otimes \sigma_B)^{\otimes 2} \otimes [11]_{R_A R_B}) = \tau_{A'B'} \checkmark$$

Target state achieved!

First result: A Catalytic Transformation

$$\forall \;\; \rho_{AB} \in \mathcal{L}(\mathbb{C}^d \otimes \mathbb{C}^d), \quad \forall \;\; n \in \mathbb{N}$$

$$\rho_{AB} \xrightarrow{\text{CLO}} \tau_{A'B'} = \frac{1}{n} (\rho_{AB})^{\otimes n} \otimes [00]_{R_A R_B} + \frac{n-1}{n} (\sigma_A \otimes \sigma_B)^{\otimes n} \otimes [11]_{R_A R_B}$$

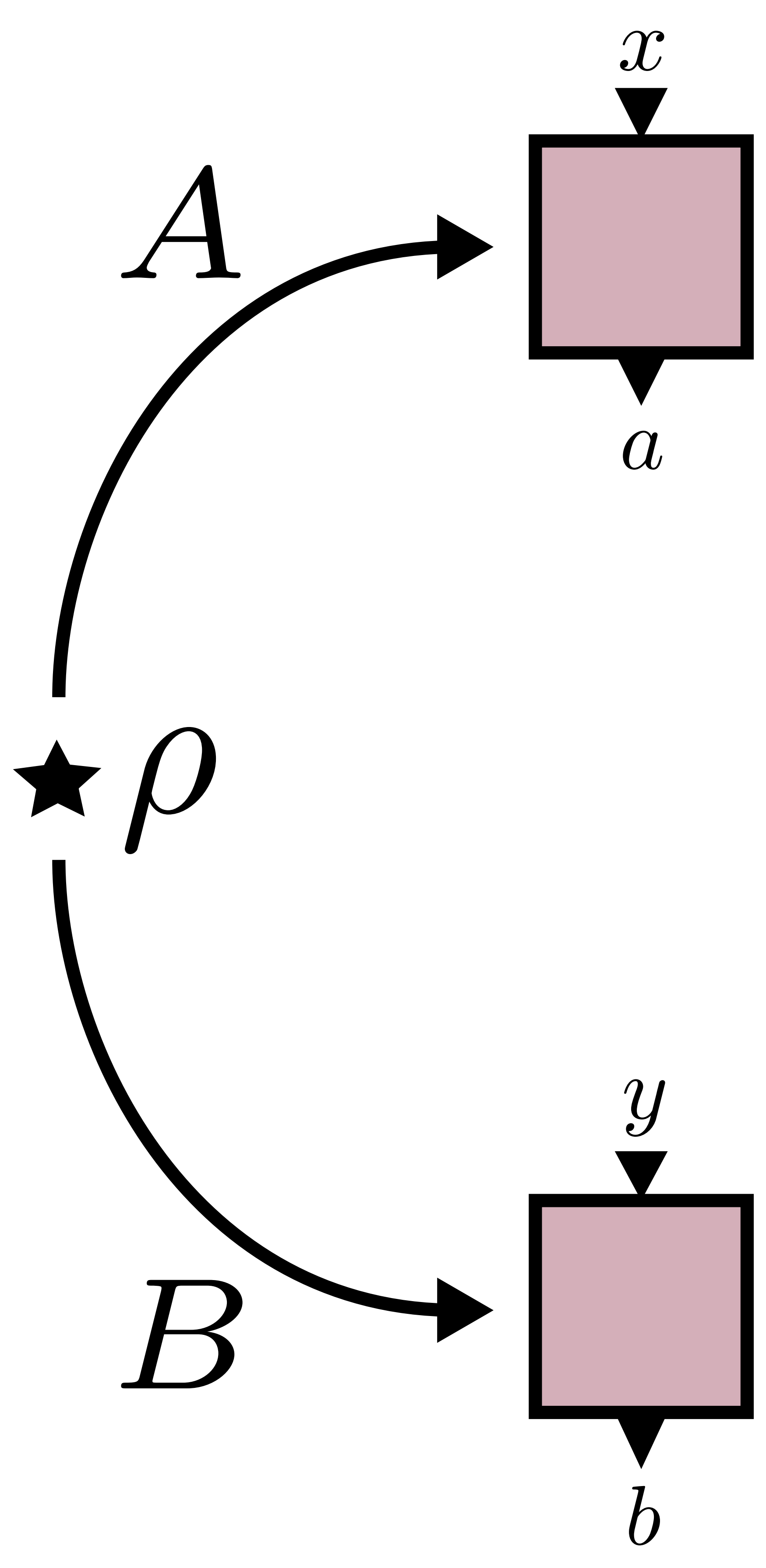
catalyst state

$$\omega_{C_A C_B} = \frac{1}{n} \sum_{i=0}^{n-1} (\rho_{\widetilde{A}\widetilde{B}})^{\otimes i} \otimes (\sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}})^{\otimes (n-1-i)} \otimes [ii]_{\widetilde{R}_A \widetilde{R}_B}$$

JB, N. Brunner, A. Girardin, P. Sekatski, P. Lipka-Bartosik, Phys. Rev. Lett. 135, 220203 (2025)

Activation of Bell Nonlocality

Bell-local quantum states



Bell scenario

Bell-local states: A bipartite quantum state ρ_{AB} is called Bell local if, for all sets of local measurements $\{M^A_{a|x}\}$ and $\{M^B_{b|y}\}$, the correlations

