

VCQ Summer School 2026

Quantum vs. Classical Correlations: Parts I and II

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Course overview

- Part I. Quantum vs. classical correlations in nonsignalling scenarios
- Part II. Quantum vs. classical correlations in network scenarios
- Part III. Quantum vs. classical correlations in signalling scenarios

Part I overview

Part I. Quantum vs. classical correlations in nonsignalling scenarios.

- The CHSH game
 - Classical strategies (local hidden variable models)
 - Quantum strategies
 - Nonsignalling strategies
 - Bell inequalities and the polytope of correlations
- Nonlocality and entanglement
 - Entanglement certification
 - Local entangled states
 - Device-independent applications
- Single-shot quantum nonlocality

The CHSH game

The CHSH game



Alice



Bob

The CHSH game



Alice



Bob

The CHSH game

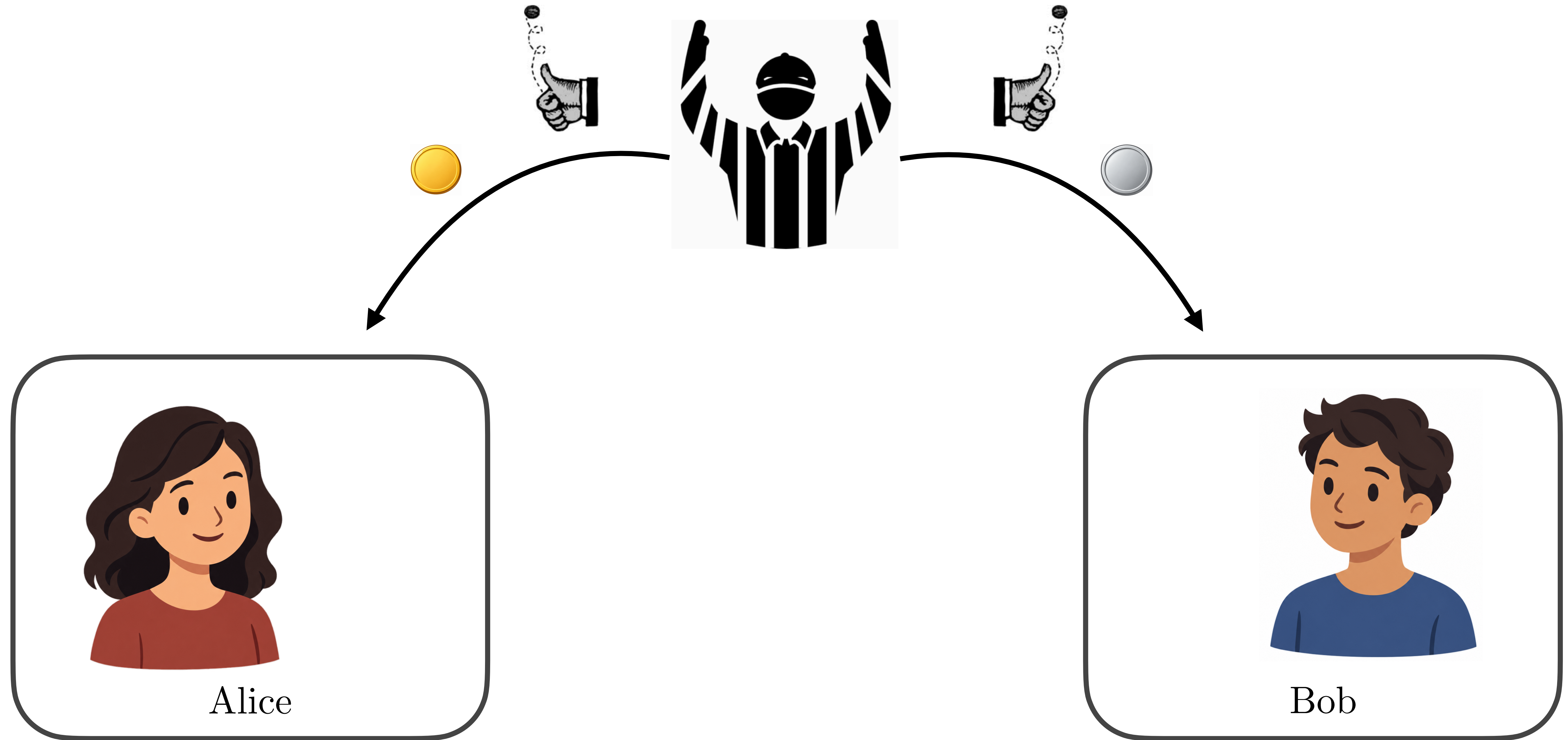


Alice

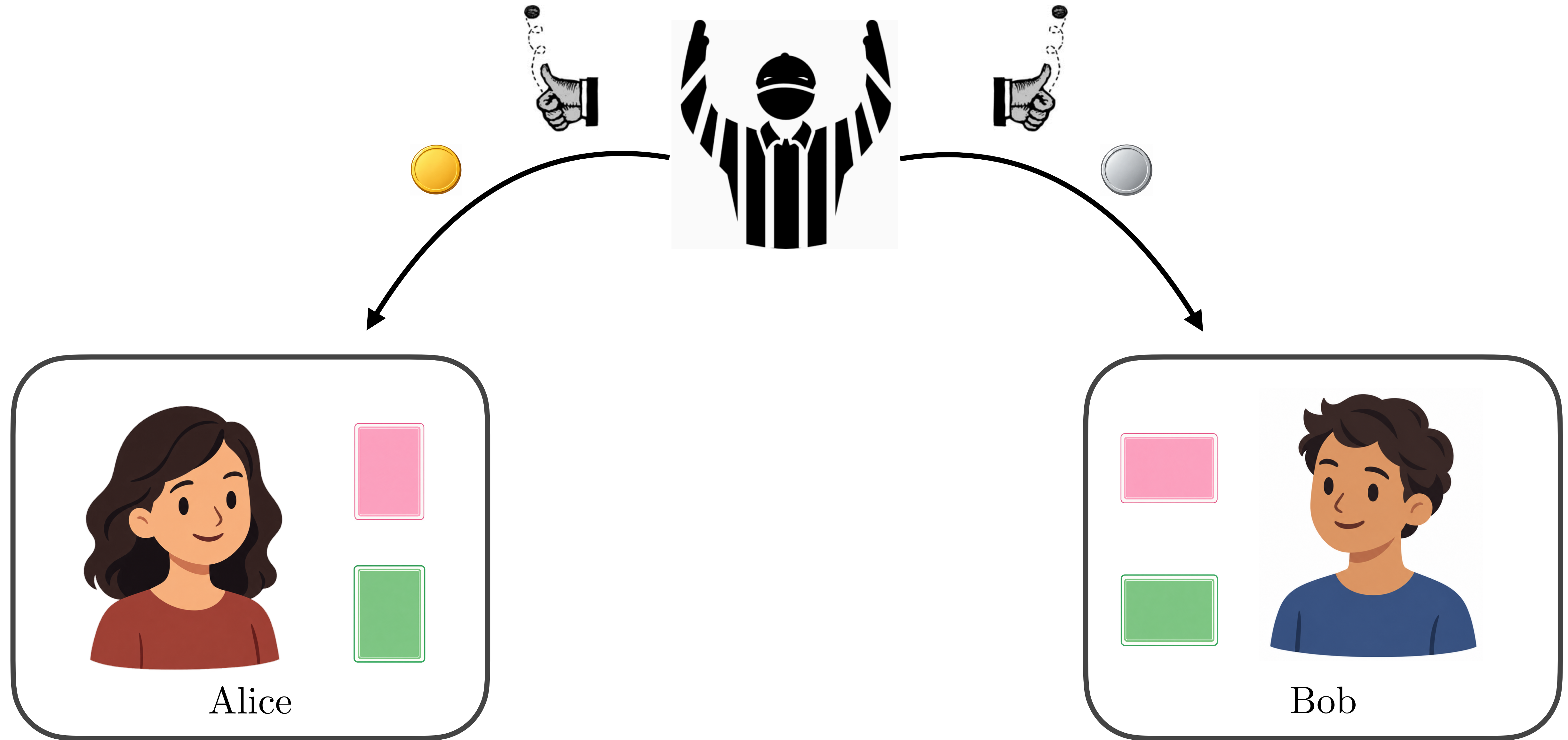


Bob

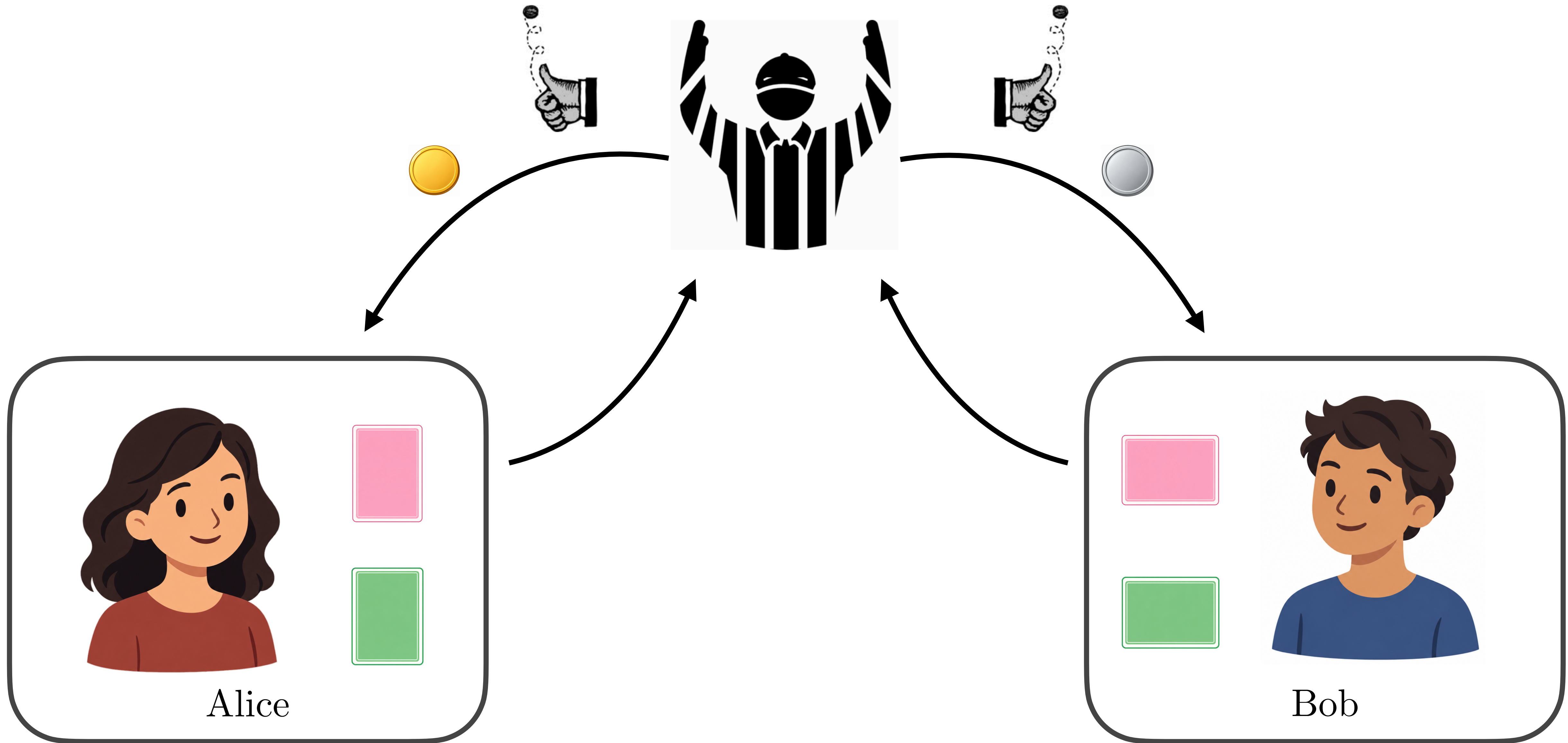
The CHSH game



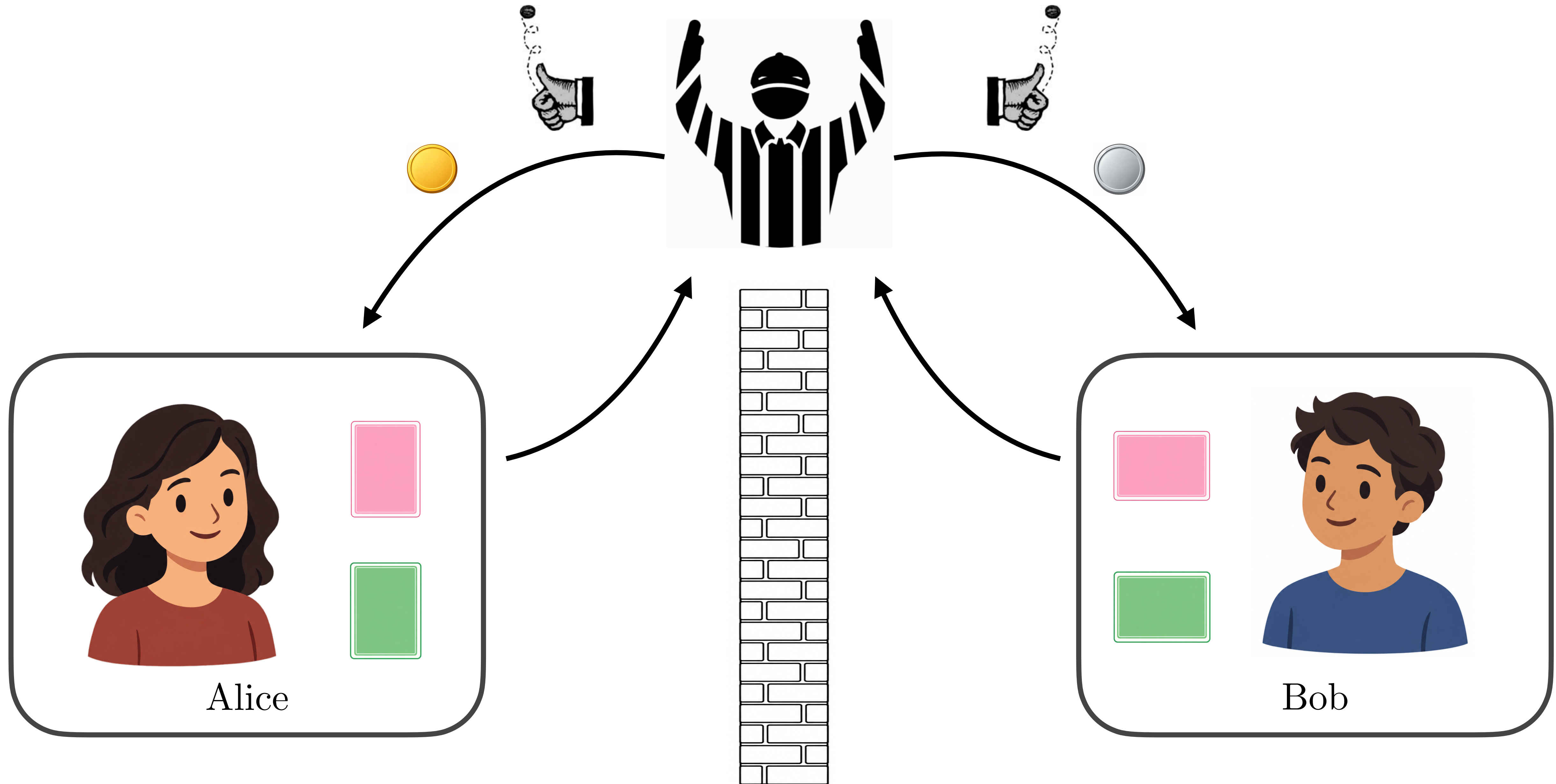
The CHSH game



The CHSH game



The CHSH game



Input

Alice

Bob



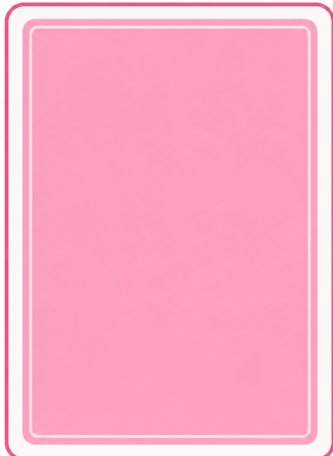
Winning conditions

Alice

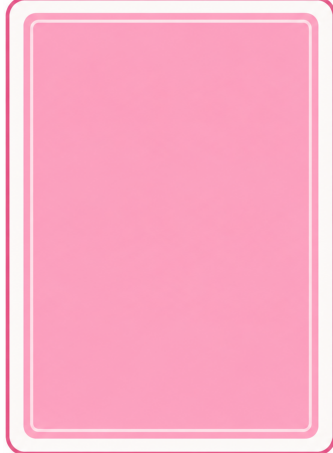
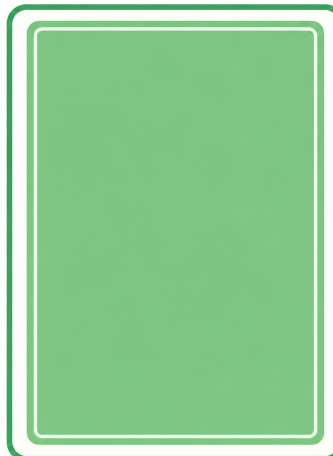
Bob

Alice

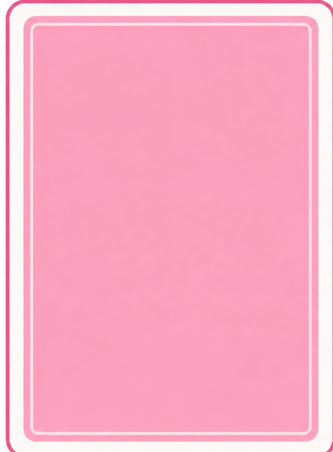
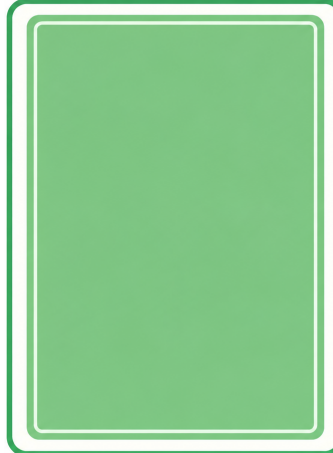
Bob



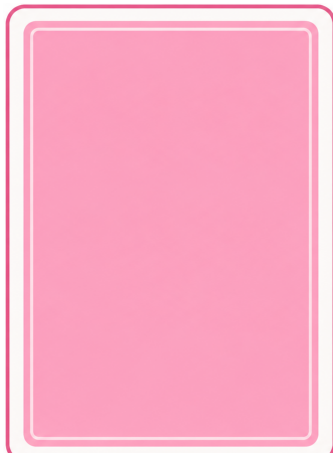
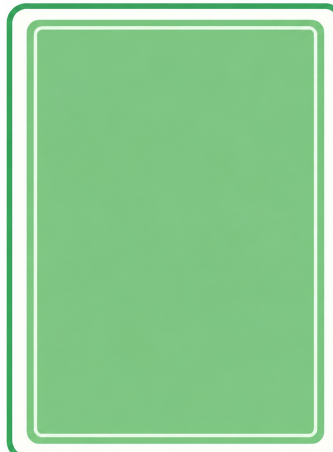
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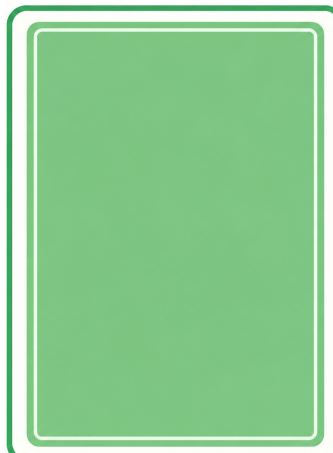
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or



