VCQ Summer School 2026

Quantum vs. Classical Correlations: Part III

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Course overview

- **Part I.** Quantum vs. classical correlations in nonsignalling scenarios
- **Part II.** Quantum vs. classical correlations in network scenarios
- **Part III.** Quantum vs. classical correlations in signalling scenarios

Part III overview

Part III. **Quantum vs. classical correlations in signalling scenarios**

- The GYNI game

  — Causal strategies

  — Process matrices strategies

  — Causal inequalities and the polytope of correlations

  — Quantum vs. classical correlations in higher-order theories

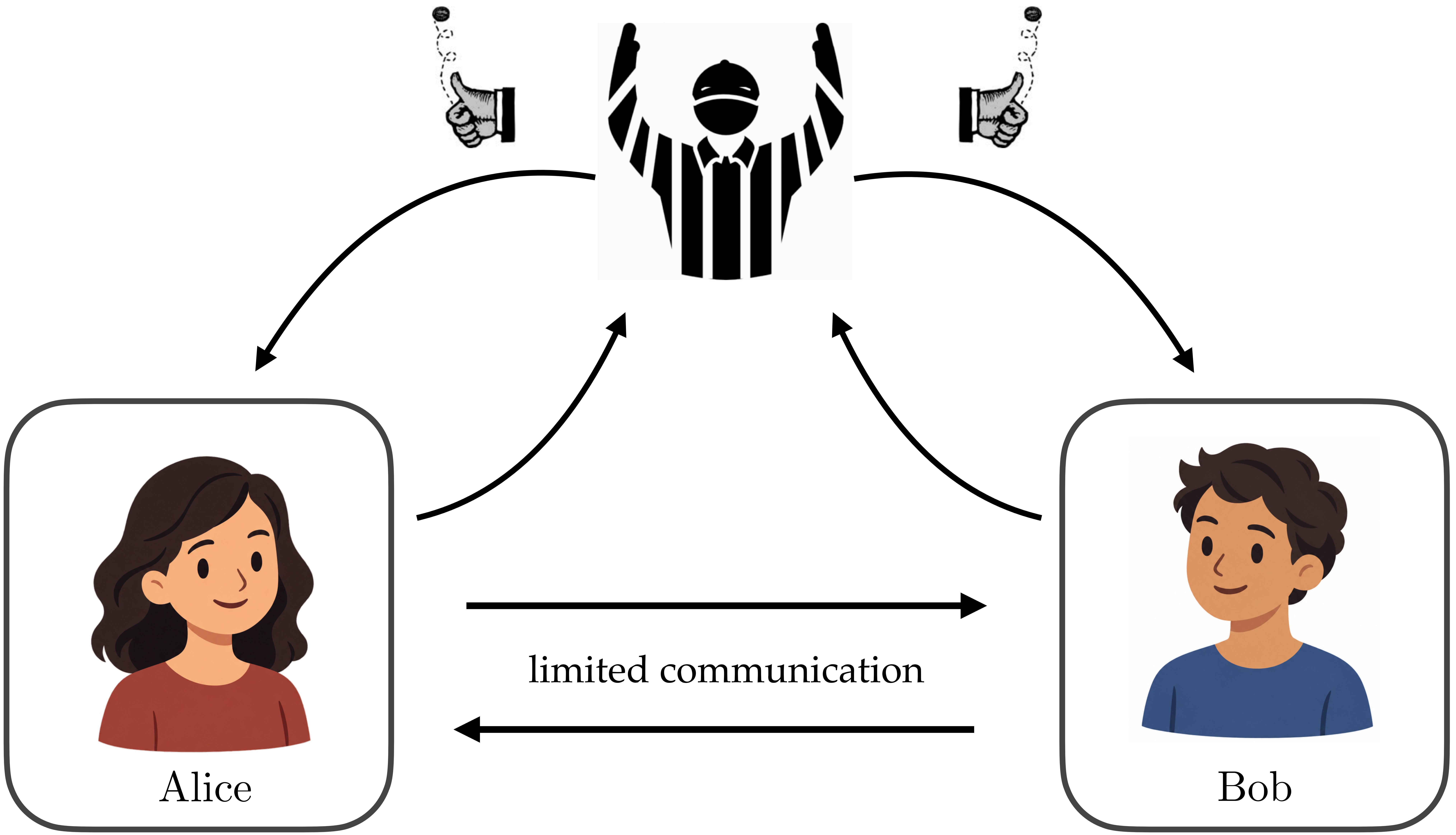
- Classical indefinite causal order

  — The AF/BW (a.k.a. Lugano) process matrix

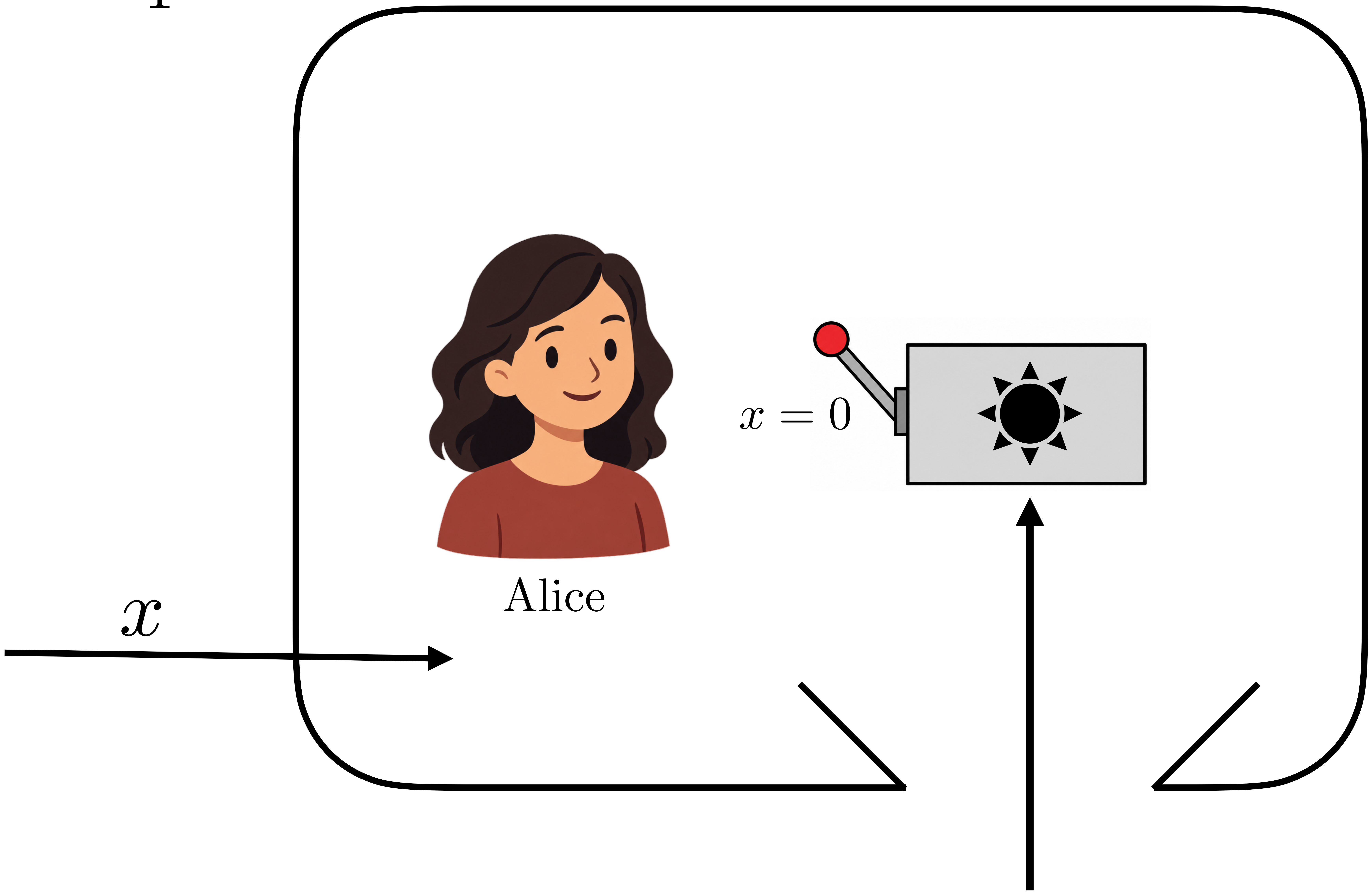
  — Perfect discrimination of the SHIFT basis