$$p(ab|xy) = \mathrm{tr}[(M^A_{a|x} \otimes M^B_{b|y})\rho_{AB}]$$

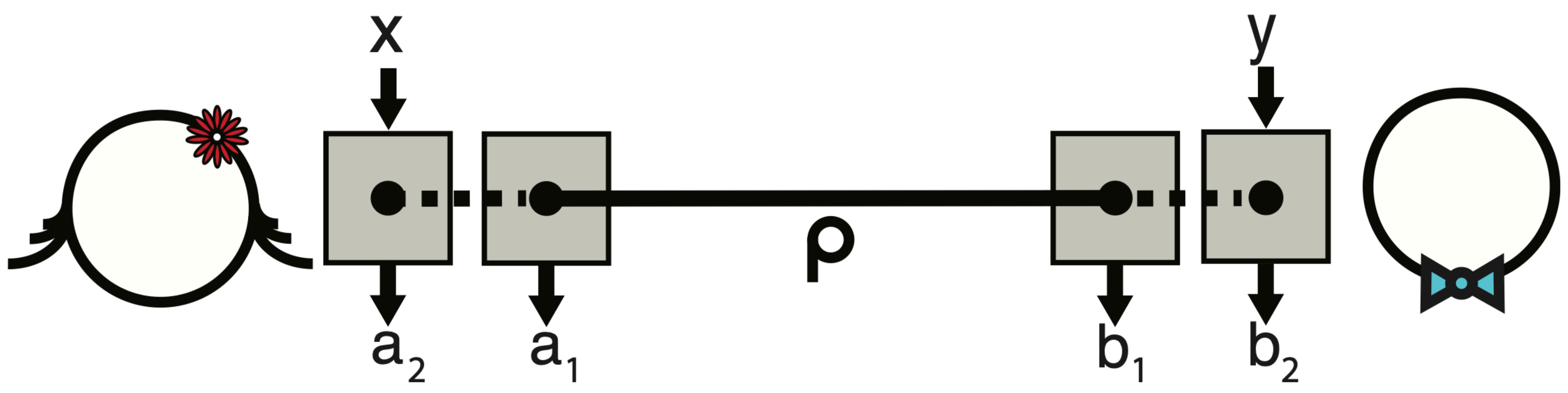
admit a local hidden variable model, according to

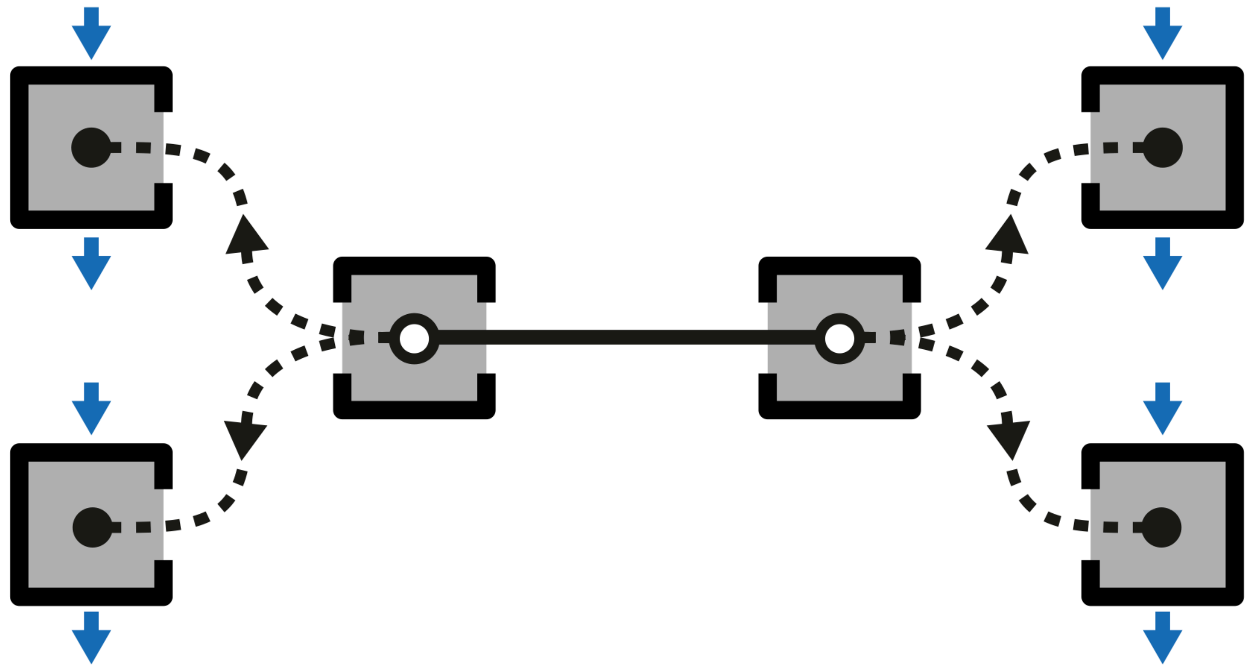
$$p(ab|xy) = \sum_\lambda \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)\,.$$

Activation of Bell nonlocality

Filtering
(SLOCC, “hidden nonlocality”)

S. Popescu, Phys. Rev. Lett. 74, 2619 (1995)
N. Gisin, Phys. Lett. A 210, 151 (1996)
…
F. Hirsch, M.T. Quintino, J. Bowles, N. Brunner,
Phys. Rev. Lett. 111, 160402 (2013)
…