or



Input

Alice

Bob



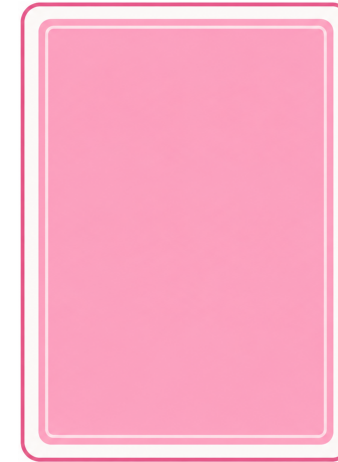
Winning conditions

Alice

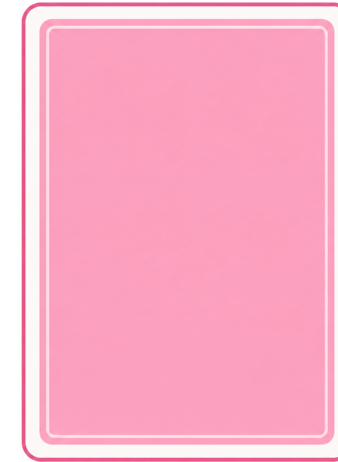
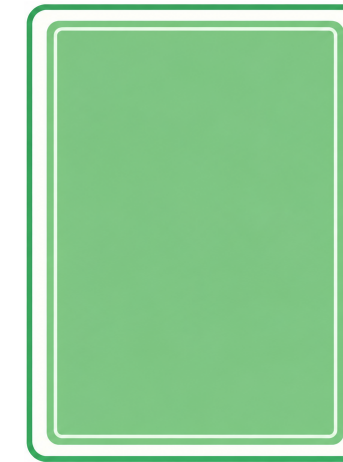
Bob

Alice

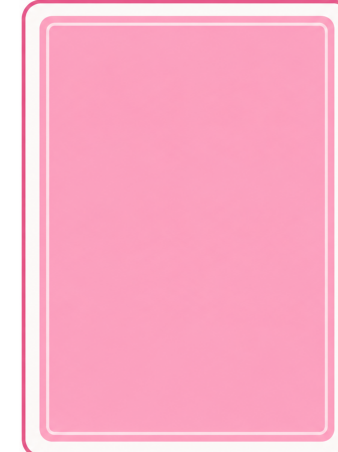
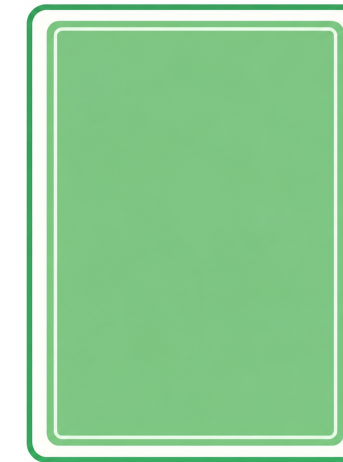
Bob



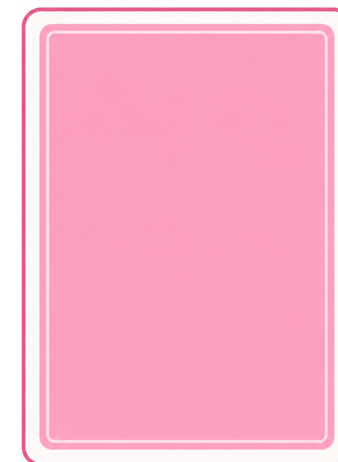
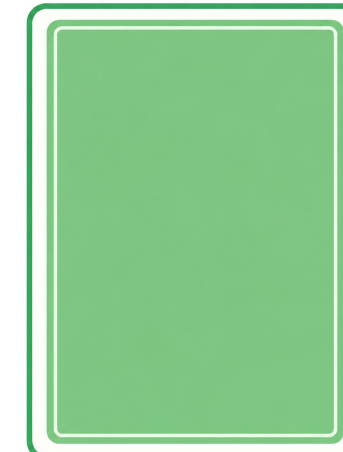
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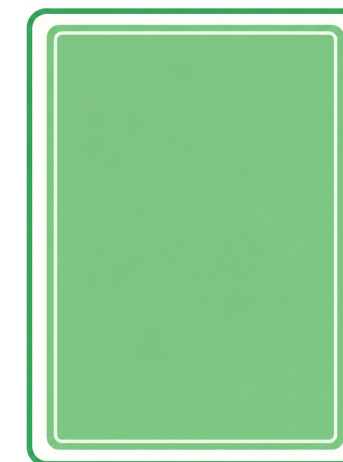
or



or



or



The CHSH game

Winning conditions

If the both receive TAILS $\longrightarrow$ output **DIFFERENT** colors

For any other input combination $\longrightarrow$ output **MATCHING** colors



Alice

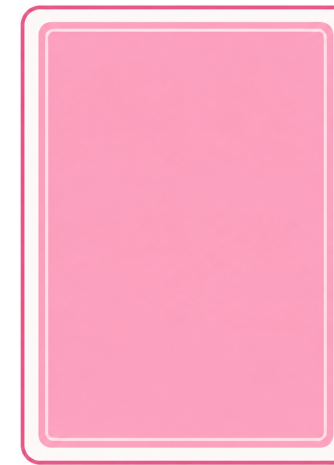
The CHSH game



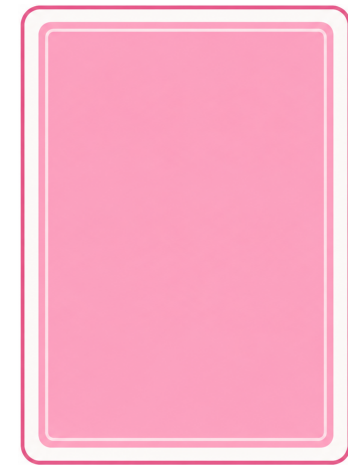
What strategy Alice and Bob can agree on before the game starts to give them the highest chance of winning each round of the game?

The CHSH game

Alice



and



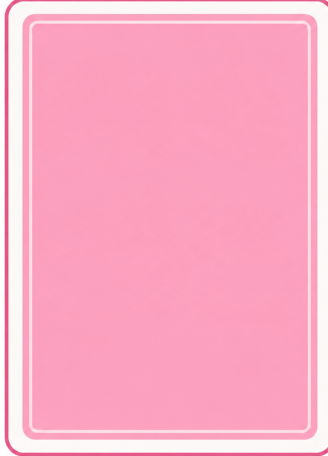
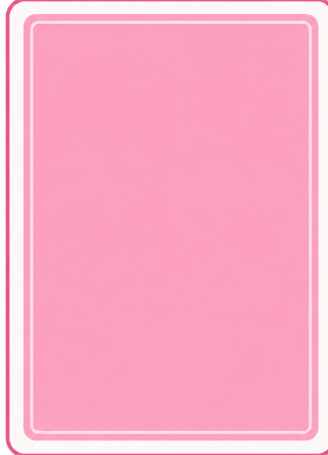
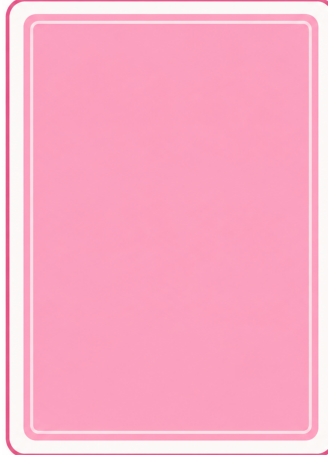
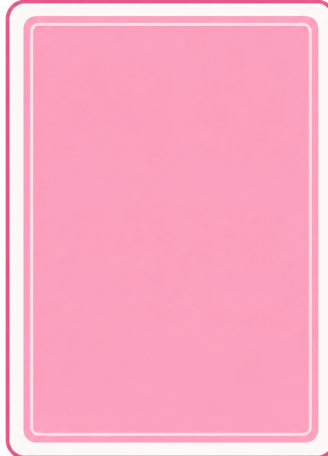
Bob



and



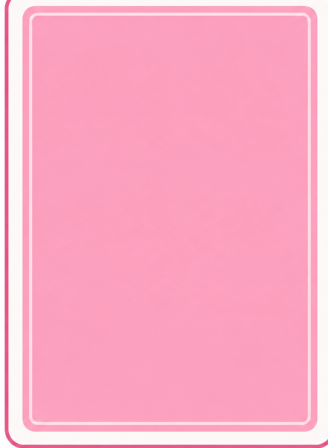
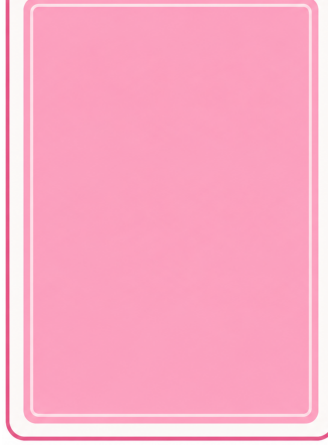
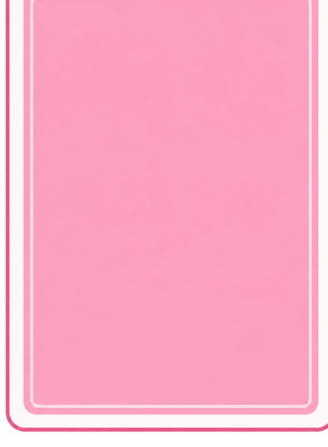
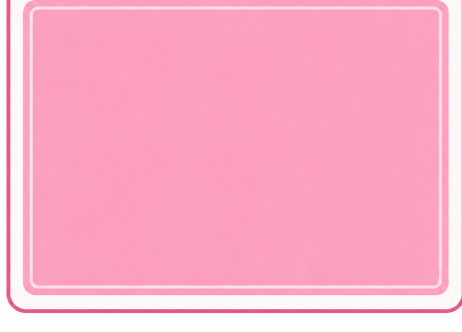
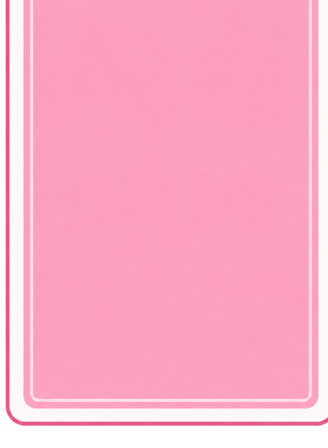
The CHSH game

Alice	Bob	Alice	Bob	
				
				
				
				

assuming fair coins:

**Win $3/4$ rounds
on average**

The CHSH game

Alice		Bob			
					assuming fair coins:
					Win $3/4$ rounds on average
					
					
					Can they do better?