The GYNI game

The GYNI game



time t_1



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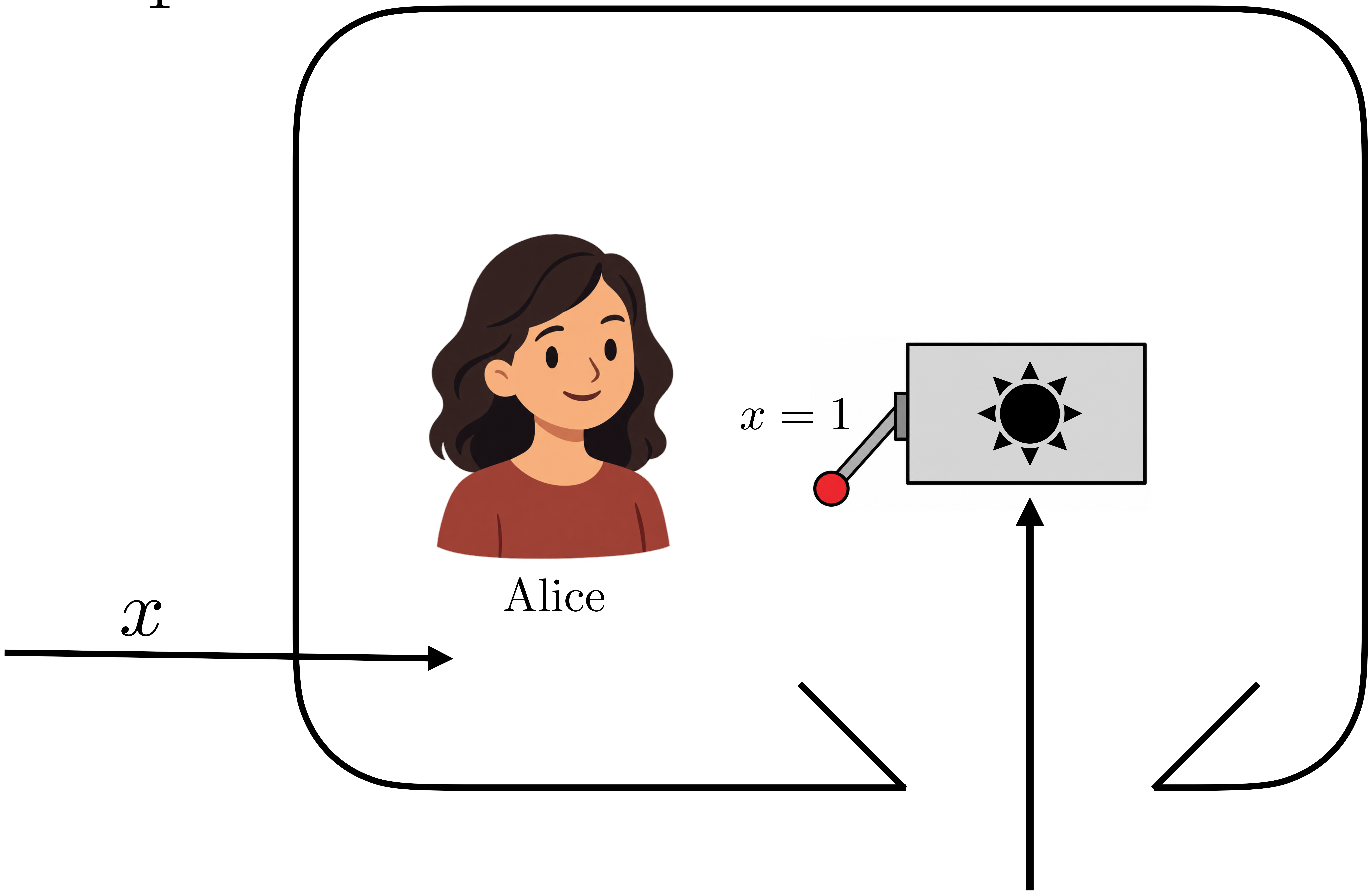
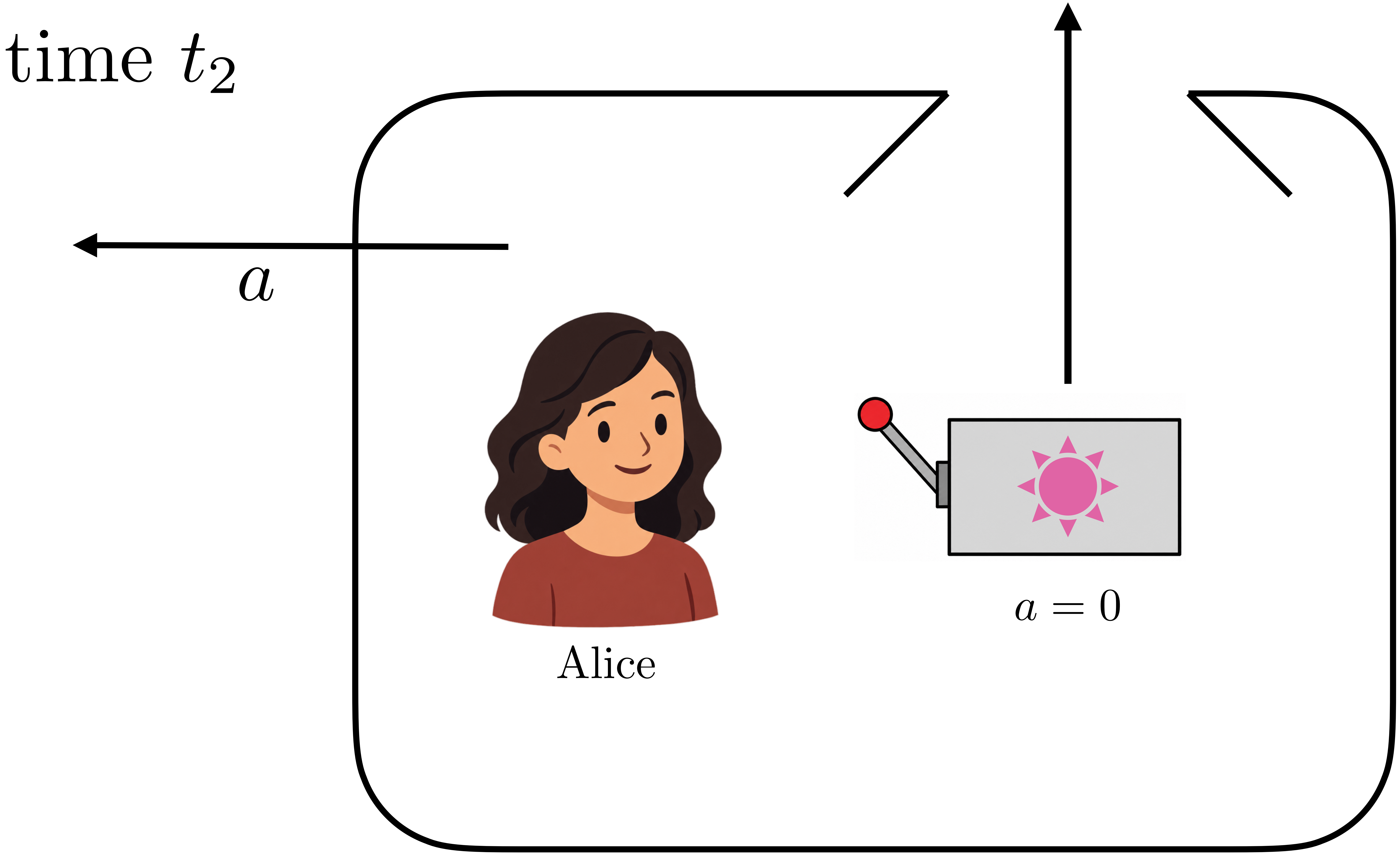
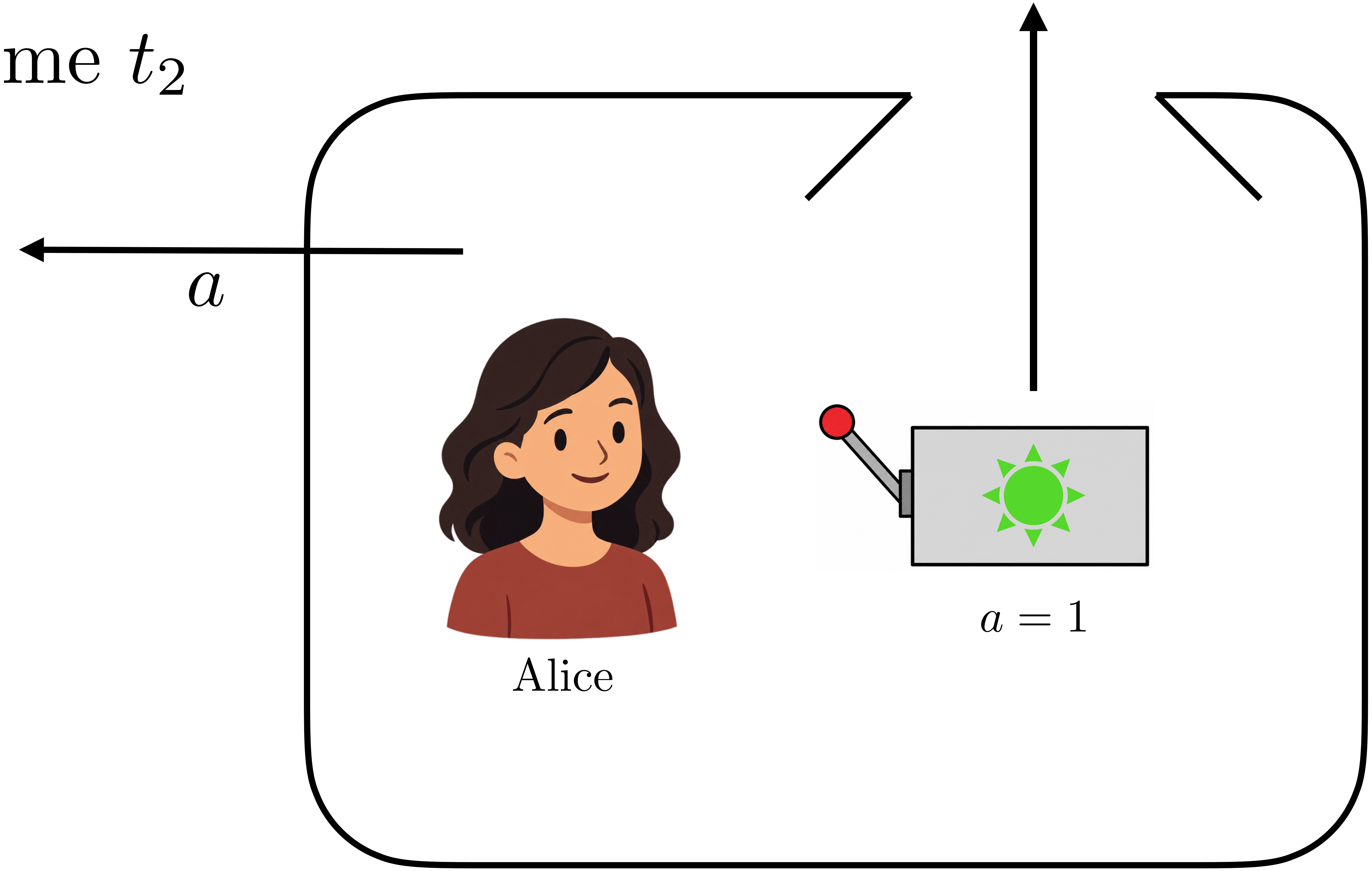


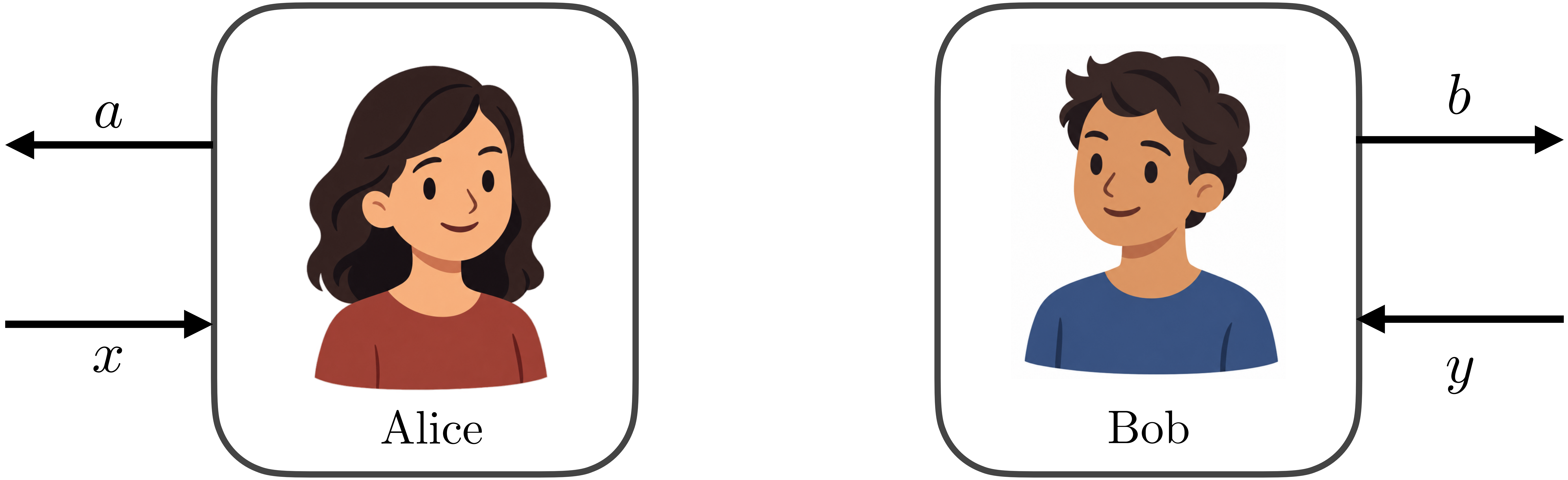


time t_2
a
Alice
$a = 0$

time t_2



The GYNI game



Rules: Each player can only open their labs **once** to receive an incoming system from the outside and **once** to sent an outgoing system to the outside.

The GYNI game

Winning conditions

$$(a = y) \text{ and } (b = x)$$

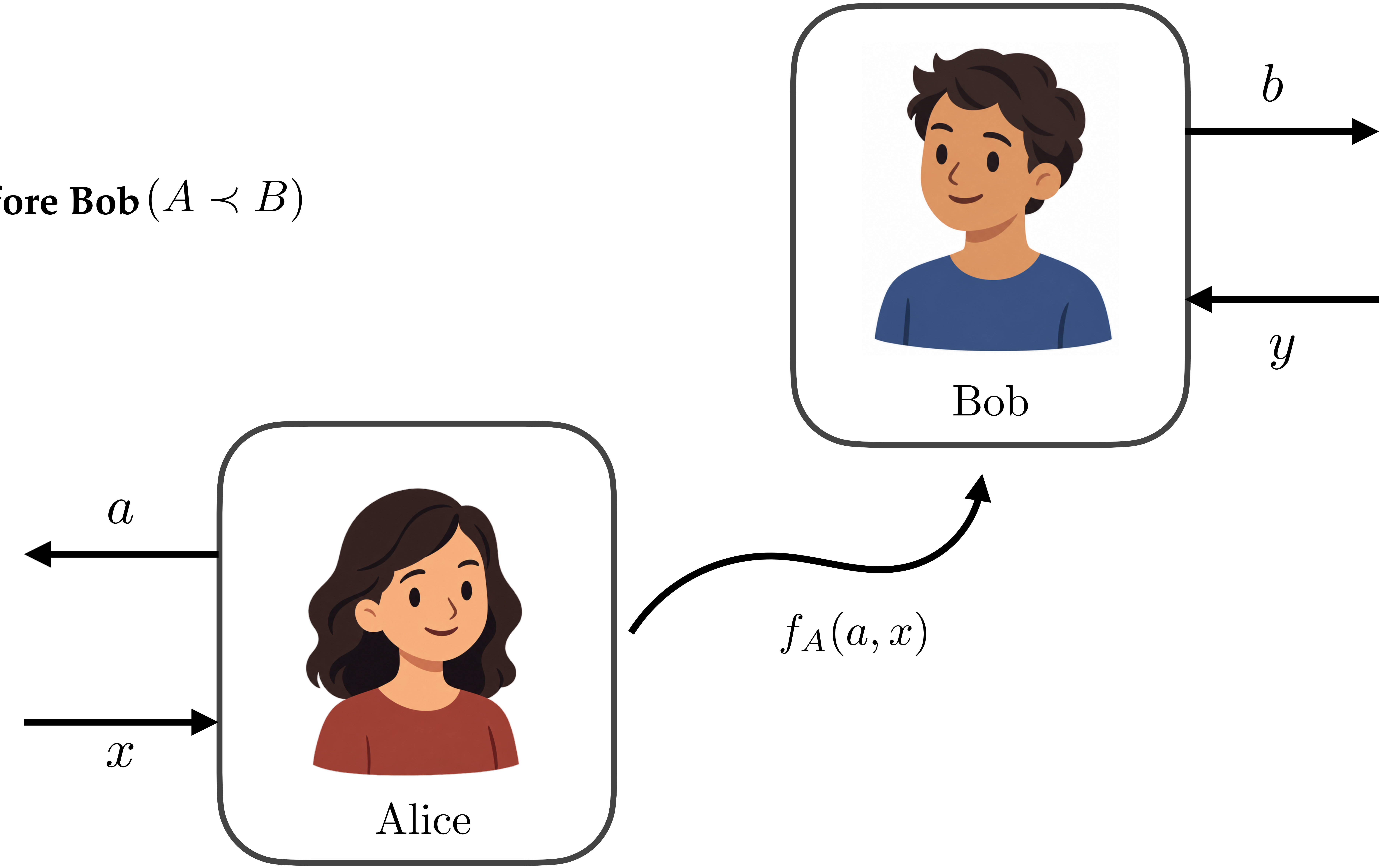
Score:

$$\mathcal{S} = \sum_{a,b,x,y} \underbrace{q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}}_{\text{settings of the game}} \underbrace{p(ab|xy)}_{\text{players' strategy}}$$