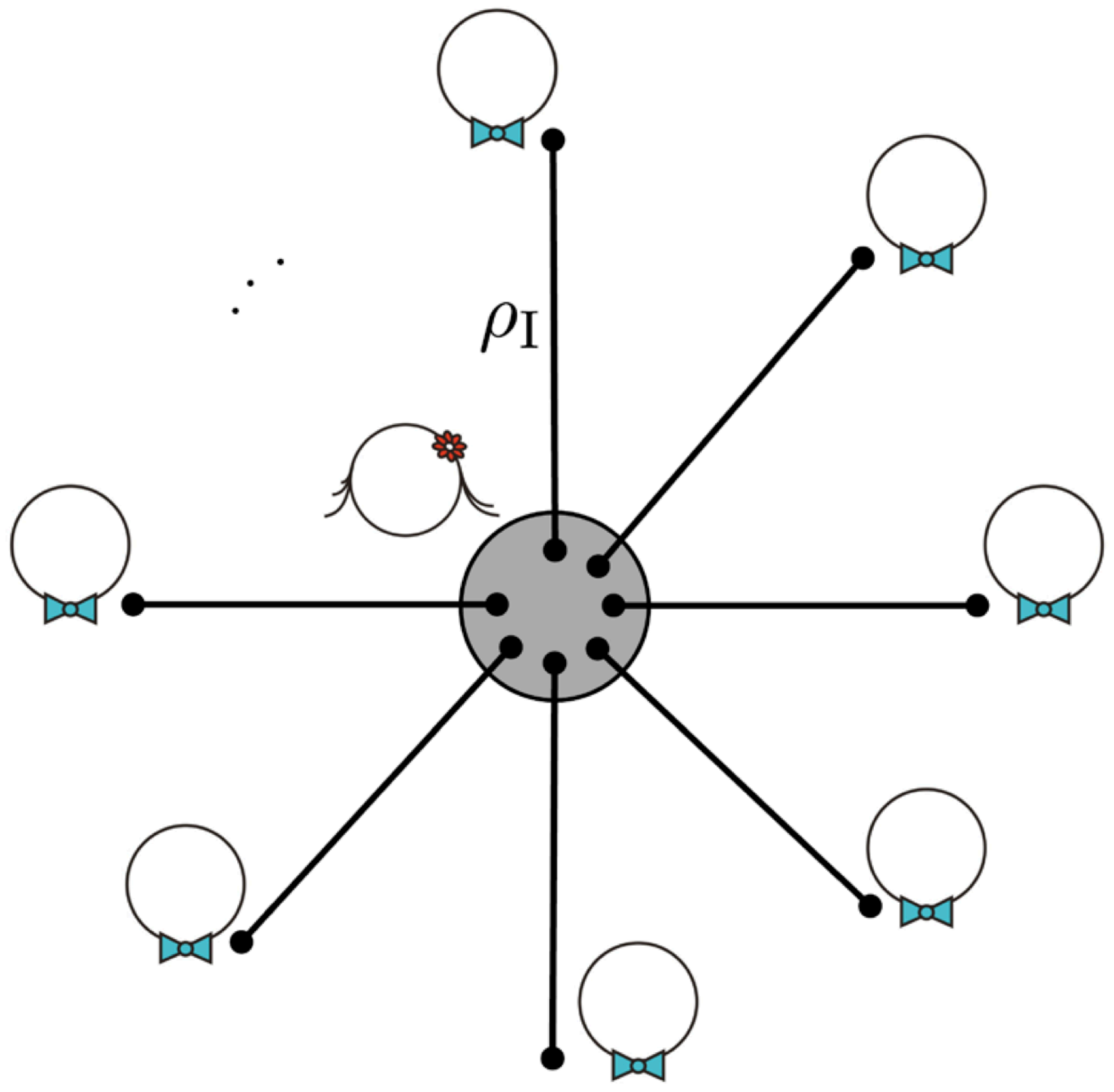


Broadcasting

J. Bowles, F. Hirsch, D. Cavalcanti, Quantum 5, 499 (2021)

…

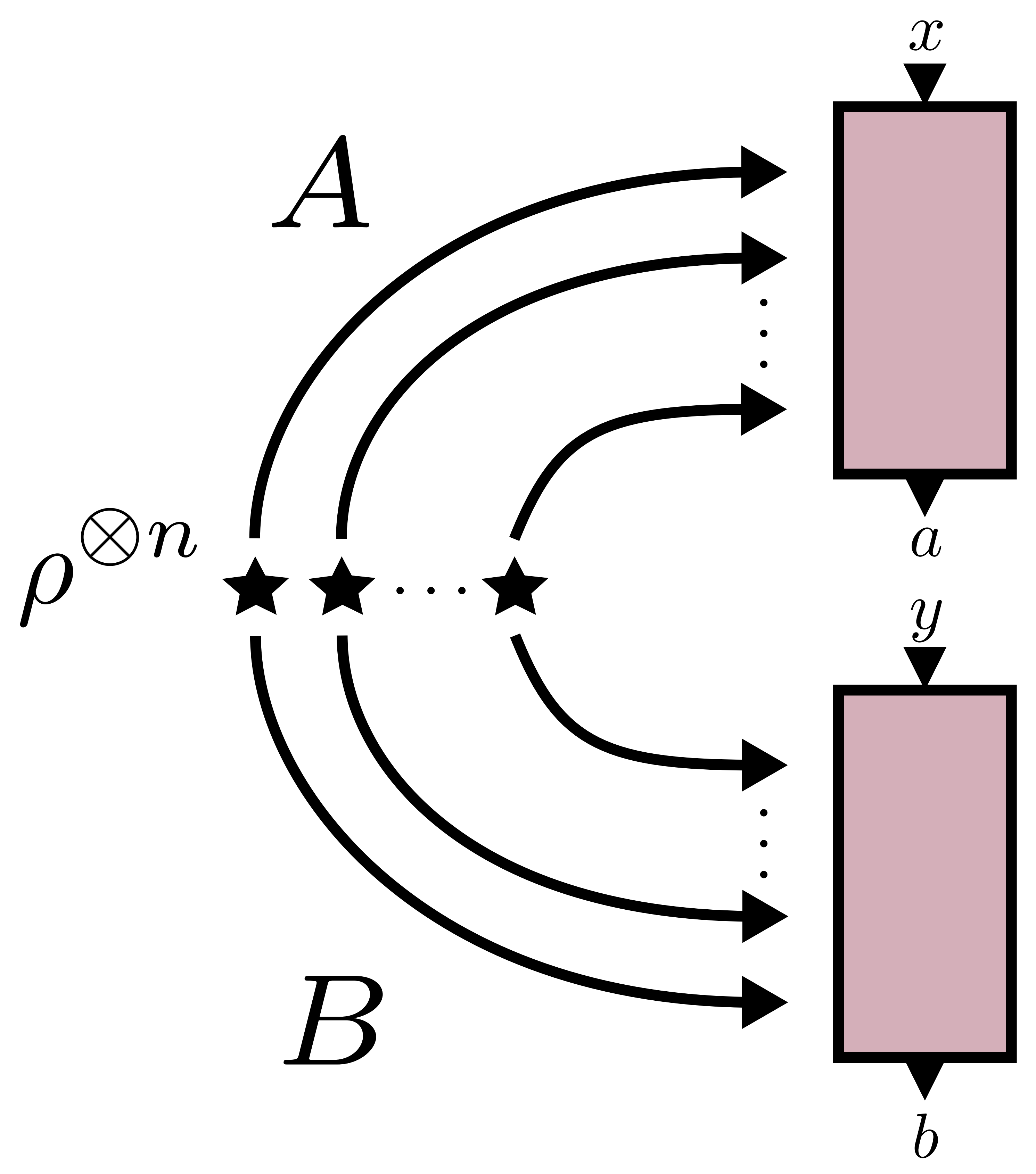
J. Pauwels, P. Sekatski, arXiv:2512.15656 [quant-ph]