The CHSH game

Can they do better?

$$x \in \{0, 1\}$$



$$x = 0$$



$$x = 1$$

$$y \in \{0, 1\}$$

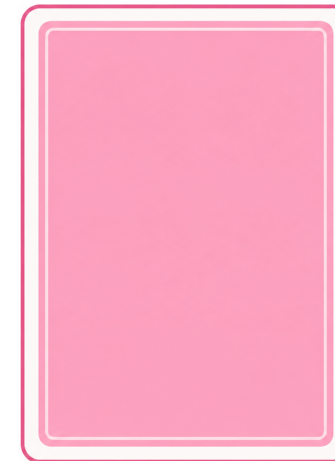


$$y = 0$$

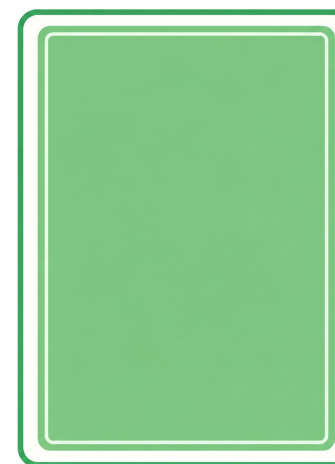


$$y = 1$$

$$a \in \{0, 1\}$$



$$a = 0$$



$$a = 1$$

$$b \in \{0, 1\}$$



$$b = 0$$



$$b = 1$$

Input

Alice

Bob



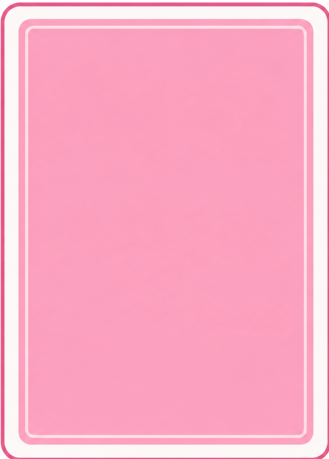
Winning conditions

Alice

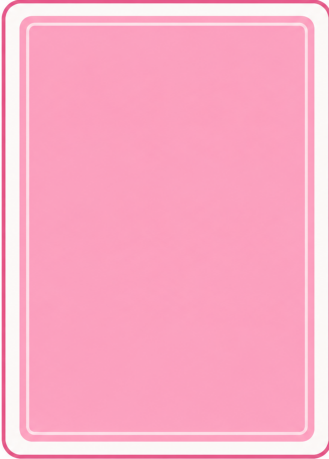
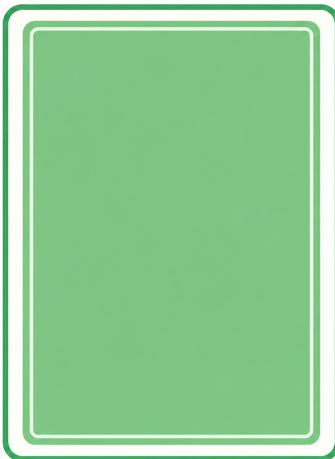
Bob

Alice

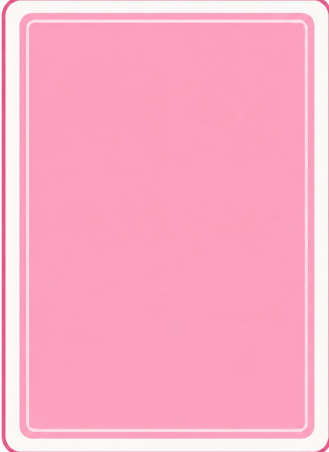
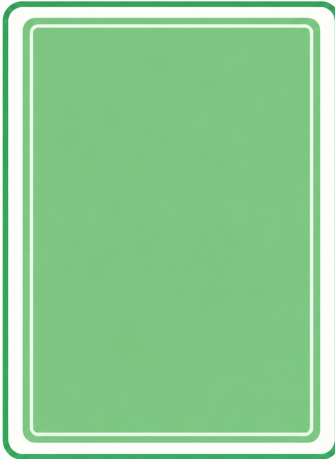
Bob



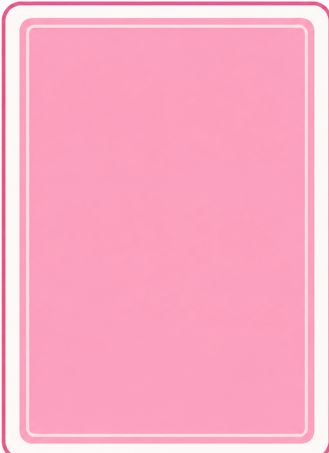
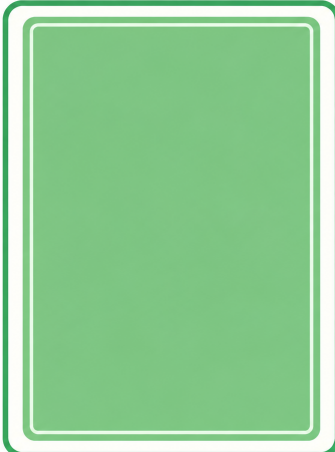
or



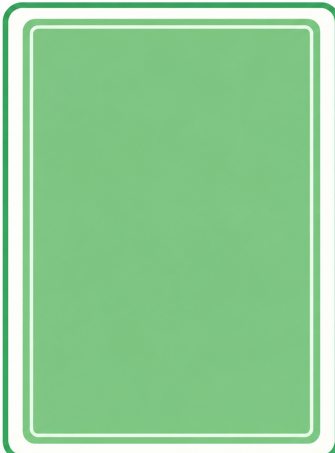
or



or



or



Input		Winning conditions			
Alice	Bob	Alice	Bob	Alice	Bob
$x = 0$	$y = 0$	$a = 0$	$b = 0$	or	$a = 1$ $b = 1$
$x = 0$	$y = 1$	$a = 0$	$b = 0$	or	$a = 1$ $b = 1$
$x = 1$	$y = 0$	$a = 0$	$b = 0$	or	$a = 1$ $b = 1$
$x = 1$	$y = 1$	$a = 0$	$b = 1$	or	$a = 1$ $b = 0$

The CHSH game

Winning condition:

$$a \oplus b = x \cdot y$$

The CHSH game

Winning condition:

$$a \oplus b = x \cdot y$$

Score:

$$\begin{aligned} \mathcal{S} = & q(x = 0, y = 0) \left[p(a = 0, b = 0 | x = 0, y = 0) + p(a = 1, b = 1 | x = 0, y = 0) \right] \\ & + q(x = 0, y = 1) \left[p(a = 0, b = 0 | x = 0, y = 1) + p(a = 1, b = 1 | x = 0, y = 1) \right] \\ & + q(x = 1, y = 0) \left[p(a = 0, b = 0 | x = 1, y = 0) + p(a = 1, b = 1 | x = 1, y = 0) \right] \\ & + q(x = 1, y = 1) \left[p(a = 0, b = 1 | x = 1, y = 1) + p(a = 1, b = 0 | x = 1, y = 1) \right] \end{aligned}$$

The CHSH game

Winning condition:

$$a \oplus b = x \cdot y$$

Score:

$$\begin{aligned}\mathcal{S} &= \sum_{x,y} q(x,y) p(a \oplus b = x \cdot y \mid xy) \\ &= \sum_{x,y} q(x,y) \sum_{a,b} \delta_{(a \oplus b, x \cdot y)} p(ab \mid xy)\end{aligned}$$

The CHSH game

Winning condition:

$$a \oplus b = x \cdot y$$

Score:

$$\mathcal{S} = \sum_{a,b,x,y} \underbrace{q(x,y) \delta_{(a \oplus b, x \cdot y)}}_{\text{settings of the game}} \underbrace{p(ab | xy)}_{\text{players' strategy}}$$

The CHSH game

Classical strategies

$$p(ab \mid xy)$$

What does it mean for Alice and Bob's strategy to be **classical**?