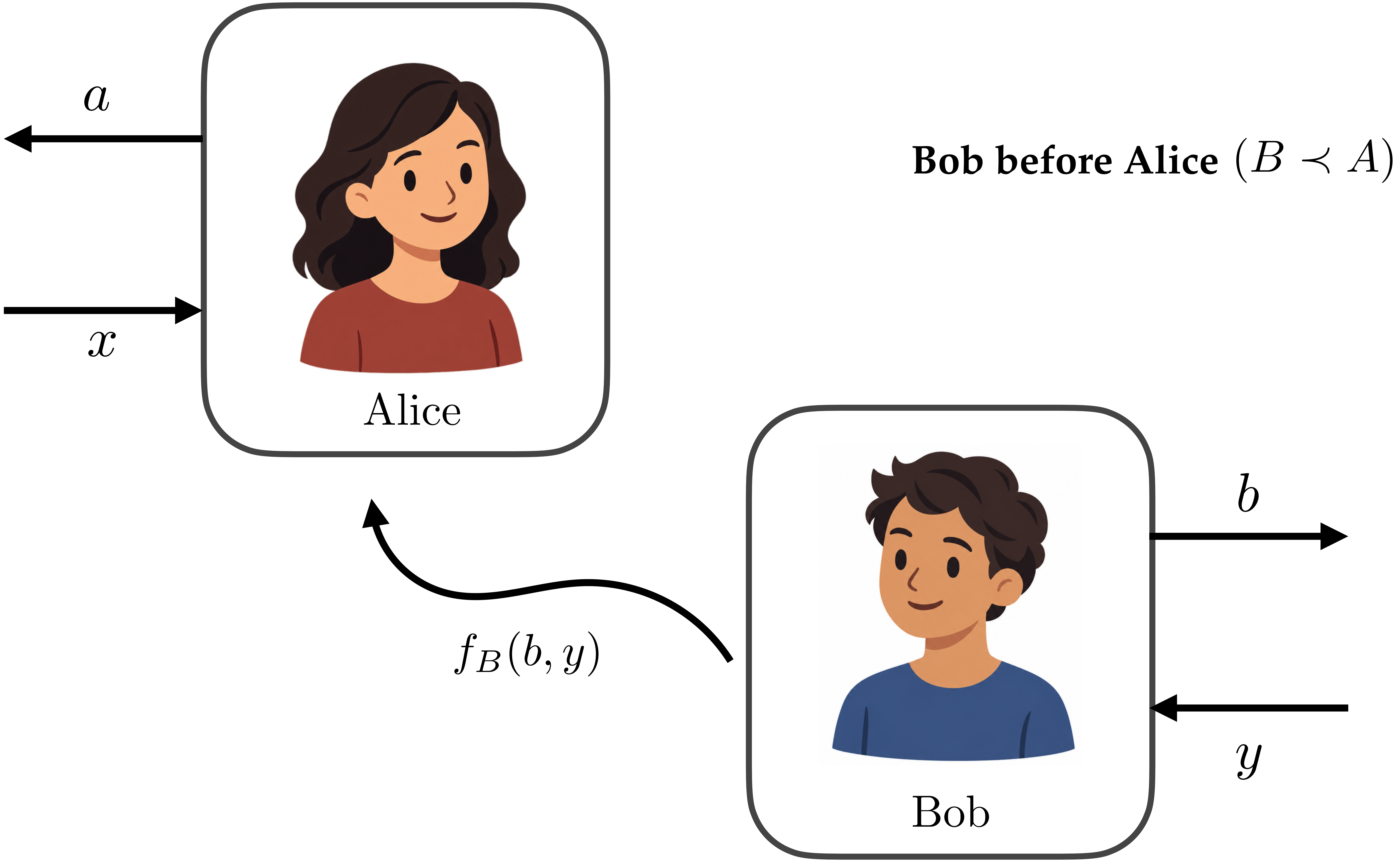
What does it mean for Alice and Bob's strategy to be **causal**?

The GYNI game

Alice before Bob $(A \prec B)$



The GYNI game

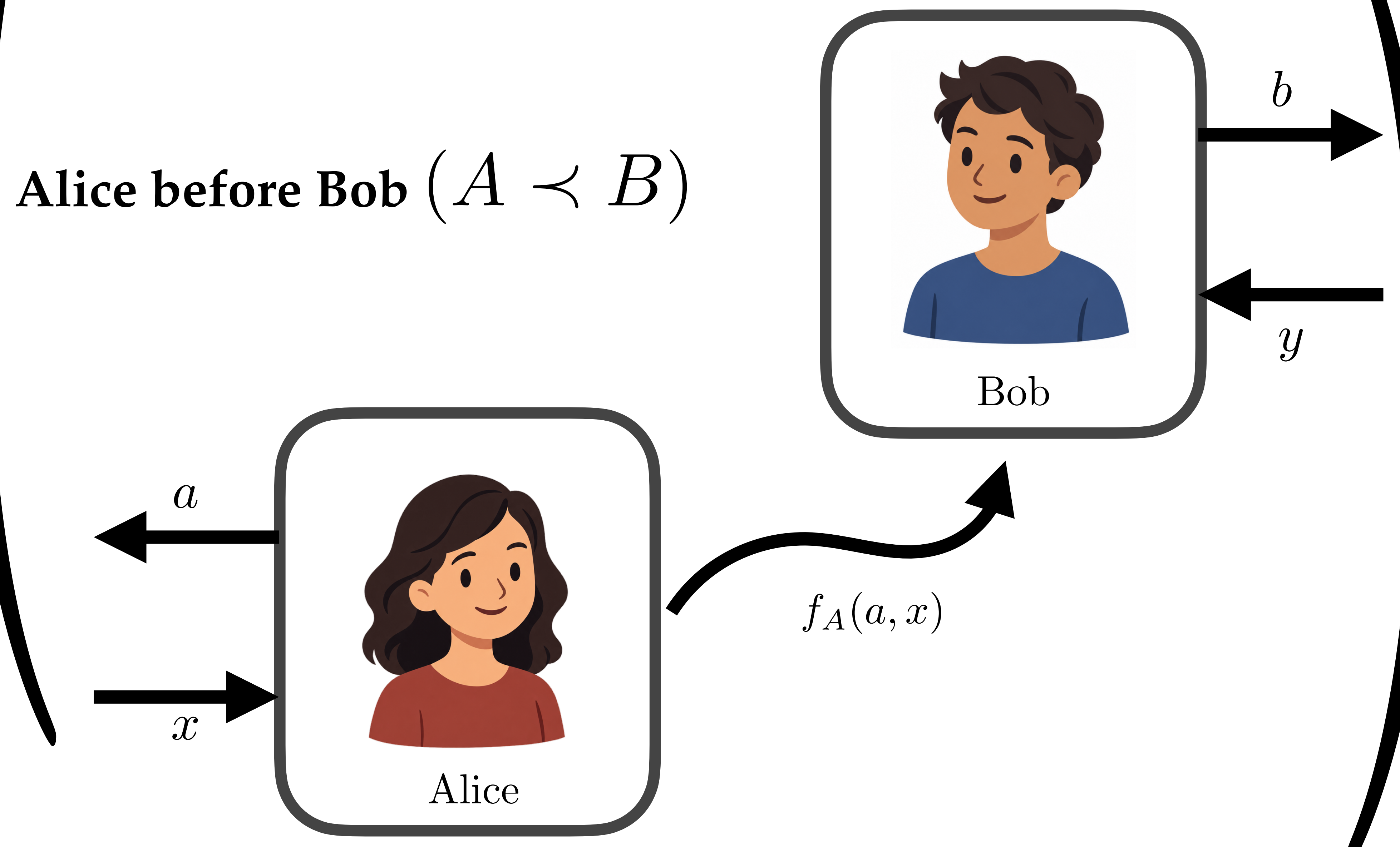


The GYNI game

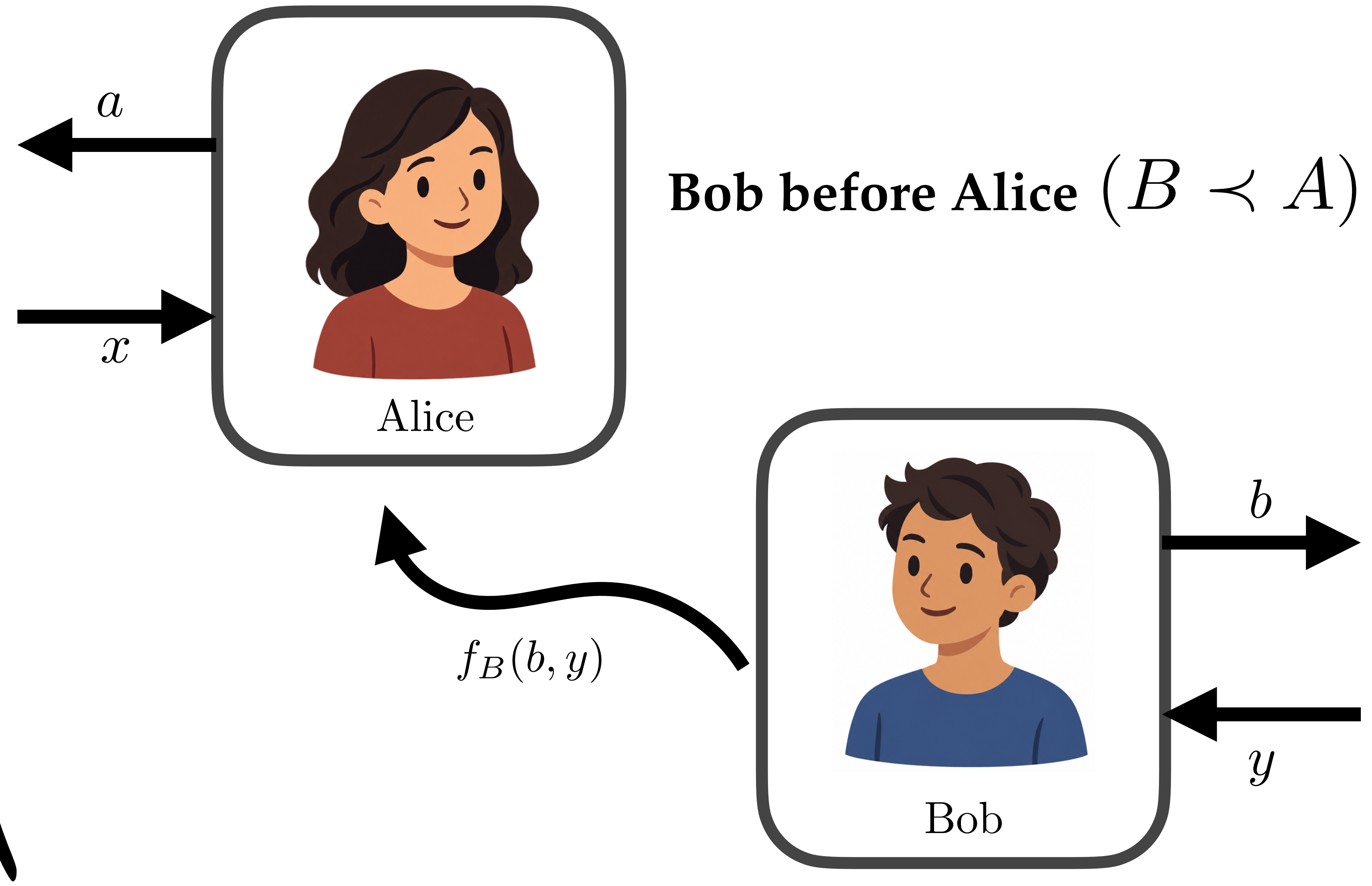
Causal strategies

$$p(ab|xy) =$$

$t\Bigg($



$\Bigg) + (1-t)\Bigg($



$\Bigg)$

The GYNI game

Causal strategies

$$\boxed{p(ab|xy) = t\, p_{A\prec B}(ab|xy) + (1-t)\, p_{B\prec A}(ab|xy)}$$

Nonsignalling from Bob to Alice:

$$\sum_b p_{A\prec B}(ab|xy) = \sum_b p_{A\prec B}(ab|xy') \quad \forall\ a, x, y, y'$$