Networks

D. Cavalcanti, M. L. Almeida, V. Scarani, A. Acin,
Nat. Commun. 2, 184 (2011)
…

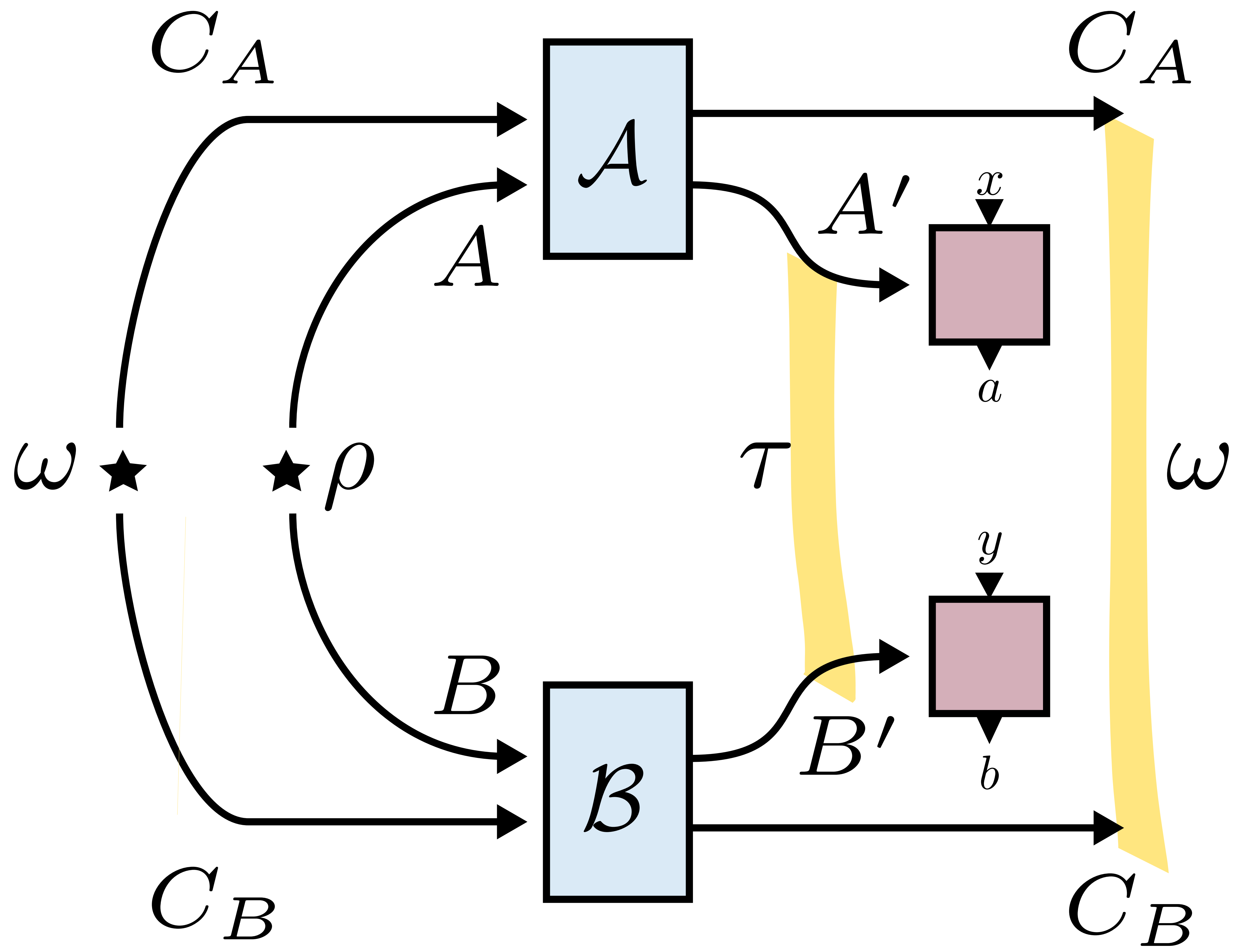
Activation of Bell nonlocality



Many-copy

C. Palazuelos, Phys. Rev. Lett. 109, 190401 (2012)

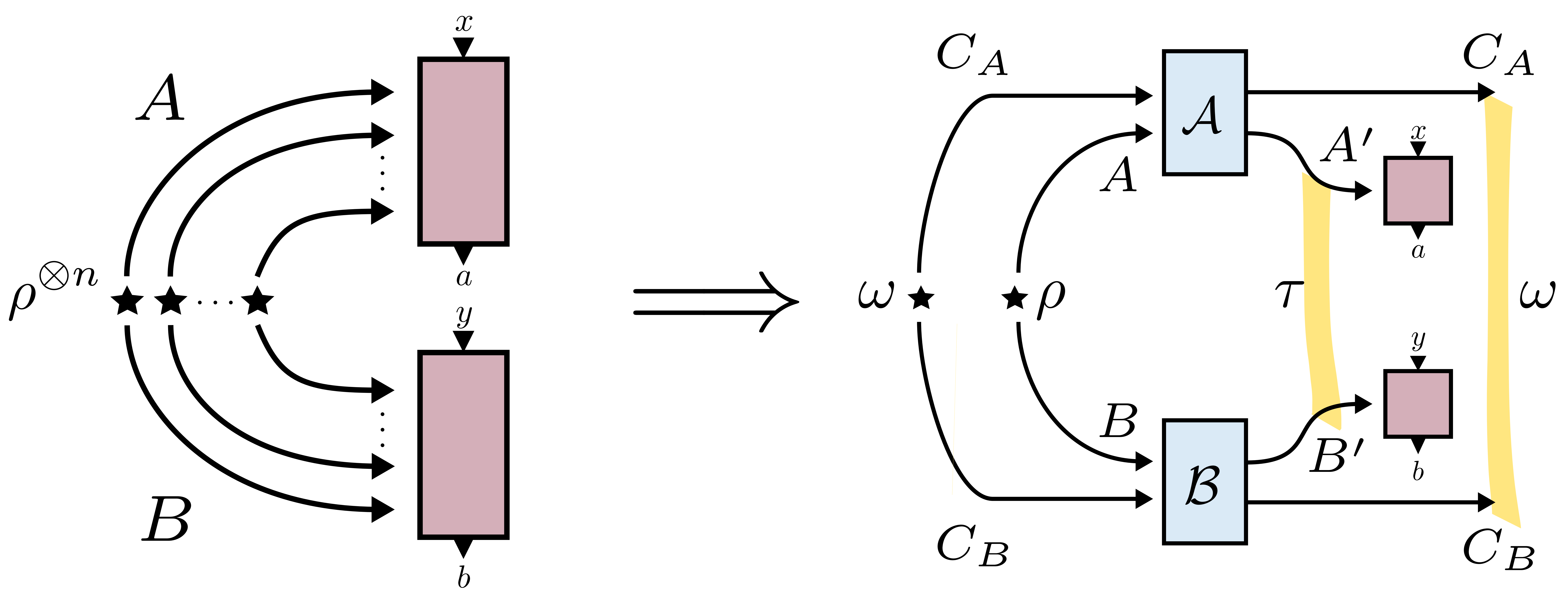
Second result: Catalytic Activation of Bell NL



Theorem 1. **Catalytic activation of Bell NL is possible.**

JB, N. Brunner, A. Girardin, P. Sekatski, P. Lipka-Bartosik, Phys. Rev. Lett. 135, 220203 (2025)

Second result: Catalytic Activation of Bell NL



Theorem 1. In particular, many-copy activation implies catalytic activation.