The CHSH game

Classical strategies

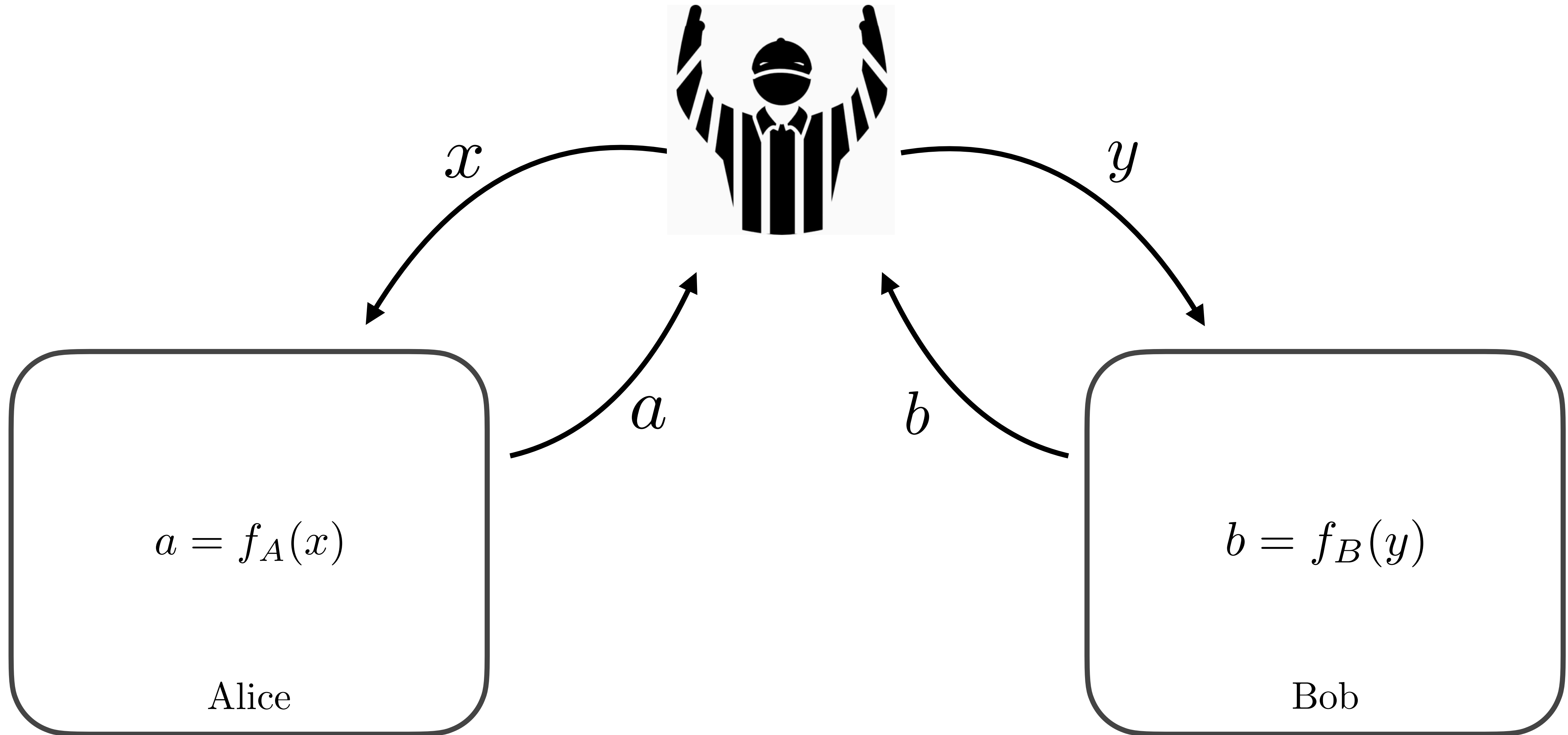
$$p(ab \mid xy)$$

What does it mean for Alice and Bob's strategy to be **classical**?

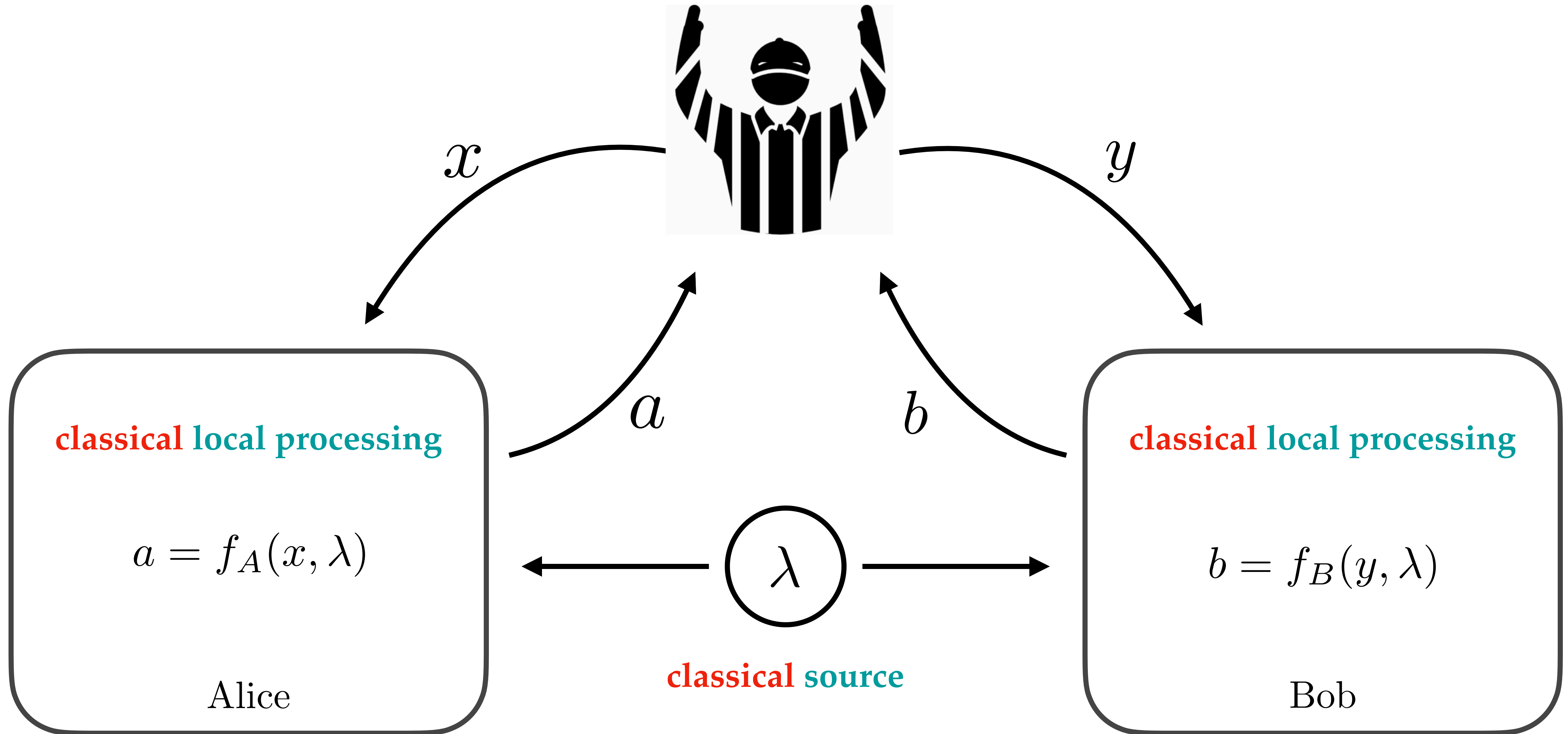
According to Bell's theorem, they must be described by a **local hidden variable model**.

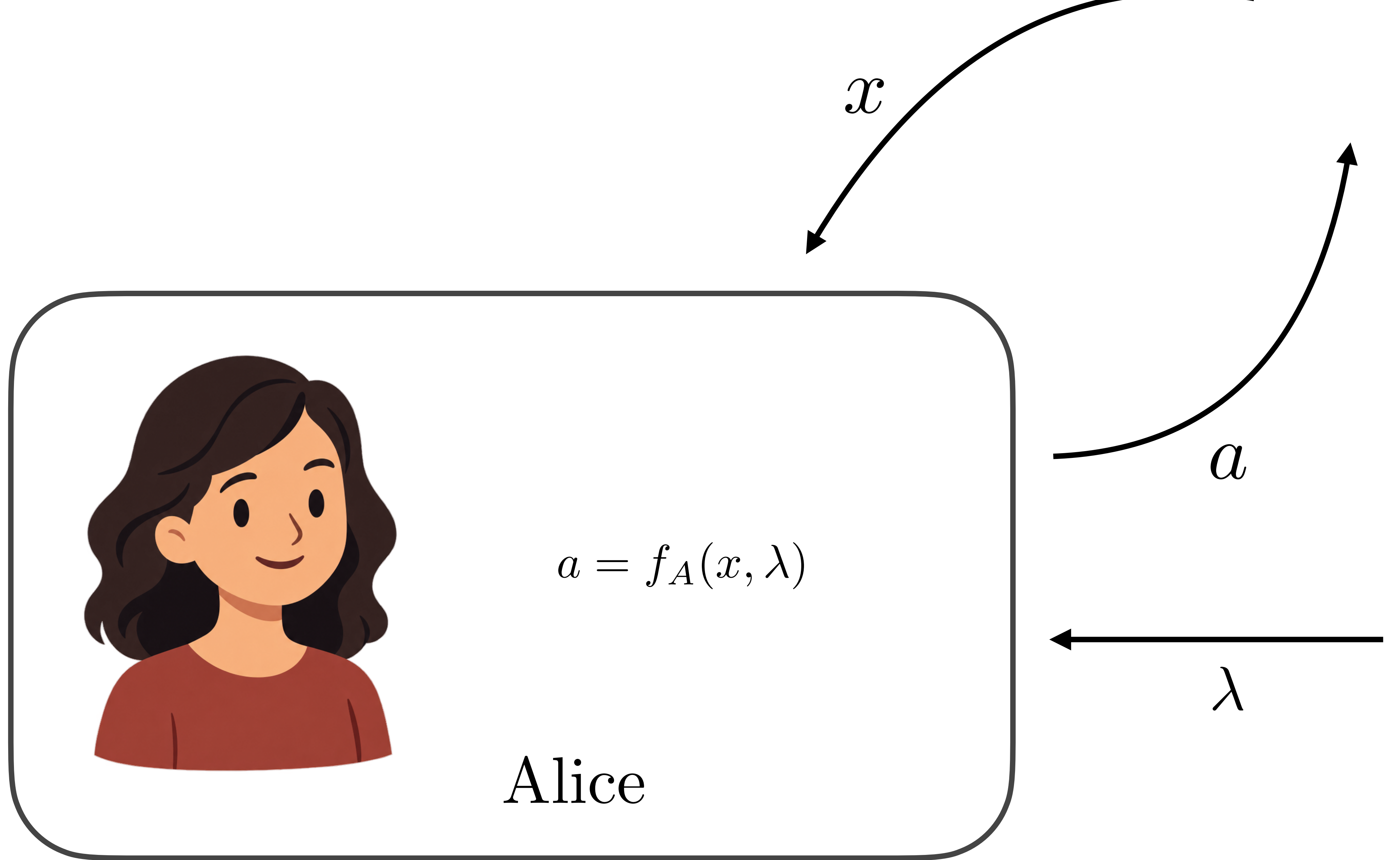
Let's understand it first as a **classical strategy**.