Nonsignalling from Alice to Bob:

$$\sum_a p_{B\prec A}(ab|xy) = \sum_a p_{B\prec A}(ab|x'y) \quad \forall\ b, y, x, x'$$

$$p(ab|xy) \in \mathrm{C}$$ **= Causal correlations**

The GYNI game

Causal strategies

$$p(ab|xy) = t\, p_{A\prec B}(ab|xy) + (1-t)\, p_{B\prec A}(ab|xy)$$

Nonsignalling from Bob to Alice:

$$p_{A\prec B}(ab|xy) = p^{(1)}_{A\prec B}(a|xy)p^{(2)}_{A\prec B}(b|xya) = p^{(1)}_{A\prec B}(a|x)p^{(2)}_{A\prec B}(b|xya)$$

Nonsignalling from Alice to Bob:

$$p_{B\prec A}(ab|xy) = p^{(1)}_{B\prec A}(b|xy)p^{(2)}_{B\prec A}(a|xyb) = p^{(1)}_{B\prec A}(b|y)p^{(2)}_{B\prec A}(a|xyb)$$

$$p(ab|xy) \in \mathrm{C}$$ **= Causal correlations**

The GYNI game

Causal strategies in the GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}\, p(ab|xy)$$

The GYNI game

Causal strategies in the GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}\, p(ab|xy)$$

$$= \sum_{a,b,x,y} q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}\, [t\, p_{A\prec B}(ab|xy) + (1-t)\, p_{B\prec A}(ab|xy)]$$

The GYNI game

Causal strategies in the GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}\,p(ab|xy)$$

$$= \sum_{a,b,x,y} q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}\,[t\,p_{A\prec B}(ab|xy) + (1-t)\,p_{B\prec A}(ab|xy)]$$

$$= \sum_{a,b,x,y} q(x,y)\,\delta(a,y)\,\delta(b,x)\left[t\,p^{(1)}_{A\prec B}(a|x)p^{(2)}_{A\prec B}(b|xya) + (1-t)\,p^{(1)}_{B\prec A}(b|y)p^{(1)}_{B\prec A}(a|xyb)\right]$$

The GYNI game

Causal strategies in the GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}\,p(ab|xy)$$

$$= \sum_{a,b,x,y} q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}\,[t\,p_{A\prec B}(ab|xy) + (1-t)\,p_{B\prec A}(ab|xy)]$$

$$= \sum_{a,b,x,y} q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}\left[t\,p^{(1)}_{A\prec B}(a|x)p^{(2)}_{A\prec B}(b|xya) + (1-t)p^{(1)}_{B\prec A}(b|y)p^{(2)}_{B\prec A}(a|xyb)\right]$$

$$= \sum_{x,y} q(x,y)\left[t\,p^{(1)}_{A\prec B}(a=y|x)p^{(2)}_{A\prec B}(b=x|xy) + (1-t)\,p^{(1)}_{B\prec A}(b=x|y)p^{(1)}_{B\prec A}(a=y|xy)\right]$$

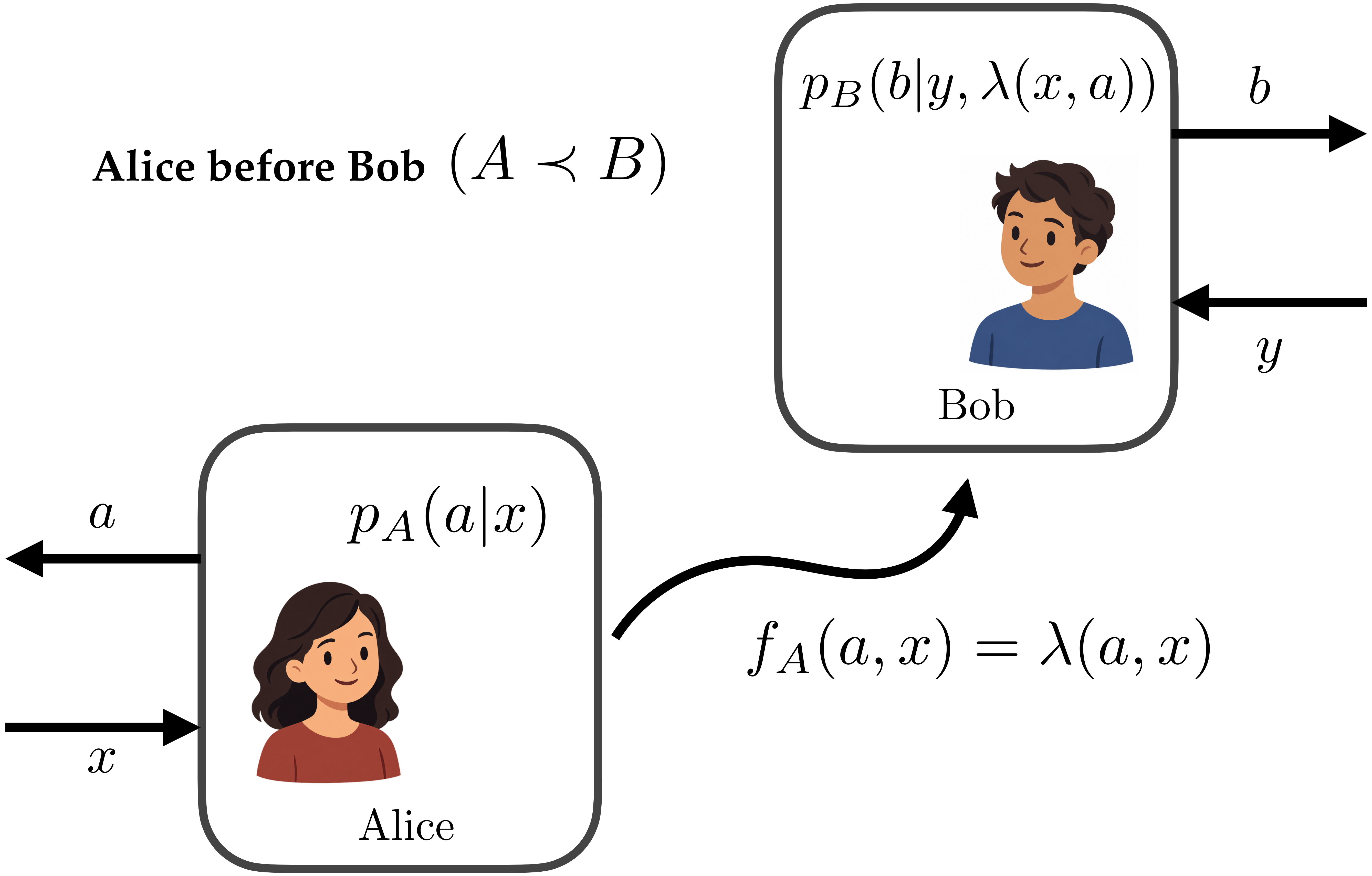
$$= \sum_{x,y} q(x,y)\left[t\,\frac{1}{2}\cdot 1 + (1-t)\,\frac{1}{2}\cdot 1\right] = \frac{1}{2}$$

The GYNI game

$$p(ab|xy) = t\, p_{A \prec B}(ab|xy) + (1-t)\, p_{B \prec A}(ab|xy)$$

Is there a classical versus quantum separation in causal correlations?

Alice before Bob $(A \prec B)$



CLASSICAL

classical local processing functions

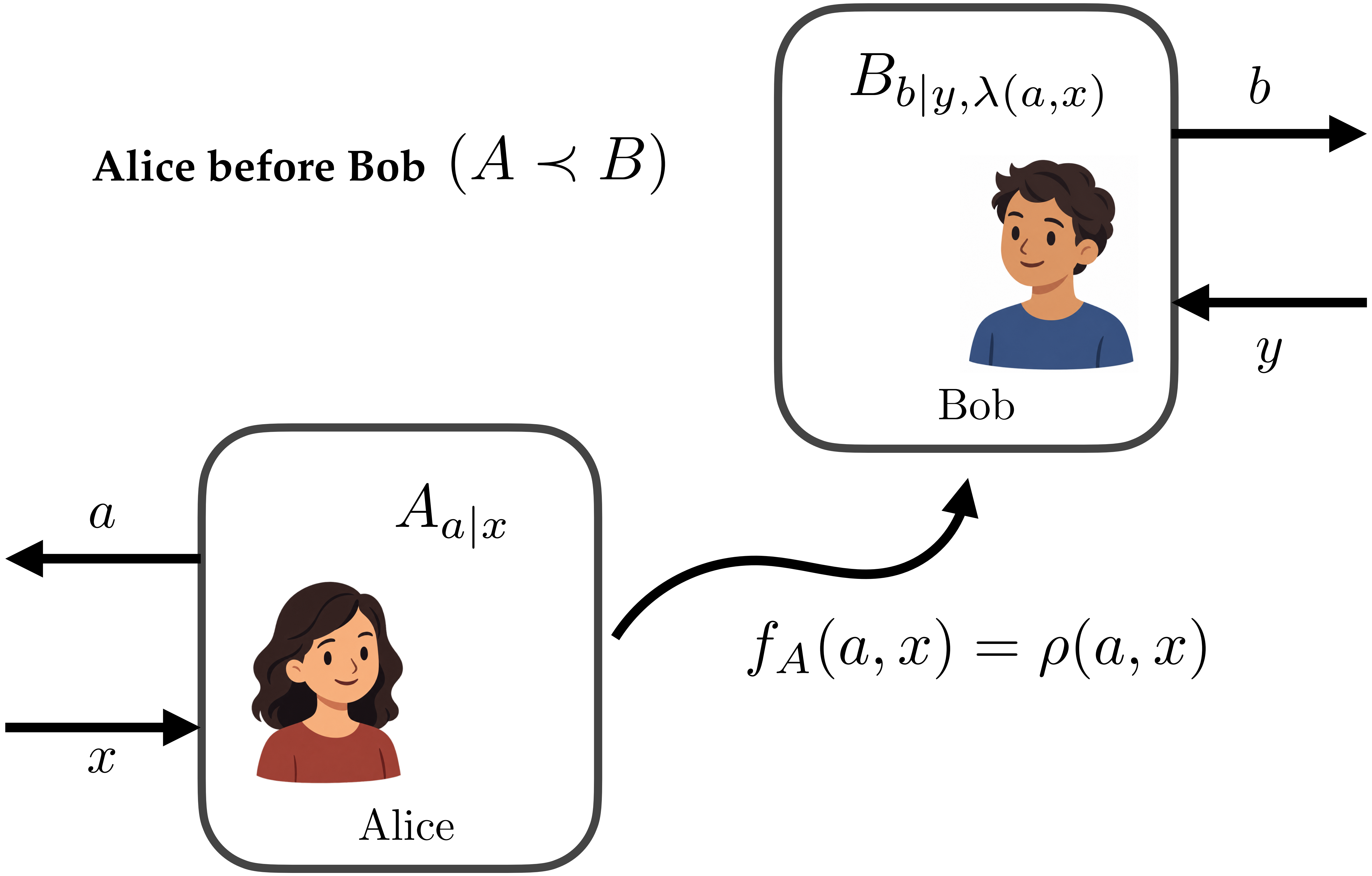
classical communication

The GYNI game

$$p(ab|xy) = t\, p_{A \prec B}(ab|xy) + (1-t)\, p_{B \prec A}(ab|xy)$$

Is there a classical versus quantum separation in causal correlations?

Alice before Bob $(A \prec B)$



QUANTUM

quantum local processing functions

quantum communication

The GYNI game

$$p(ab|xy) = t\, p_{A \prec B}(ab|xy) + (1-t)\, p_{B \prec A}(ab|xy)$$

Is there a classical versus quantum separation in causal correlations?