JB, N. Brunner, A. Girardin, P. Sekatski, P. Lipka-Bartosik, Phys. Rev. Lett. 135, 220203 (2025)

Second result: Catalytic Activation of Bell NL

Observation 1. If $\rho_{AB}^{\otimes n}$ is Bell nonlocal, $\tau_{A'B'}$ is also Bell nonlocal.

$$\rho_{AB} \xrightarrow{\text{CLO}} \tau_{A'B'} = \frac{1}{n}\left(\rho_{AB}\right)^{\otimes n} \otimes [00]_{R_A R_B} + \frac{n-1}{n}\left(\sigma_A \otimes \sigma_B\right)^{\otimes n} \otimes [11]_{R_A R_B}$$

If the NL of a Bell-local state is activatable with *n* copies,
it can also be catalytically activatable, "consuming" only 1 copy.

Second result: Catalytic Activation of Bell NL

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If the NL of a Bell-local state is activatable with *n* copies,
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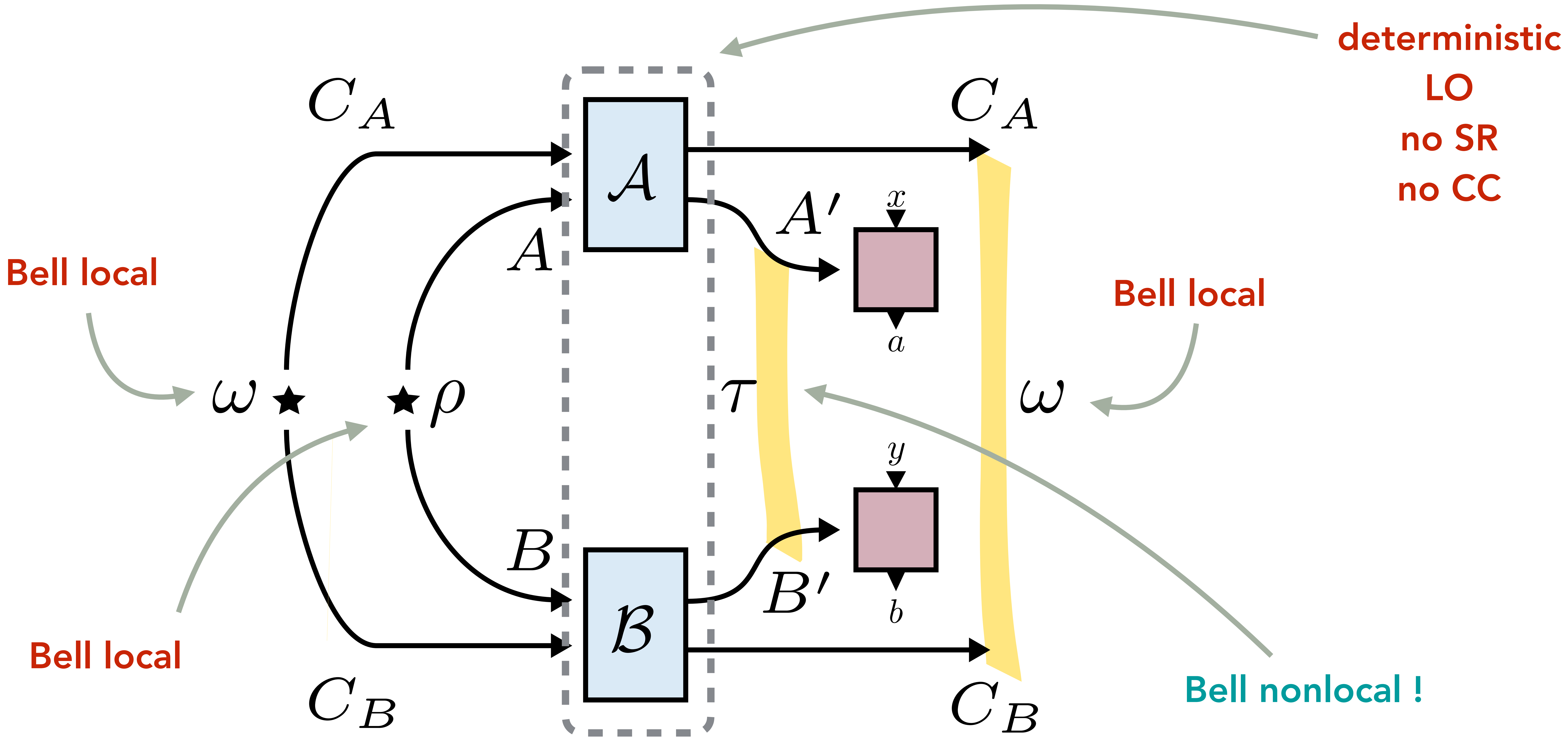
Observation 2. If $\rho_{AB}^{\otimes (n-1)}$ is Bell local, $\omega_{C_A C_B}$ is also Bell local.

$$\omega_{C_A C_B} = \frac{1}{n}\sum_{i=0}^{n-1}(\rho_{\widetilde{A}\widetilde{B}})^{\otimes i} \otimes (\sigma_{\widetilde{A}} \otimes \sigma_{\widetilde{B}})^{\otimes (n-1-i)} \otimes [ii]_{\widetilde{R}_A \widetilde{R}_B}$$

Catalytic activation of Bell NL is possible even with a local catalyst!

Second result: Catalytic Activation of Bell NL



Catalytic super-activation of Bell NL is possible.

Quantum state transformations

CLO(d)

CLO(d): Catalytic Local Operations. The class of dimension-bounded catalytic local operations contains all transformations of a bipartite state into another that can be implemented via local CPTP maps with the assistance of a catalyst state whose marginal is preserved and which has Schmidt number respecting

$$\mathrm{SN}(\omega_{C_A C_B}) \leq d\,.$$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

SLOCCQ(d)

SLOCCQ(d): Stochastic Local Operations with Classical Communication and Quantum communication. This class of transformations contains all sequences of stochastic local operations, classical communication, and dimension-bounded quantum communication, where the dimension of the quantum communication is bounded in the following way. Let d_i be the dimension of the quantum system sent from one party to another in round i. We require that

$$\prod_i d_i \leq d\,.$$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

State transformations

Observation 3. $\mathrm{CLO}^{(d)} \subseteq \mathrm{LOSE}^{(d)} \subseteq \mathrm{SLOCCQ}^{(d)}$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