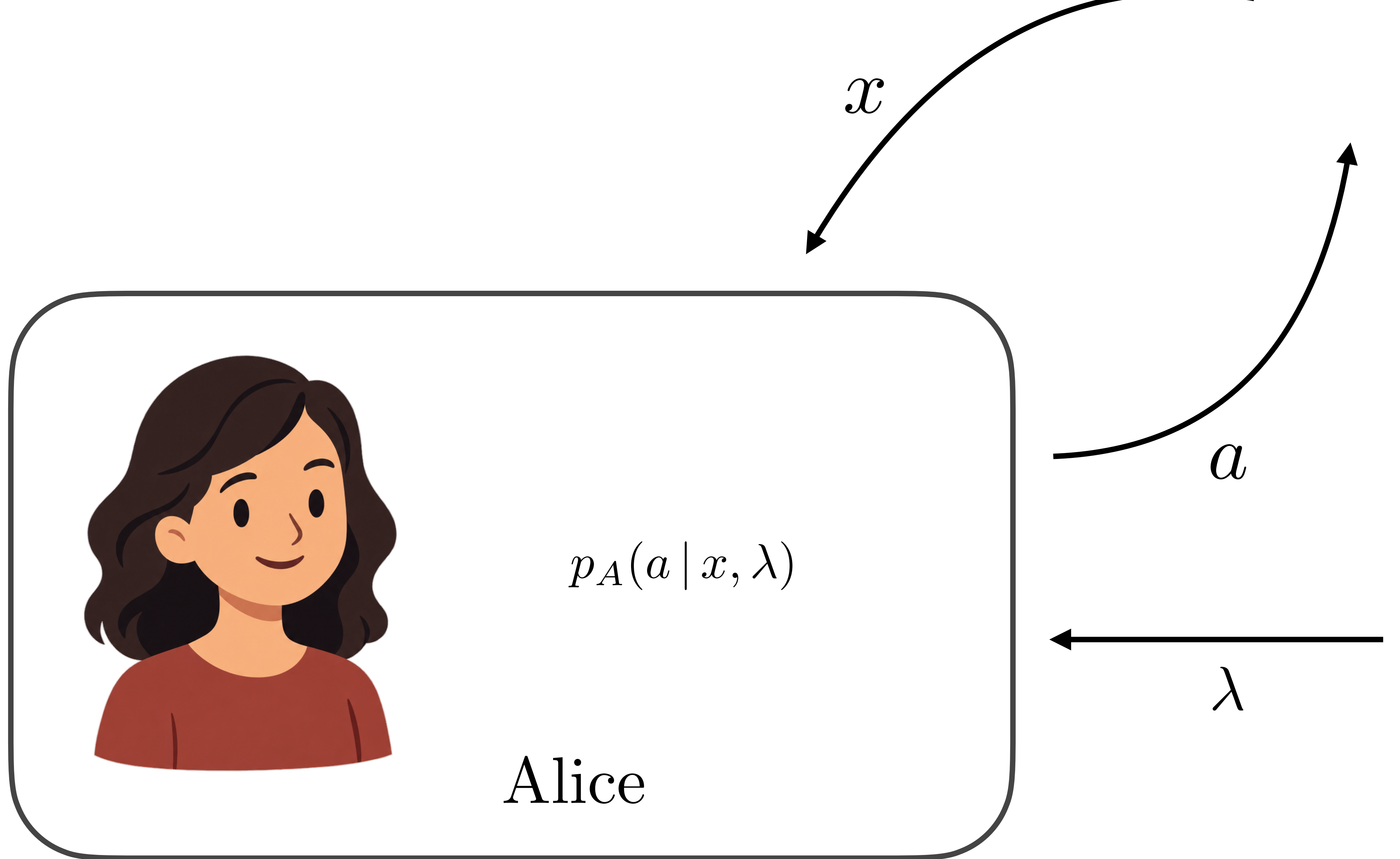
The CHSH game



The CHSH game







The CHSH game

Classical strategies

$$p(ab \mid xy) = \sum_{\lambda} \pi(\lambda) p_A(a \mid x, \lambda) p_B(b \mid y, \lambda)$$

The CHSH game

Classical strategies

$$p(ab \mid xy) = \sum_{\lambda} \pi(\lambda) p_A(a \mid x, \lambda) p_B(b \mid y, \lambda)$$

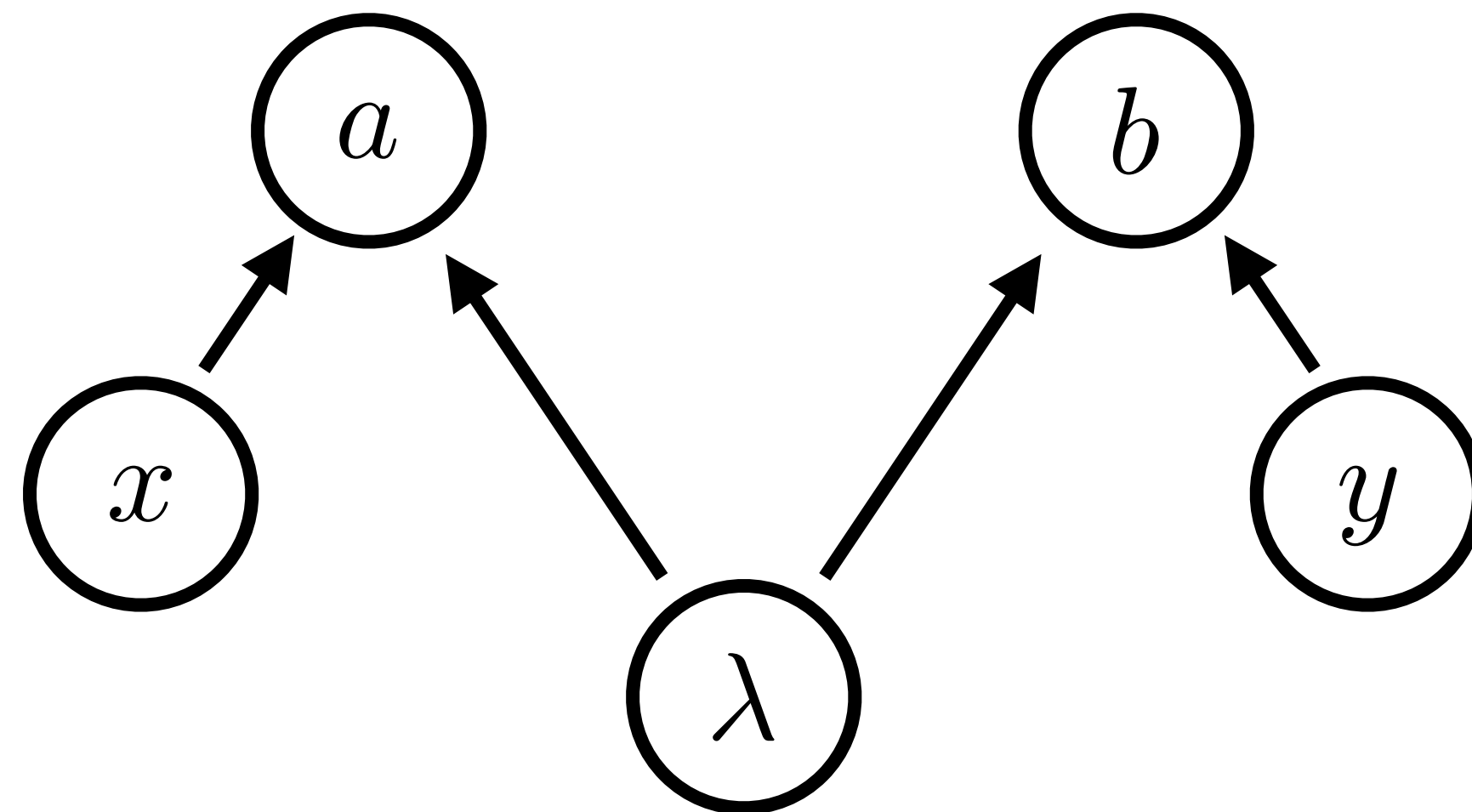
$$\pi(\lambda) \neq \pi(\lambda \mid xy)$$

The CHSH game

Classical strategies

$$p(ab \mid xy) = \sum_{\lambda} \pi(\lambda) p_A(a \mid x, \lambda) p_B(b \mid y, \lambda)$$