$\boxed{\textbf{No.}}$

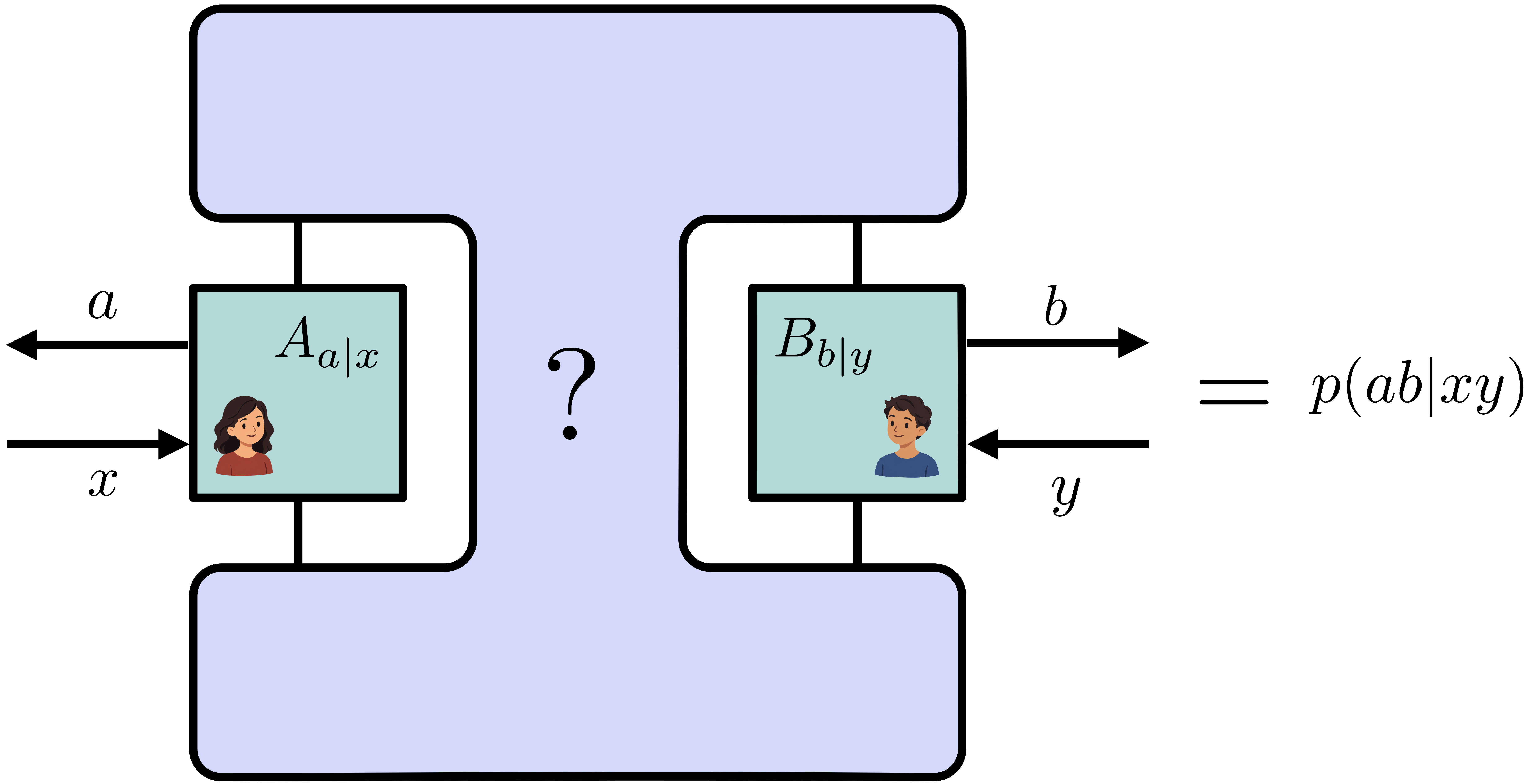
There is no classical vs. quantum separation in causal communication scenarios:
All causal correlations can be produced by
classical local processing functions and classical communication.

$\Longrightarrow$ **No quantum advantage in causal scenarios.**

The GYNI game

What about more general scenarios?

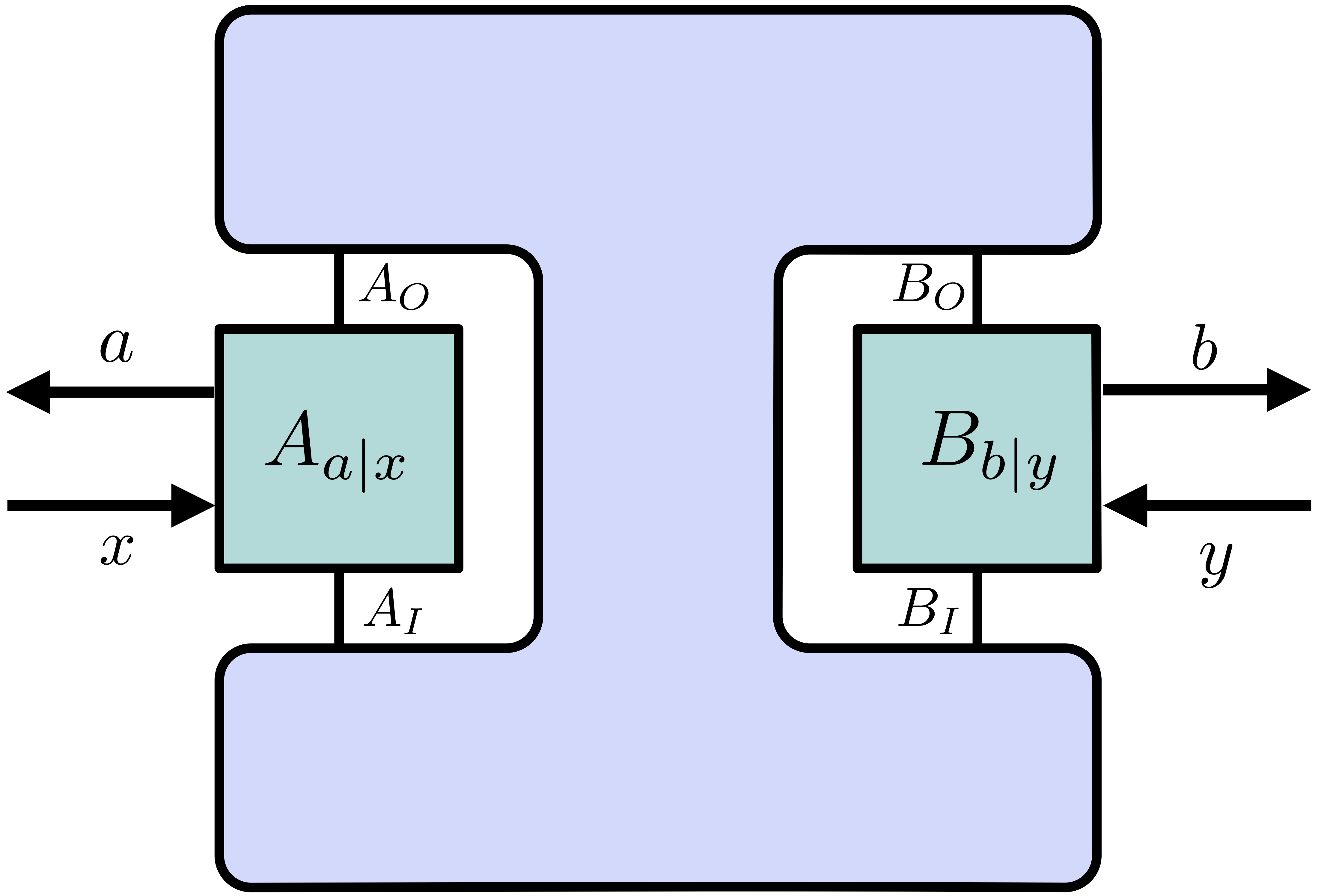
Higher-order transformations



The GYNI game

Higher-order quantum transformations

Let us start by assuming quantum theory locally but not imposing a fixed order between Alice and Bob.



$$= p(ab|xy)$$

Most general quantum local operations:

Quantum instruments

$$A_{a|x} \in \mathcal{L}(\mathcal{H}_{A_I} \otimes \mathcal{H}_{A_O})$$

$$A_{a|x} \geq 0,$$

$$\sum_a A_{a|x} = C_x^A \text{ is CPTP}$$

$$B_{b|y} \in \mathcal{L}(\mathcal{H}_{B_I} \otimes \mathcal{H}_{B_O})$$

$$B_{b|y} \geq 0,$$

$$\sum_b B_{b|y} = C_y^B \text{ is CPTP}$$

= Probabilistic quantum channels

Oreshkov et al., Nat. Commun. (2012)

The GYNI game

Higher-order quantum transformations

Let us start by assuming quantum theory locally but not imposing a fixed order between Alice and Bob.

What is the most general bi-linear function f that maps all sets of independent instruments into a probability distribution?

$$f : \mathcal{L}(\mathcal{H}_{A_I} \otimes \mathcal{H}_{A_O} \otimes \mathcal{H}_{B_I} \otimes \mathcal{H}_{B_O}) \to \mathbb{C}$$

$$p(ab|xy) = f(A_{a|x} \otimes B_{b|y})$$

$$\Big\Downarrow \quad \textbf{(Riesz representation lemma)}$$

$$p(ab|xy) = \mathrm{tr}\left[(A_{a|x} \otimes B_{b|y})\, W\right] \qquad \textbf{(Generalized Born rule)}$$

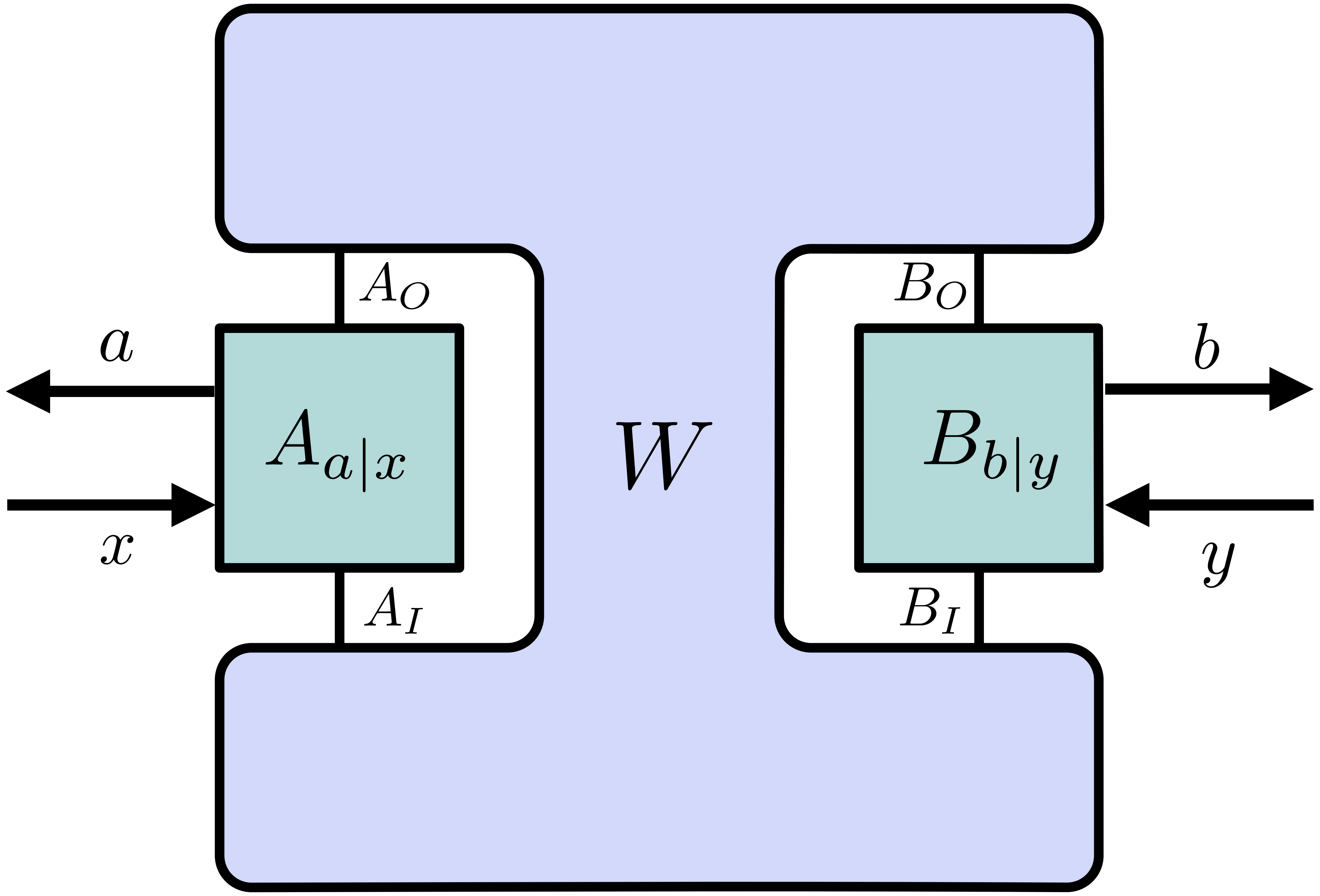
$$W \in \mathcal{L}(\mathcal{H}_{I_A} \otimes \mathcal{H}_{O_A} \otimes \mathcal{H}_{I_B} \otimes \mathcal{H}_{O_B})$$

Oreshkov et al., Nat. Commun. (2012)

The GYNI game

Higher-order quantum transformations

Let us start by assuming quantum theory locally but not imposing a fixed order between Alice and Bob.