State transformations

Observation 3. $\mathrm{CLO}^{(d)} \subseteq \mathrm{LOSE}^{(d)} \subseteq \mathrm{SLOCCQ}^{(d)}$

Observation 4. $\mathrm{CLO}^{(1)} = \mathrm{LOSR}$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

State transformations

Observation 3. $\mathrm{CLO}^{(d)} \subseteq \mathrm{LOSE}^{(d)} \subseteq \mathrm{SLOCCQ}^{(d)}$

Observation 4. $\mathrm{CLO}^{(1)} = \mathrm{LOSR}$

Observation 5. $\mathrm{LOCC} \not\subseteq \mathrm{CLO}^{(d)}$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

State transformations

Observation 3. $\mathrm{CLO}^{(d)} \subseteq \mathrm{LOSE}^{(d)} \subseteq \mathrm{SLOCCQ}^{(d)}$

Observation 4. $\mathrm{CLO}^{(1)} = \mathrm{LOSR}$

Observation 5. $\mathrm{LOCC} \not\subset \mathrm{CLO}^{(d)}$

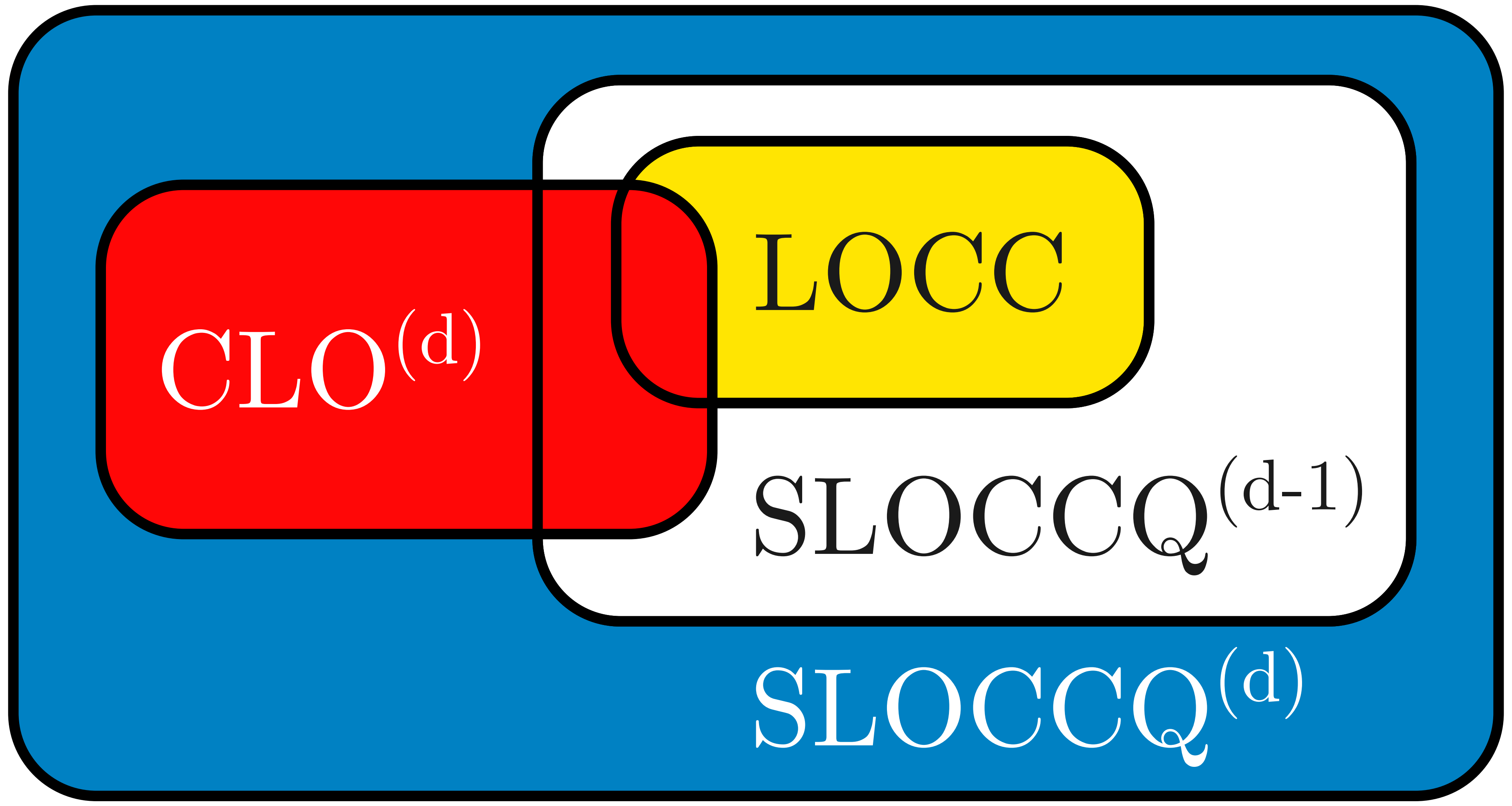
$$\rho_{ABC} = \sum_{i=0}^{1} \frac{1}{2} [0]_A \otimes [i]_B \otimes [i]_C \quad \begin{matrix} \xrightarrow{\text{LOCC}} \\ \overset{\text{LOSE}}{\not\longrightarrow} \end{matrix} \quad \sigma_{ABR} = \sum_{i=0}^{1} \frac{1}{2} [i]_A \otimes [0]_B \otimes [i]_C$$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

Third result: State transformations

Theorem 2.