$$\pi(\lambda) \neq \pi(\lambda \mid xy)$$



$$p(ab \mid xy) \in \mathsf{L}$$

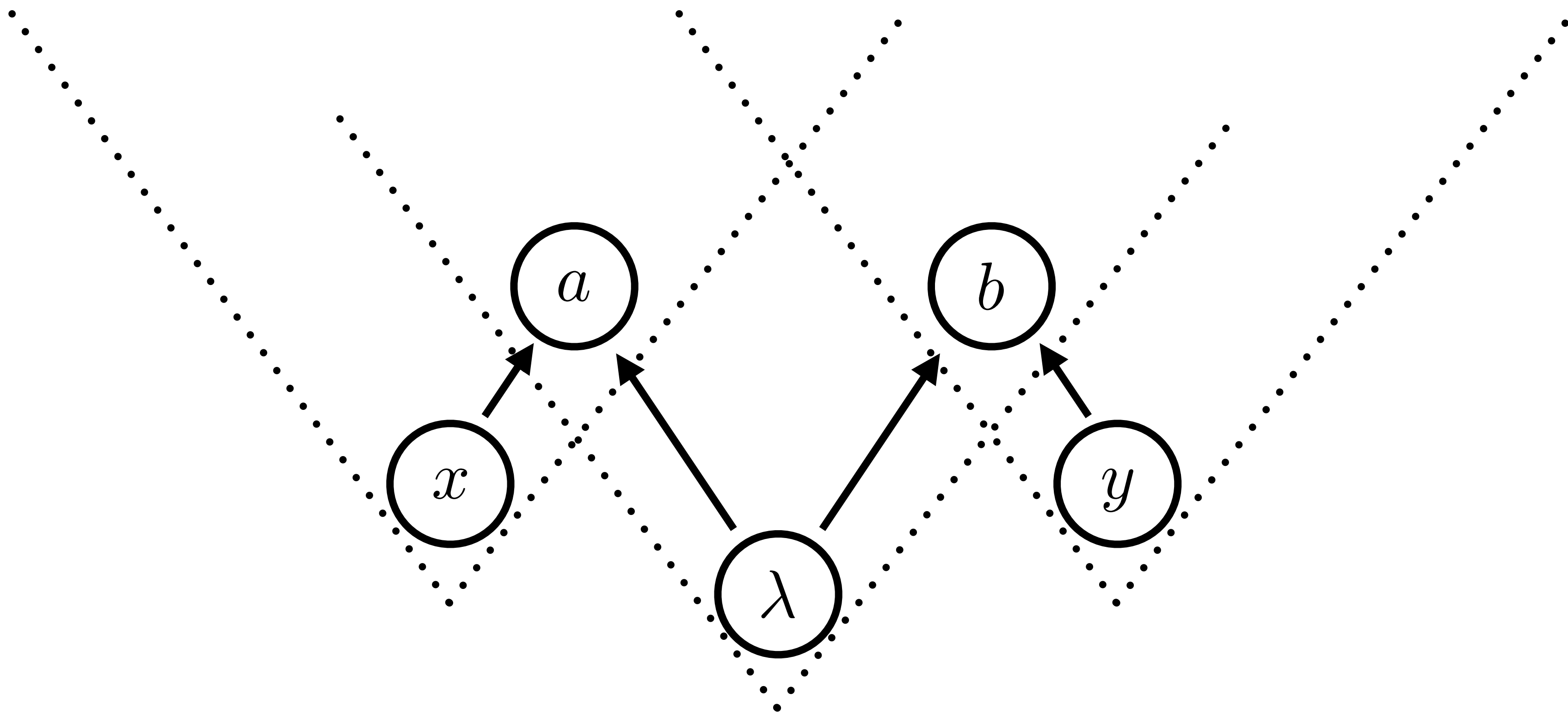
The CHSH game

Classical strategies

$$p(ab | xy) = \sum_{\lambda} \pi(\lambda) p_A(a | x, \lambda) p_B(b | y, \lambda)$$

$$\pi(\lambda) \neq \pi(\lambda | xy)$$

LHV



The CHSH game

Score:

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab \mid xy)$$

Maximum classical score:

The CHSH game

Score:

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy)$$

Maximum classical score:

$$\max_{p \in \mathcal{L}} \mathcal{S} = \max_{p \in \mathcal{L}} \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy)$$

The CHSH game

Score:

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy)$$

Maximum classical score:

$$\begin{aligned} \max_{p \in \mathcal{L}} \mathcal{S} &= \max_{p \in \mathcal{L}} \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy) \\ &= \max_{\{\pi, p_A, p_B\}} \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda) \end{aligned}$$

The CHSH game

Score:

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy)$$

Maximum classical score:

$$\begin{aligned} \max_{p \in \mathcal{L}} \mathcal{S} &= \max_{p \in \mathcal{L}} \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy) \\ &= \max_{\{\pi, p_A, p_B\}} \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda) \\ &= \frac{3}{4} =: \mathcal{B}_L \quad \checkmark \end{aligned}$$

Assuming fair coins:

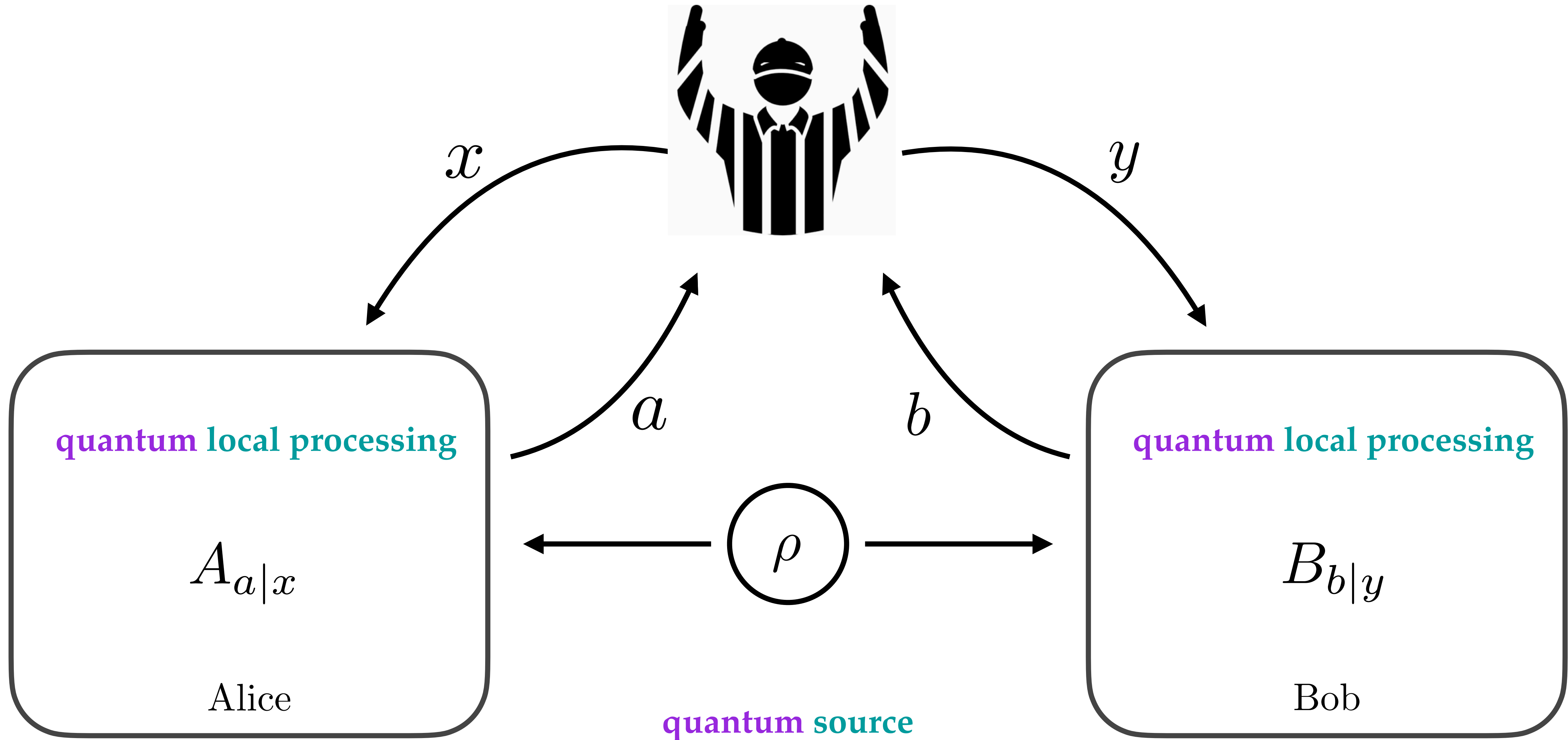
The CHSH game

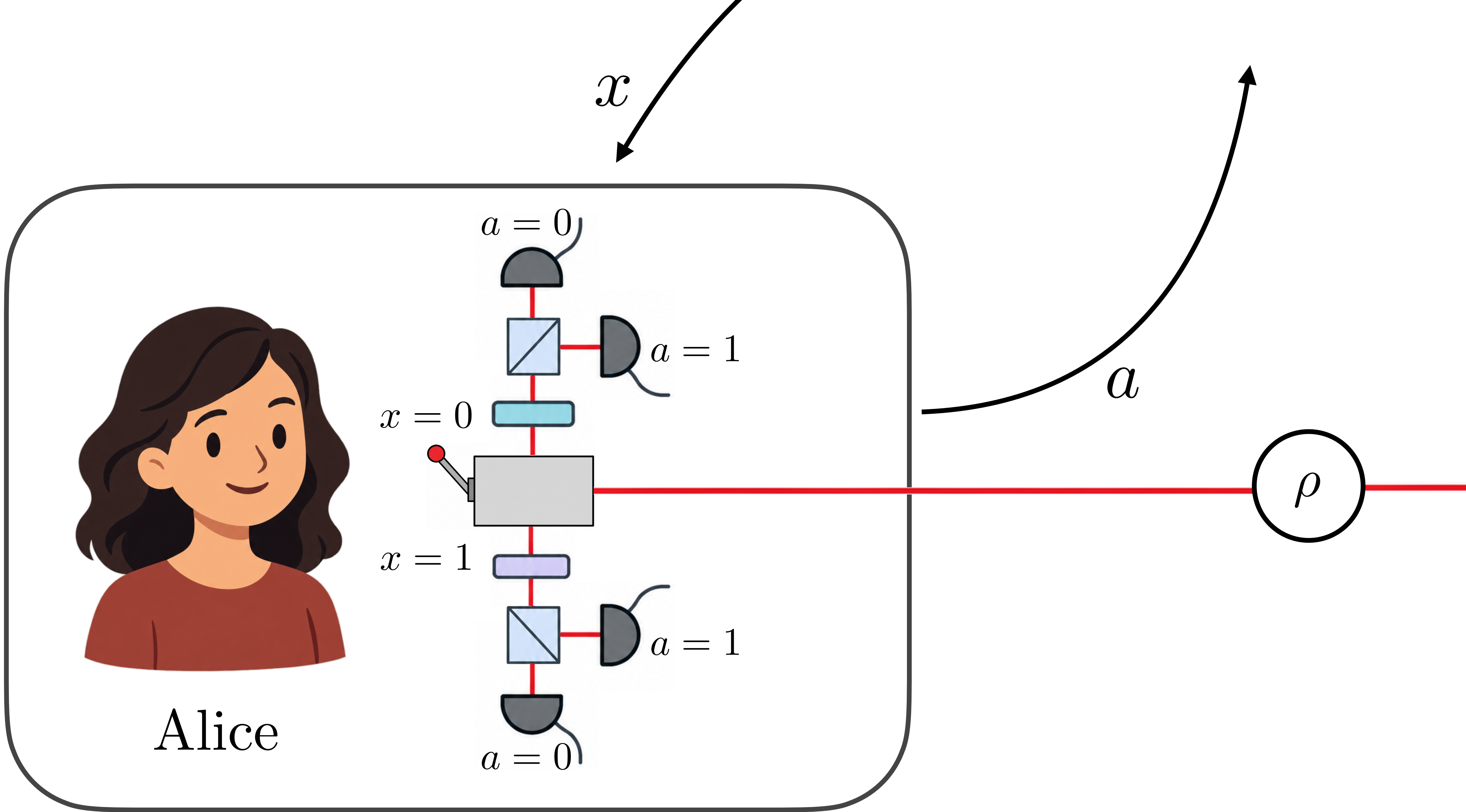
Can quantum strategies do better?