$= p(ab|xy)$

$$W \in \mathcal{L}(\mathcal{H}_{I_A} \otimes \mathcal{H}_{O_A} \otimes \mathcal{H}_{I_B} \otimes \mathcal{H}_{O_B})$$

$$W \geq 0$$

$$\mathrm{tr}(W) = d_{A_O} d_{B_O}$$

$$W = L_V(W)$$

Process matrix

$${}_X W := \mathrm{tr}_X(W) \otimes \frac{\mathbb{1}}{d_X}, \qquad L_V(W) = {}_{A_O B_I B_O}W - {}_{B_I B_O}W + {}_{A_I A_O B_O}W - {}_{A_I A_O}W + {}_{A_O}W + {}_{B_O}W - {}_{B_O A_O}W$$

Araújo et al., NJP (2015)

The GYNI game

Higher-order quantum transformations

Can a strategy that involves **local quantum instruments** for Alice and Bob and whose environment in is described by a **process matrix** win the GYNI with a higher probability than **causal strategies**?

$$p(ab|xy) = \mathrm{tr}\left[\left(A_{a|x} \otimes B_{b|y}\right) W\right]$$

$$p(ab|xy) \in \mathrm{QW}$$

The GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}\, p(ab|xy)$$

$$= \sum_{a,b,x,y} q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}\, \mathrm{tr}\left[\left(A_{a|x} \otimes B_{b|y}\right) W\right]$$

$$= \dots$$ [Branciard et al., NJP (2016)]

$$\approx 0.6218 > \frac{1}{2} \quad \Longrightarrow$$

Process matrix strategies can violate the causal bound

Branciard et al., NJP (2016)

The GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\, \delta_{(a,y)}\, \delta_{(b,x)}\, p(ab|xy) \leq \frac{1}{2} < 0.62\ldots$$

for causal correlations

achievable by process matrix correlations

Is there a strategy that can win the game perfectly?
$(\mathcal{S} = 1)$

The GYNI game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}\,p(ab|xy) \le \underset{\substack{= \\ \frac{1}{2}}}{\mathcal{B}_{\mathrm{C}}} \le \underset{\substack{\le \\ 0.76}}{\mathcal{B}_{\mathrm{QW}}} \le 1$$

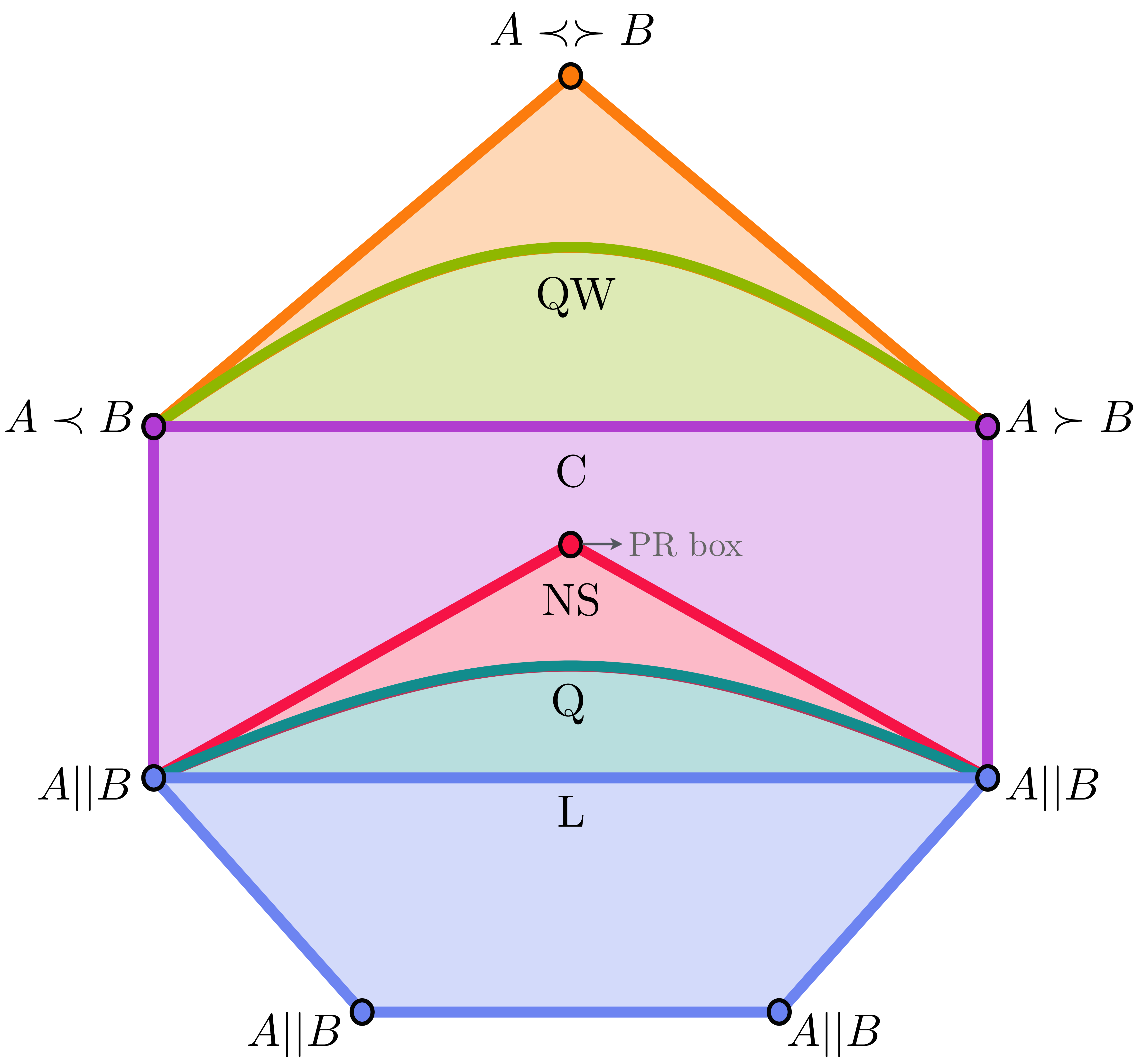
causal: $\mathcal{B}_{\mathrm{C}} := \max_{p \in \mathrm{C}} \mathcal{S}$ $\qquad p(ab|xy) = t\,p_{A \prec B}(ab|xy) + (1-t)\,p_{B \prec A}(ab|xy)$

process matrix: $\mathcal{B}_{\mathrm{QW}} := \max_{p \in \mathrm{QW}} \mathcal{S}$ $\qquad p(ab|xy) = \mathrm{tr}\left[(A_{a|x} \otimes B_{b|y})\,W\right]$

Winning with probability 1 is **not** possible even with indefinite causal order.

Bavaresco et al., Quantum (2019); Zixuan & Chiribella, Nat. Commun. (2025)

The polytope of all correlations



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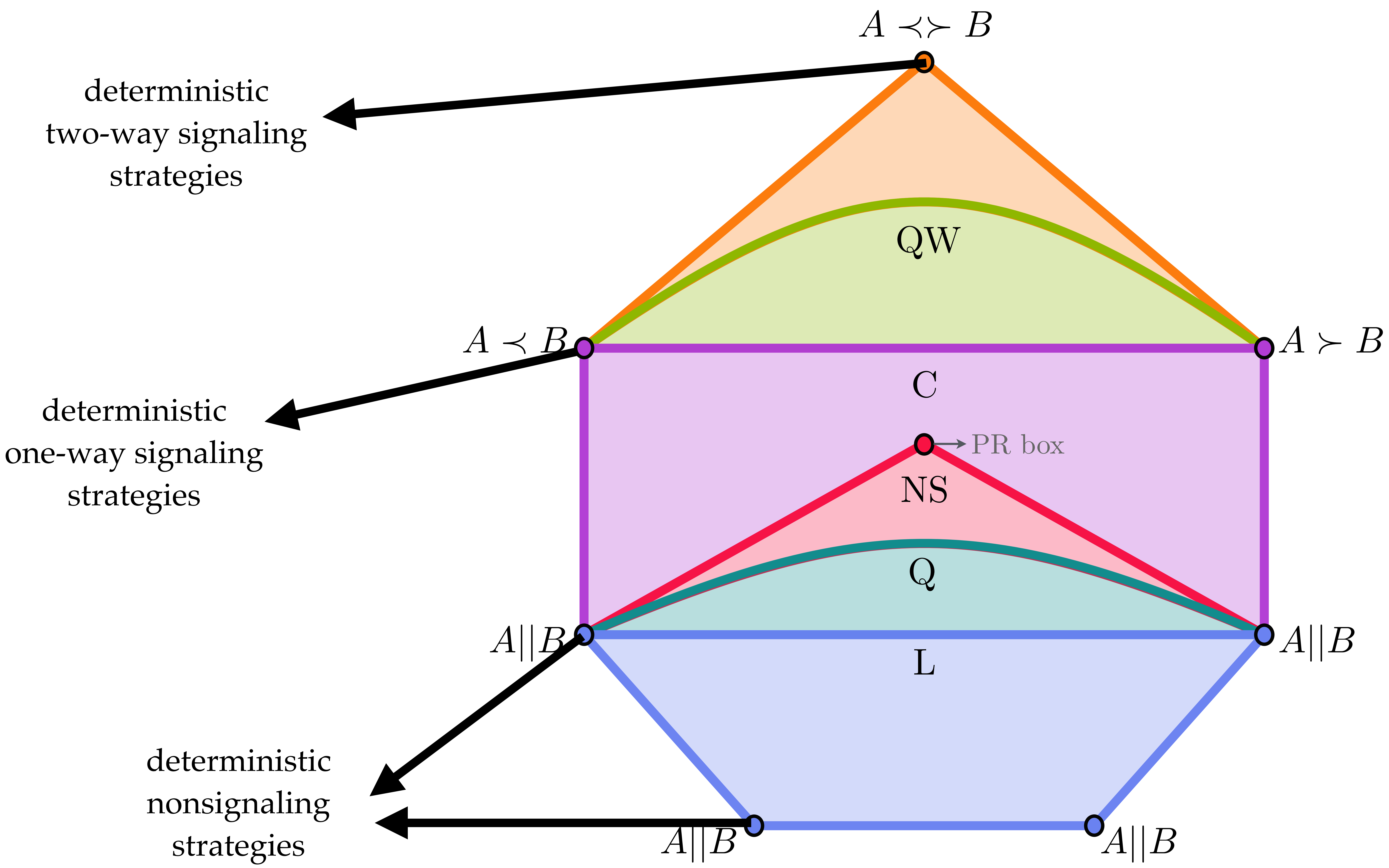
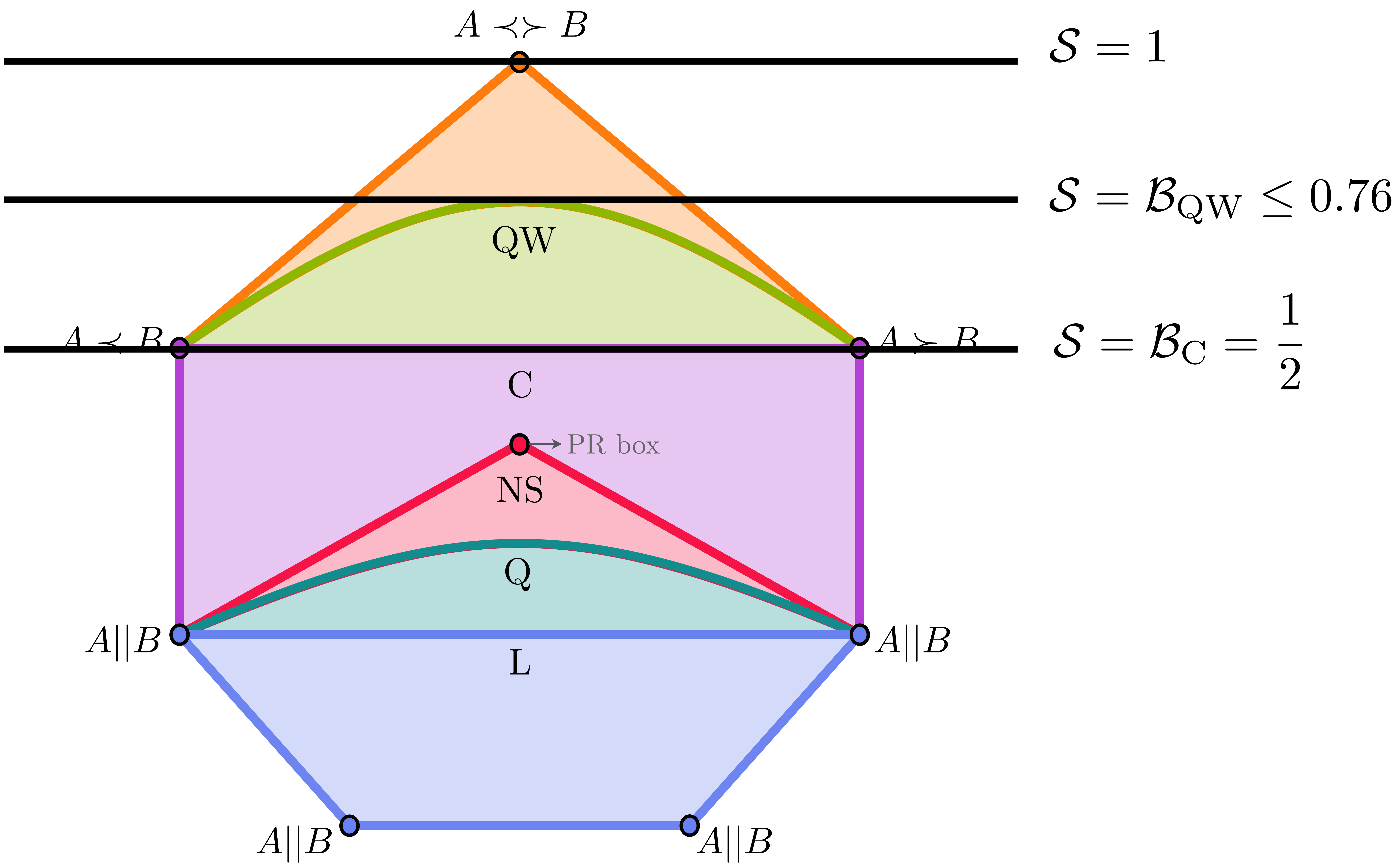