$$\mathrm{CLO}^{(d)} \not\subset \mathrm{SLOCCQ}^{(d-1)}$$

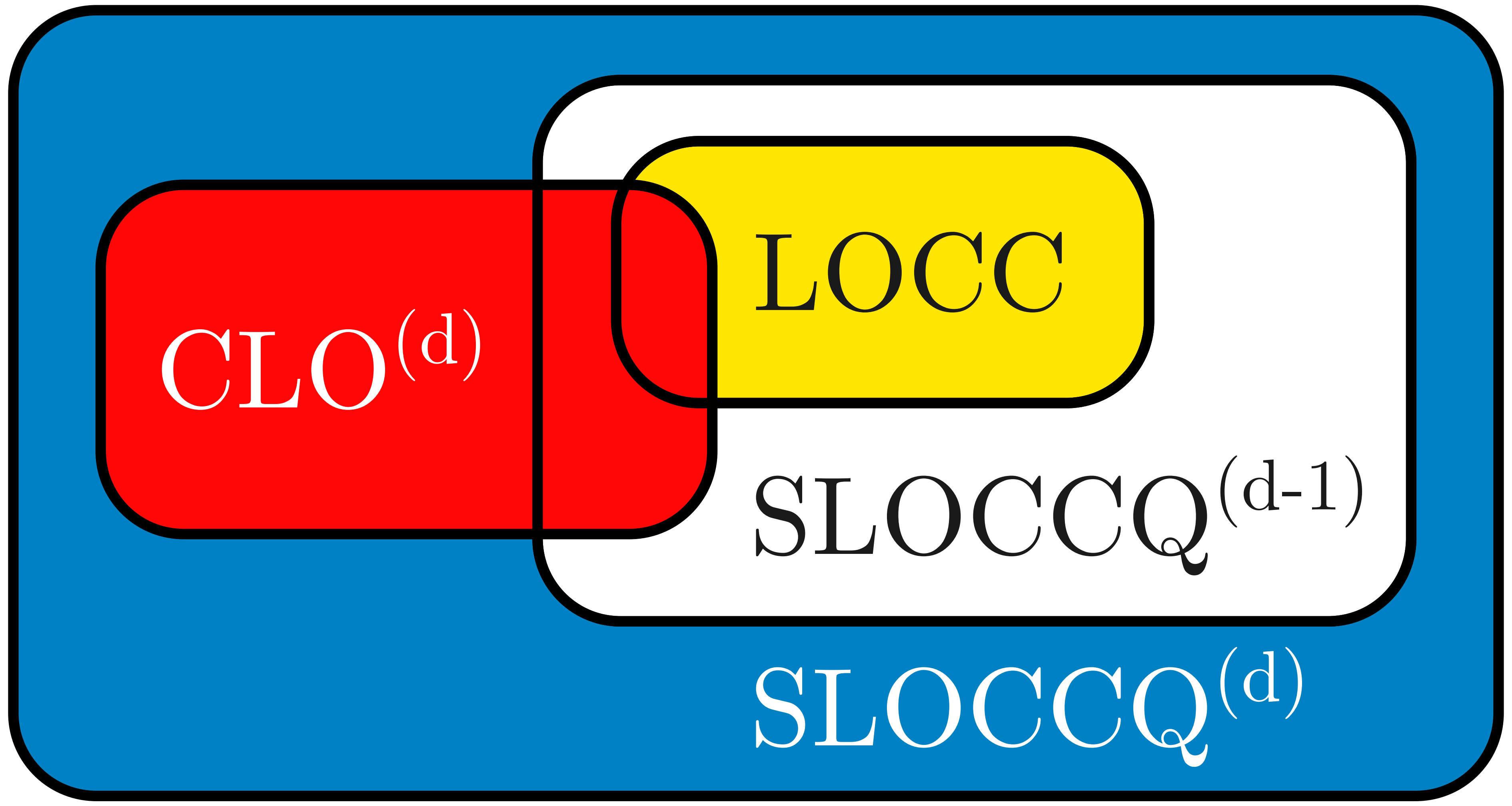


P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

Third result: State transformations

Theorem 2.

$$\mathrm{CLO}^{(d)} \not\subset \mathrm{SLOCCQ}^{(d-1)}$$



Take $\rho_{AB} = |\Phi^+\rangle\langle\Phi^+|$ with $\mathrm{SN}(\rho_{AB}) = 2$.

Via $\mathrm{CLO}^{(2^n)}$ it can be transformed into $\tau_{A'B'}$ with $\mathrm{SN}(\tau_{A'B'}) = 2^{n+1}$.

$$\frac{\mathrm{SN}(\rho_{AB})}{\mathrm{SN}(\tau_{A'B'})} = 2^n > d = 2^n - 1 \implies \text{not possible with } \mathrm{SLOCCQ}^{(2^n-1)}$$

P. Lipka-Bartosik, J.B., N. Brunner, P. Sekatski, arXiv:2510.08320 [quant-ph]

Summary

Lemma 1. $\rho_{AB} \xrightarrow{\text{CLO}} \tau_{A'B'} = \frac{1}{n}(\rho_{AB})^{\otimes n} \otimes [00]_{R_A R_B} + \frac{n-1}{n}(\sigma_A \otimes \sigma_B)^{\otimes n} \otimes [11]_{R_A R_B}$

Observation 1. If $\rho_{AB}^{\otimes n}$ is Bell nonlocal, $\tau_{A'B'}$ is also Bell nonlocal.

Observation 2. If $\rho_{AB}^{\otimes (n-1)}$ is Bell local, $\omega_{C_A C_B}$ is also Bell local.

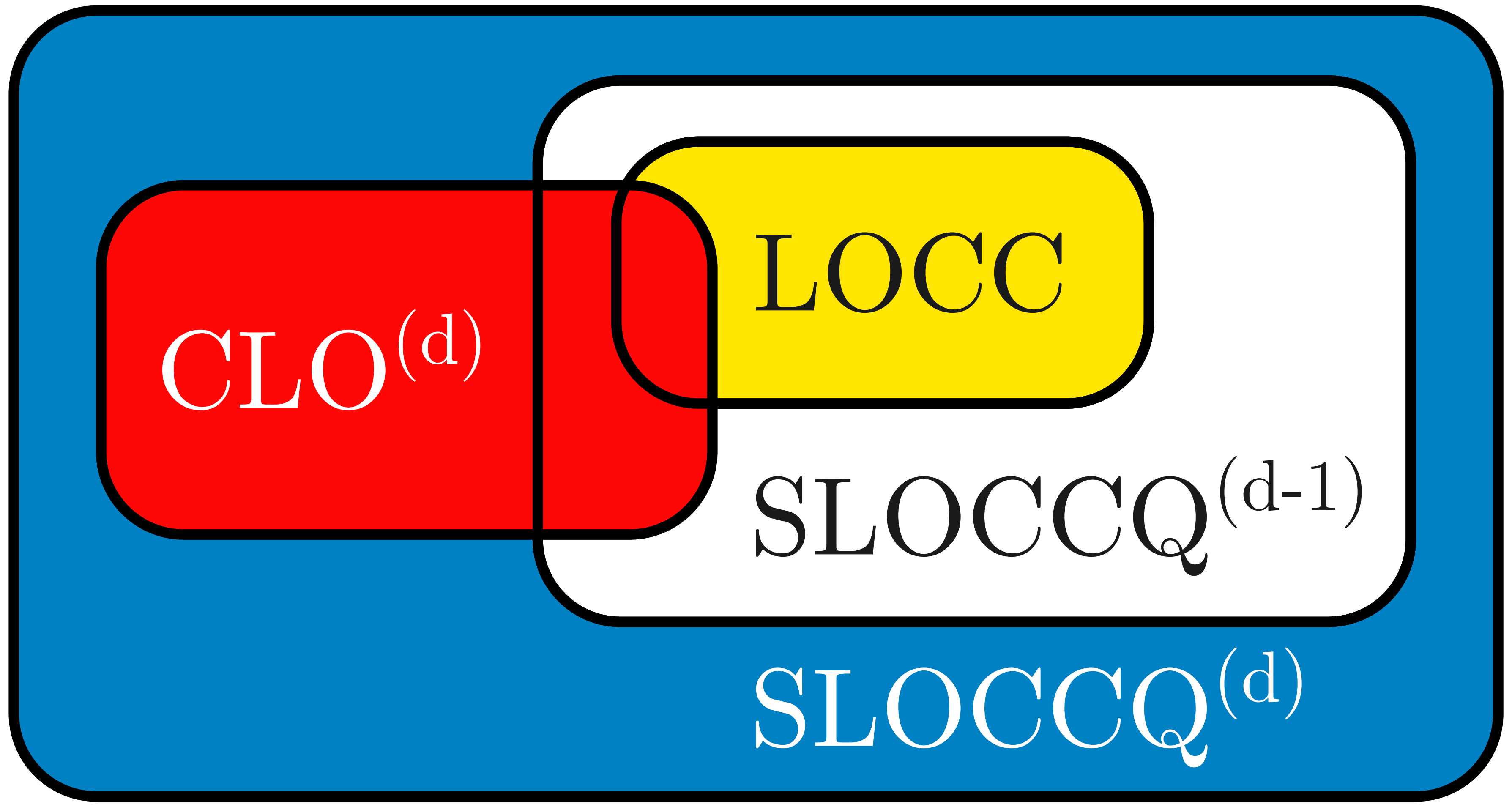
Theorem 1. Many-copy activation implies catalytic activation with a local catalyst.