The polytope of all correlations



Causal inequalities

GYNI inequality:

$$\sum_{a,b,x,y} \underbrace{q(x,y)\,\delta_{(a,y)}\,\delta_{(b,x)}}_{} \, p(ab|xy) \leq \frac{1}{2}$$

$\big\downarrow$ coefficients can be generalized

General causal inequalities:

$$\boxed{\mathcal{S} = \sum_{a,b,x,y} \alpha_{xy}^{ab}\, p(ab\,|\,xy) \leq \mathcal{B}_{\mathrm{C}}}$$

$$\alpha_{xy}^{ab} \in \mathbb{R} \ \ \forall\ a,b,x,y$$

$$\mathcal{B}_{\mathrm{C}} := \max_{p \in \mathrm{C}} \sum_{a,b,x,y} \alpha_{xy}^{ab}\, p(ab|xy)$$

Causal vs. noncausal correlations

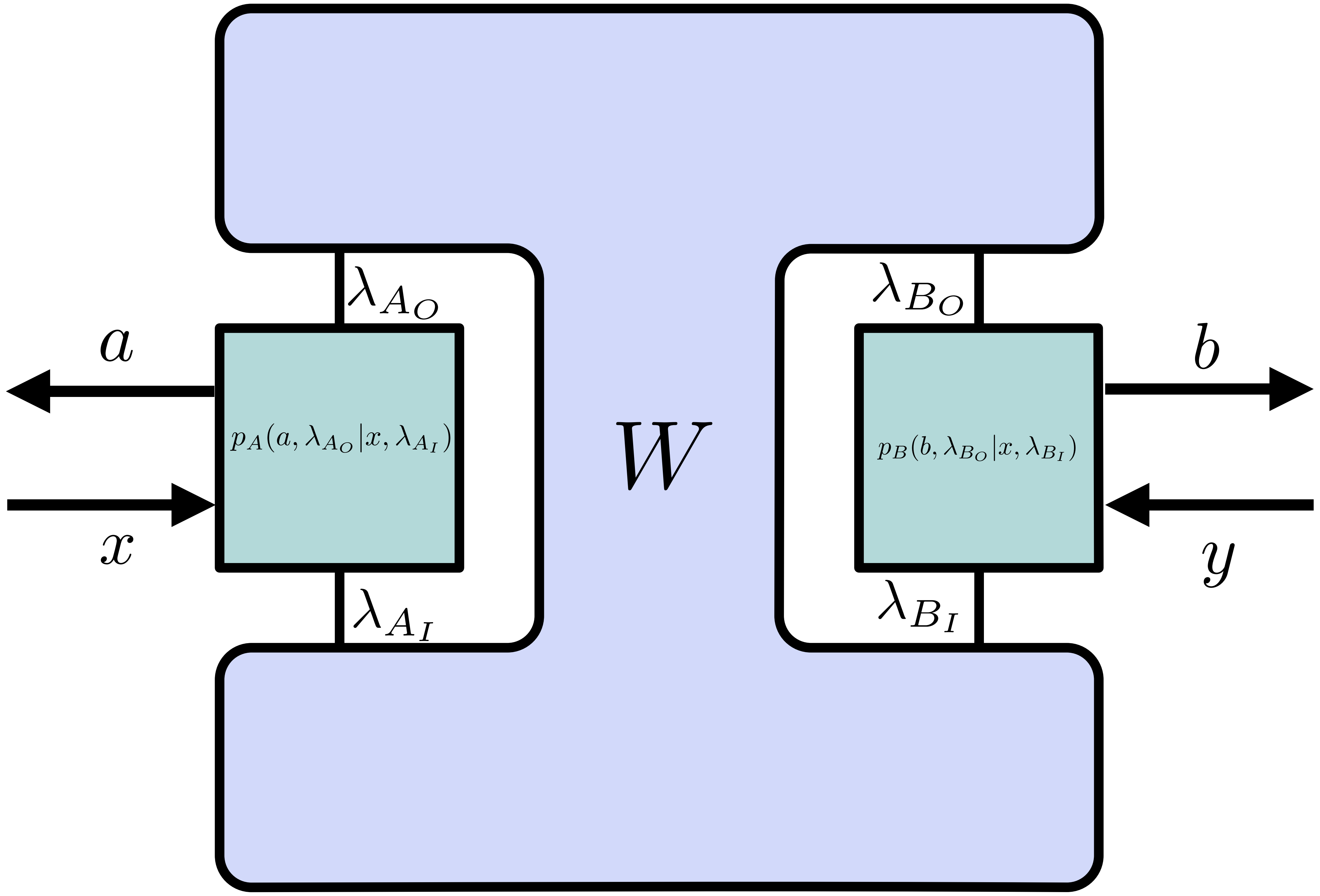
There is a separation between causal correlations and correlations from process matrix strategies.

But how are these related to quantum vs. classical correlations?

Causal vs. noncausal correlations

Higher-order classical transformations

Let us consider a scenario where Alice and Bob apply classical local processing:



$= \; p(ab|xy)$

For classical local processing,
all process matrices are effectively
diagonal in a local product bases.

classical process matrix

=

diagonal quantum process matrix

Causal vs. noncausal correlations

classical process matrix
+ classical local processing $\Longrightarrow$ always causal correlations

$\boxed{\text{classical} \Longrightarrow \text{causal}}$

quantum process matrix
+ quantum local processing $\Longrightarrow$ achievable noncausal correlations

$\boxed{\text{quantum} \Longleftarrow \text{noncausal}}$

Causal vs. noncausal correlations

The causal inequalities can be used to witness
a separation between **classical** and **quantum** correlations
in signaling, **higher-order** scenarios.

Causal vs. noncausal correlations

The causal inequalities can be used to witness
a separation between **classical** and **quantum** correlations
in signaling, **higher-order** scenarios.

Higher-order quantum strategy = quantum strategy ?

Higher-order classical strategy = classical strategy ?

Causal vs. noncausal correlations

The causal inequalities can be used to witness
a separation between **classical** and **quantum** correlations
in signaling, **higher-order** scenarios.

Is this the case beyond bipartite scenarios?

Classical indefinite causal order

Classical indefinite causal order

classical process matrix
+ classical local processing $\Longrightarrow$ always
causal correlations

In BIPARTITE scenarios

Classical indefinite causal order

classical process matrix
+ classical local processing $\not\Longrightarrow$ **always causal correlations**

In TRIPARTITE scenarios

In scenarios with at least three parties, assuming
classical local processing and classical process matrixes (locally diagonal)
is **not sufficient** to guarantee that the correlations will be causal

Baumeler & Wolf, NJP (2016)

Classical indefinite causal order

classical process matrix
+ classical local processing

$\not\Longrightarrow$

always
causal correlations

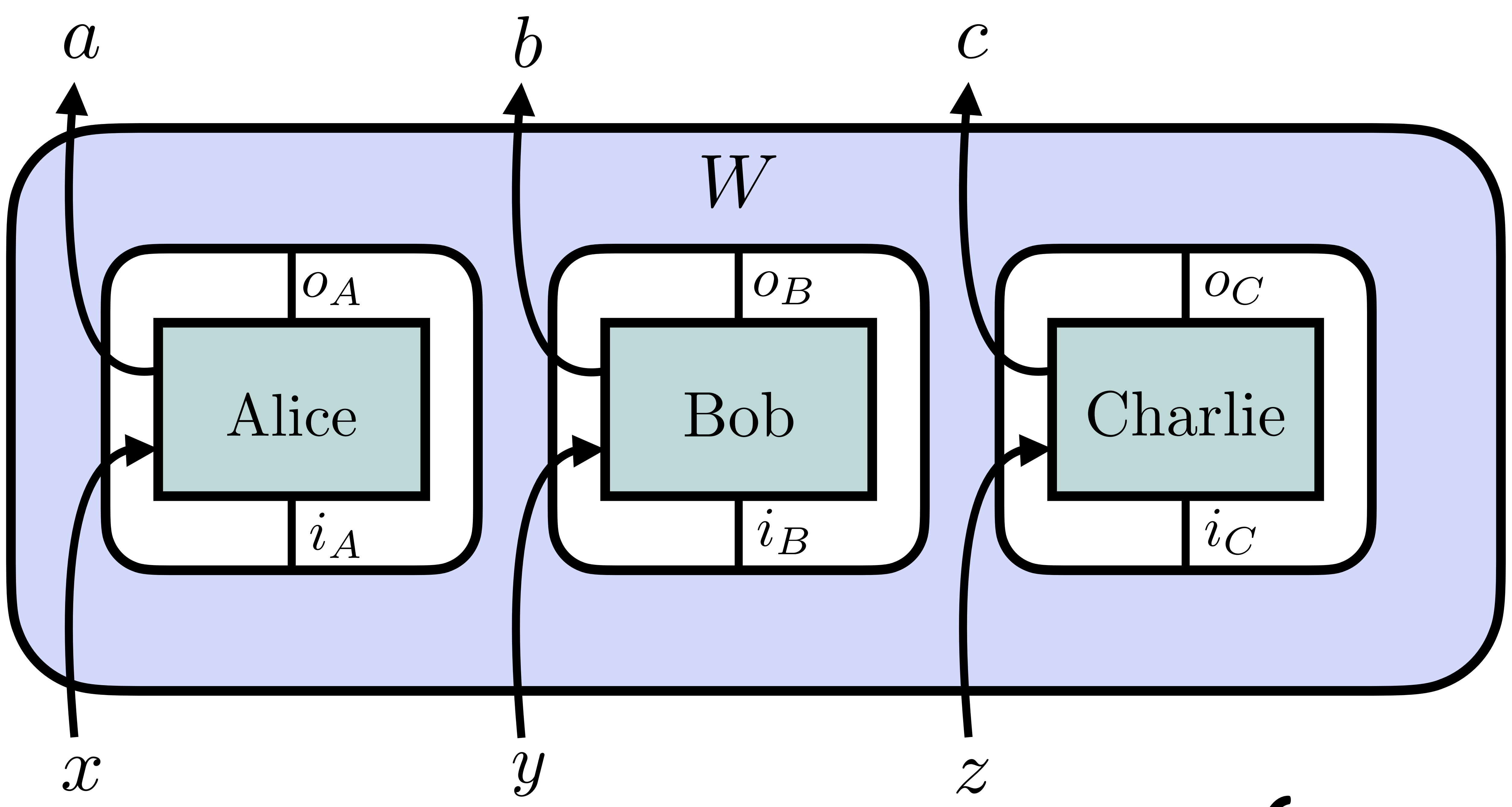
In TRIPARTITE scenarios

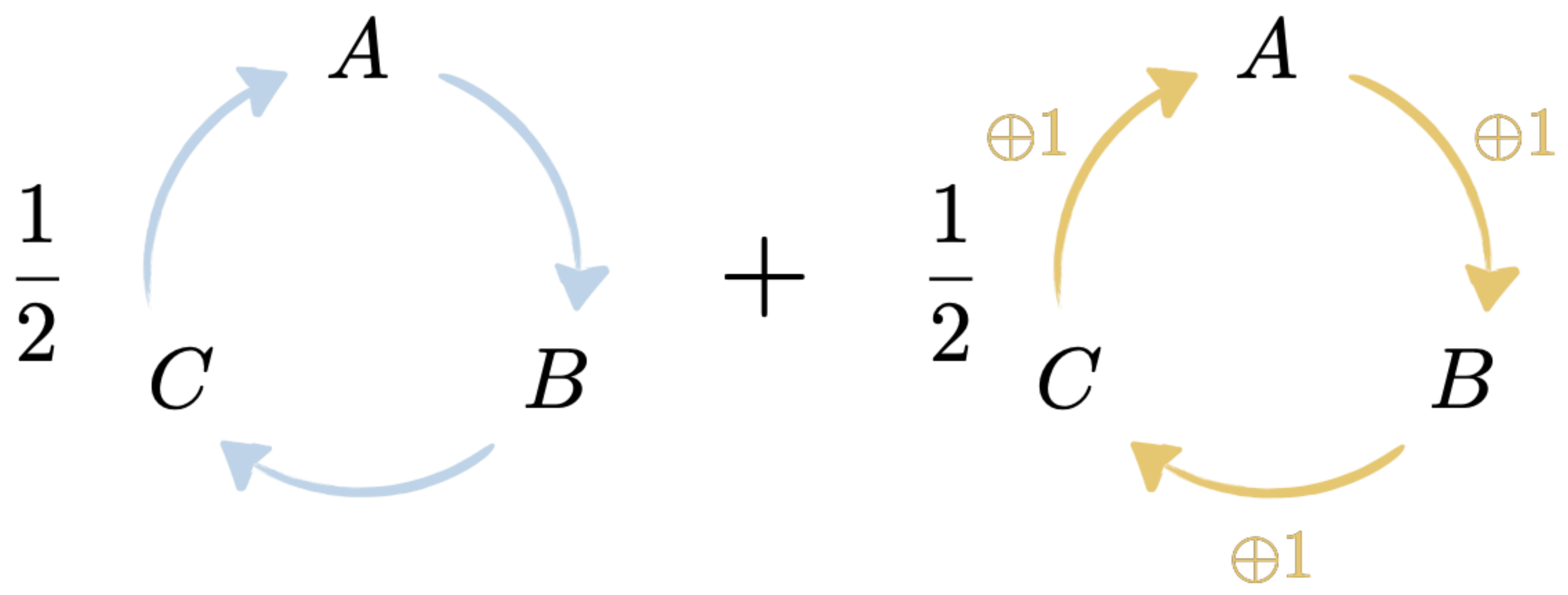
causal inequalities can be classically violated

In scenarios with at least three parties, assuming
classical local processing and classical process matrixes (locally diagonal)
is **not sufficient** to guarantee that the correlations will be causal

Baumeler & Wolf, NJP (2016)

AF/BW "Lugano" process



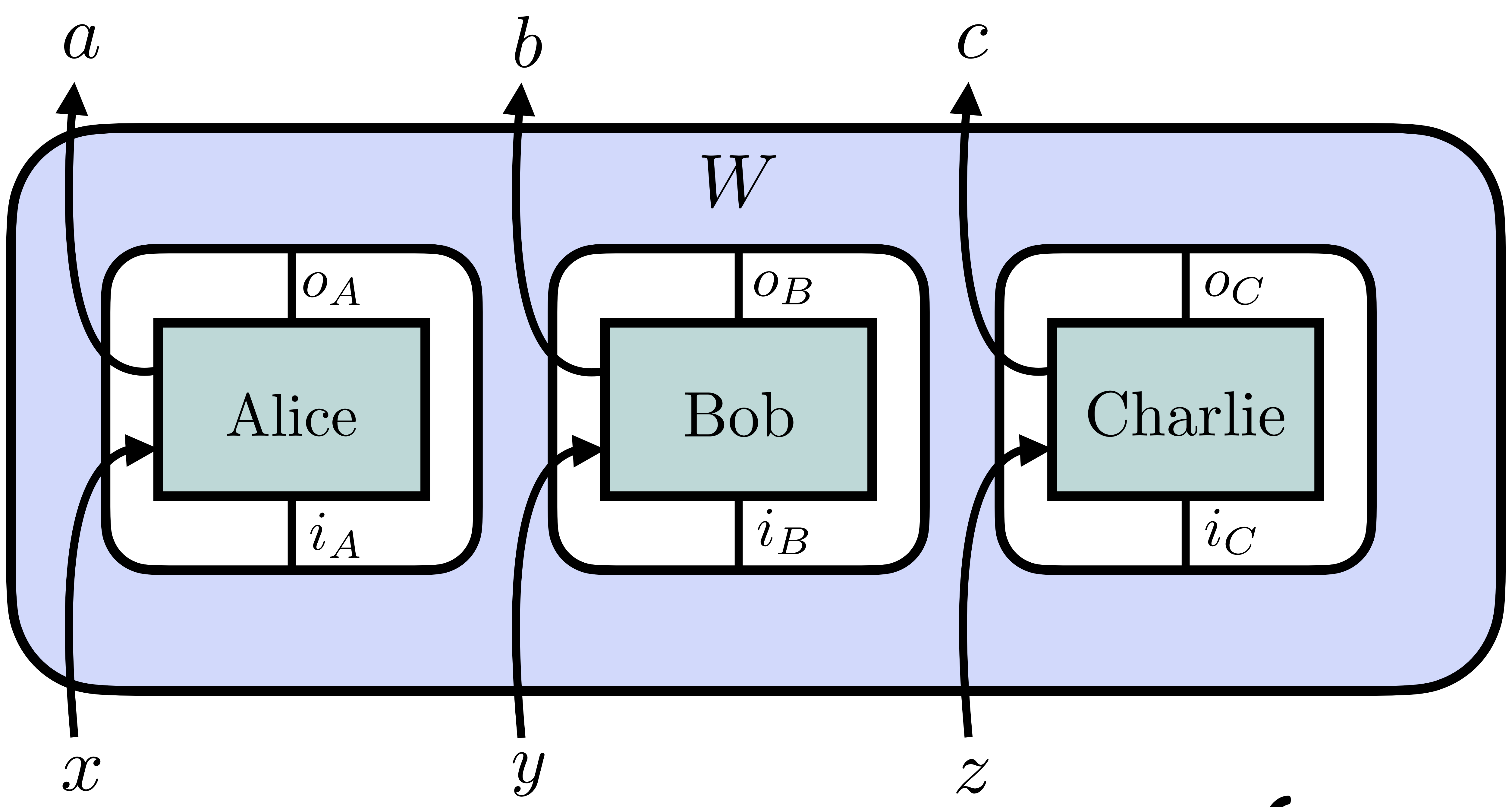


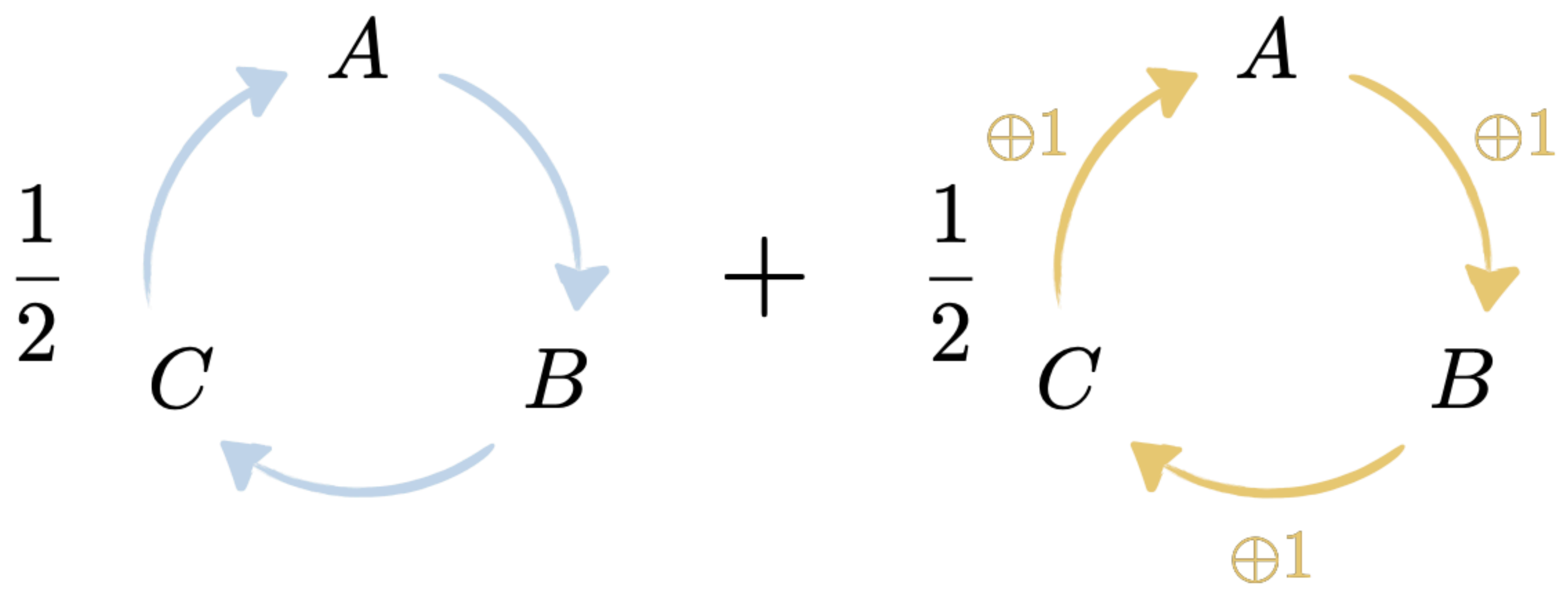
$$W = p(i_A, i_B, i_C | o_A, o_B, o_C) = \begin{cases} 1/2 & \text{if } i_A = o_C, i_B = o_A, i_C = o_B \\ 1/2 & \text{if } i_A = \bar{o}_C, i_B = \bar{o}_A, i_C = \bar{o}_B \\ 0 & \text{otherwise} \end{cases}$$

This classical process matrix can violate causal inequalities

Araújo et al., Quantum (2017); Baumeler & Wolf, NJP (2016)

AF/BW "Lugano" process





$$W = p(i_A, i_B, i_C | o_A, o_B, o_C) = \begin{cases} 1/2 & \text{if } i_A = o_C, i_B = o_A, i_C = o_B \\ 1/2 & \text{if } i_A = \bar{o}_C, i_B = \bar{o}_A, i_C = \bar{o}_B \\ 0 & \text{otherwise} \end{cases}$$

Classical communication with indefinite causal order

Araújo et al., Quantum (2017); Baumeler & Wolf, NJP (2016)

Perfect discrimination of the SHIFT basis

$$\{|000\rangle, |111\rangle, |{+01}\rangle, |{-01}\rangle, |1+0\rangle, |1-0\rangle, |01{+}\rangle, |01{-}\rangle\}$$

CANNOT be perfectly discriminated with measurements implemented via Local Operations and Classical Communication (LOCC)

CAN be perfectly discriminated with measurements implemented via Local Operations and Classical Communication with indefinite causal order (LOCC*)

Bennet et al., PRA (1998); Kunjwal & Baumeler, PRL (2023)

Quantum vs. classical separation in signaling scenarios

Bipartite scenarios:

Causal = classical

Noncausal = quantum

Tripartite scenarios:

Causal = classical

Noncausal = $\left\{ \begin{array}{l} \textbf{Noncausal classical communication} \\ \textbf{Noncausal quantum communication} \end{array} \right.$

Is there still a separation?

Yes.

Kunjwal & Oreshkov, arXiv (2023)

Part III. Quantum vs. classical correlations in signalling scenarios

MAIN MESSAGE:

Quantum and classical correlations can be distinguished in signaling scenarios.

However, classicality is not necessarily related to causality.

Thank you!