Observation 3. $\mathrm{CLO}^{(d)} \subseteq \mathrm{LOSE}^{(d)} \subseteq \mathrm{SLOCCQ}^{(d)}$

Observation 4. $\mathrm{CLO}^{(1)} = \mathrm{LOSR}$

Observation 5. $\mathrm{LOCC} \not\subset \mathrm{CLO}^{(d)}$

Theorem 2. $\mathrm{CLO}^{(d)} \not\subset \mathrm{SLOCCQ}^{(d-1)}$

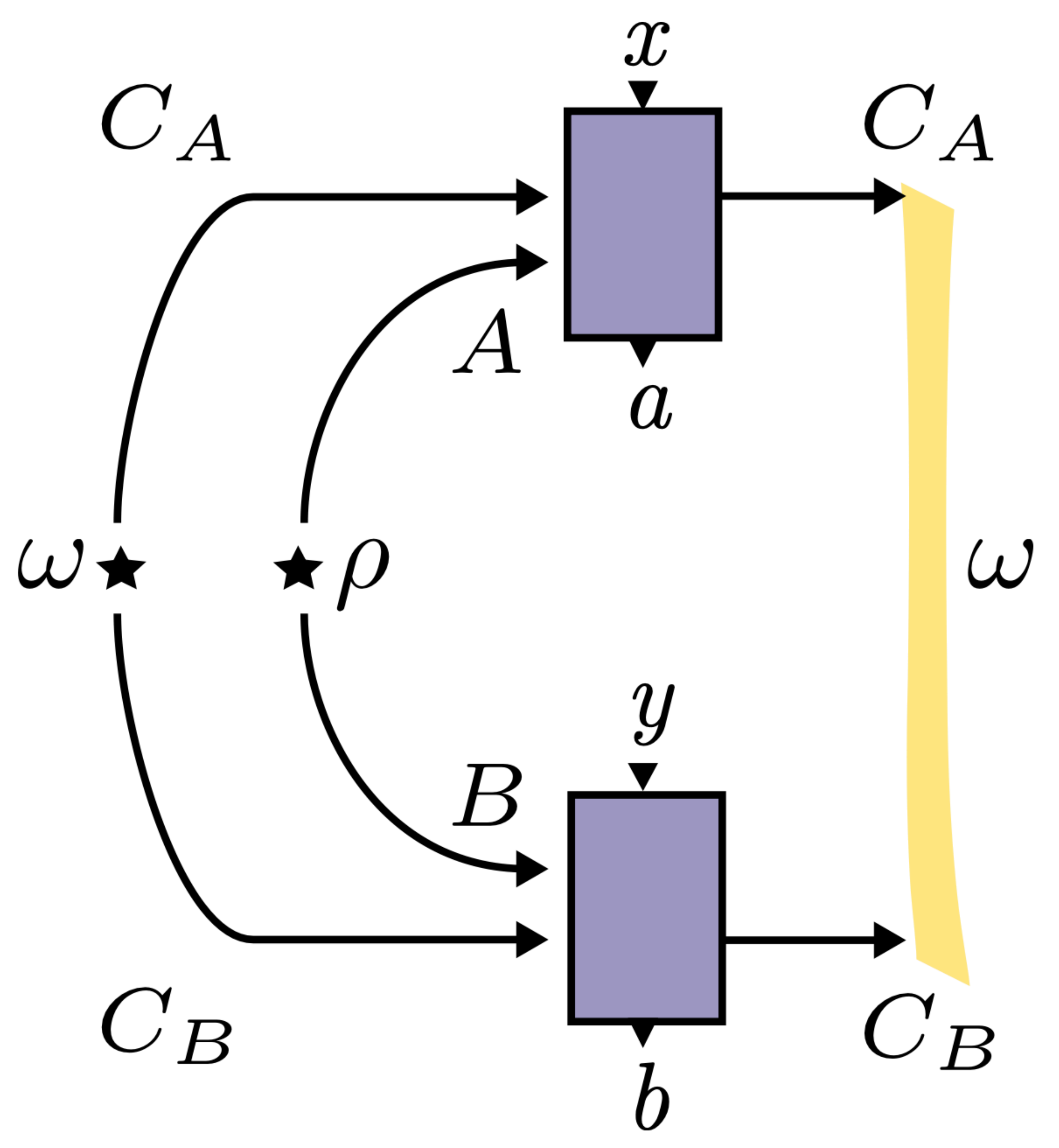


Future directions

- Bell nonlocality

- **State activation:** can quantum states be catalytically activated even when they cannot be many-copy activatable?

- **Other forms of catalysis:** can other quantum states be activated in different catalysis activation scenarios?



- Entanglement theory

- **Monotones:** which entanglement measures are monotonic under CLO and which (such as the Schmidt number) are not?

Thank you!