What does it mean for Alice and Bob's strategy to be **quantum**?

$$p(ab \mid xy)$$

The CHSH game





The CHSH game

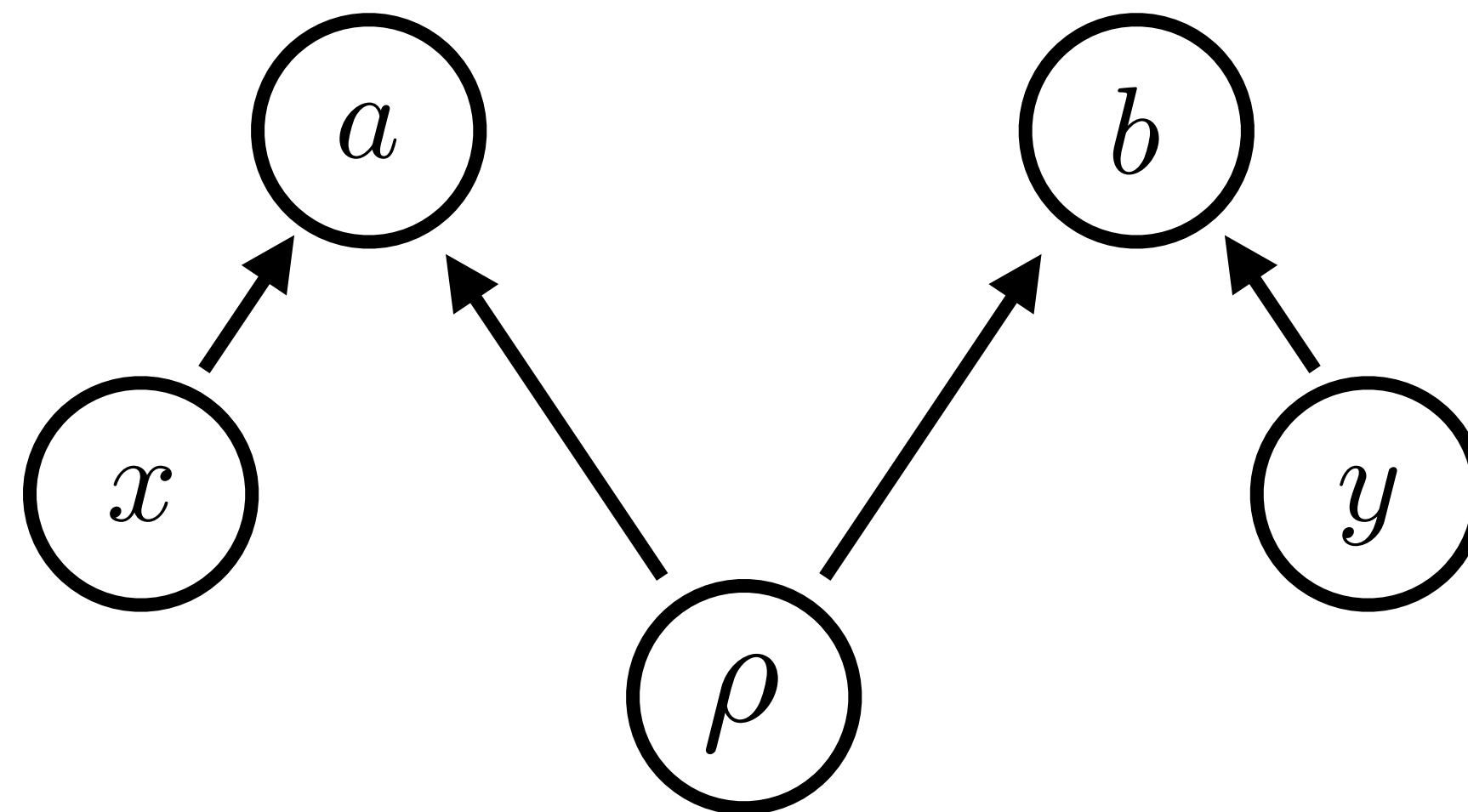
Quantum strategies

$$p(ab \mid xy) = \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \rho \right] \quad \longrightarrow \quad \text{Born rule}$$

The CHSH game

Quantum strategies

$$p(ab | xy) = \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \rho \right] \quad \longrightarrow \quad \text{Born rule}$$



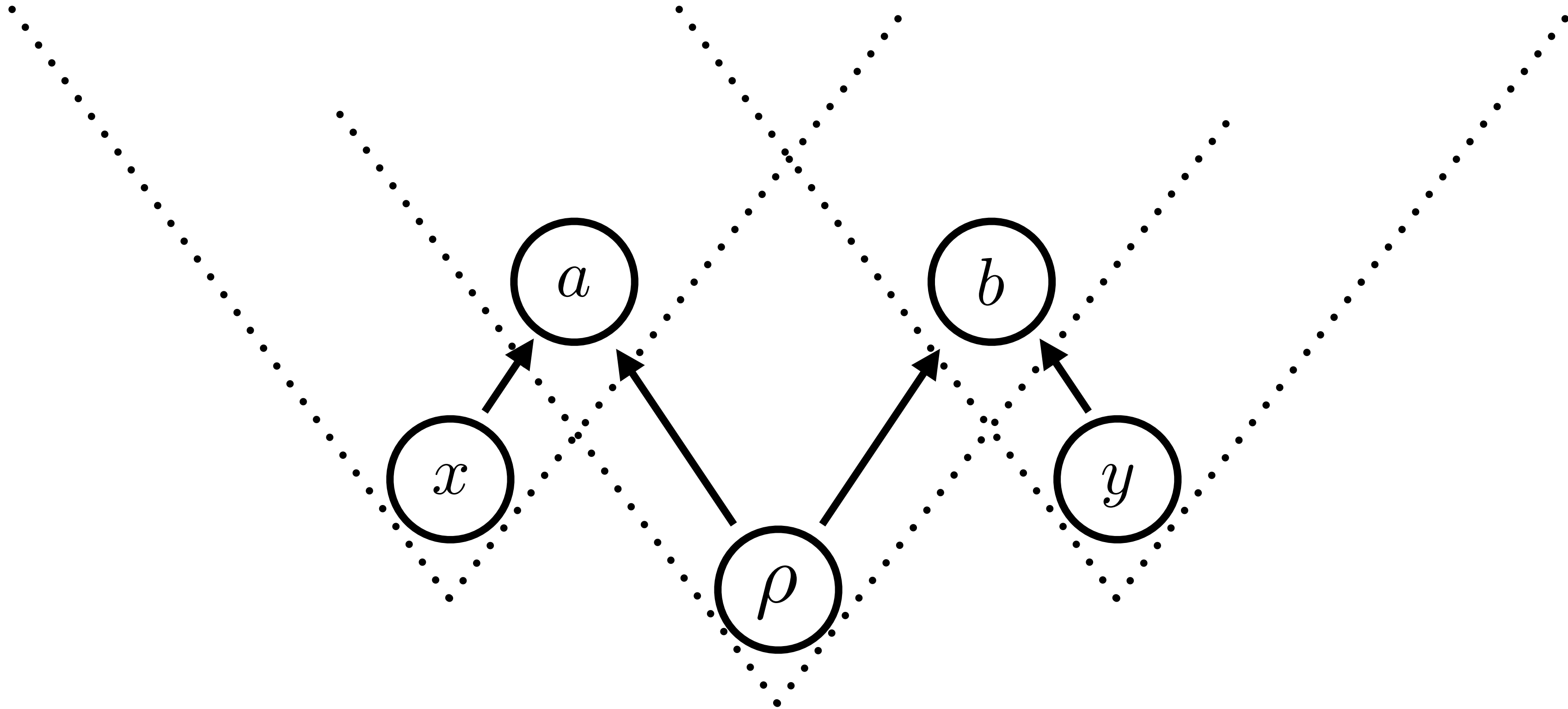
$$\rho \neq f(x, y)$$

$$p(ab | xy) \in \mathbb{Q}$$

The CHSH game

Quantum strategies

$$p(ab | xy) = \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \rho \right] \quad \longrightarrow \quad \text{Born rule}$$



The CHSH game

Quantum strategies

$$p(ab \mid xy) = \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \rho \right]$$

Alice's measurements:

$$A_{a|x} \geq 0 \quad \forall a, x \quad \sum_a A_{a|x} = \mathbb{1} \quad \forall x$$

Bob's measurements:

$$B_{b|y} \geq 0 \quad \forall b, y \quad \sum_b B_{b|y} = \mathbb{1} \quad \forall y$$

shared quantum state:

$$\rho \geq 0 \quad \text{tr}(\rho) = 1$$

The CHSH game

Quantum strategies

$$p(ab \mid xy) = \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \rho \right]$$

Alice's measurements:

$$A_{a|x} \geq 0 \quad \forall a, x \quad \sum_a A_{a|x} = \mathbb{1} \quad \forall x$$

Bob's measurements:

$$B_{b|y} \geq 0 \quad \forall b, y \quad \sum_b B_{b|y} = \mathbb{1} \quad \forall y$$

shared quantum state:

$$\rho \geq 0 \quad \text{tr}(\rho) = 1$$

$$\mathcal{S} > \frac{3}{4} ?$$

The CHSH game

Quantum strategies

Quantum state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\rho = |\Phi^+\rangle\langle\Phi^+|$$

Quantum measurements

$$|m(\theta)\rangle = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

$$A_{0|0} = |m(0)\rangle\langle m(0)|, \quad A_{1|0} = \mathbb{1} - A_{0|0}$$

$$A_{0|1} = |m(\pi/4)\rangle\langle m(\pi/4)|, \quad A_{1|1} = \mathbb{1} - A_{0|1}$$

$$B_{0|0} = |m(\pi/8)\rangle\langle m(\pi/8)|, \quad B_{1|0} = \mathbb{1} - B_{0|0}$$

$$B_{0|1} = |m(-\pi/8)\rangle\langle m(-\pi/8)|, \quad B_{1|1} = \mathbb{1} - B_{0|1}$$

The CHSH game

Quantum strategies

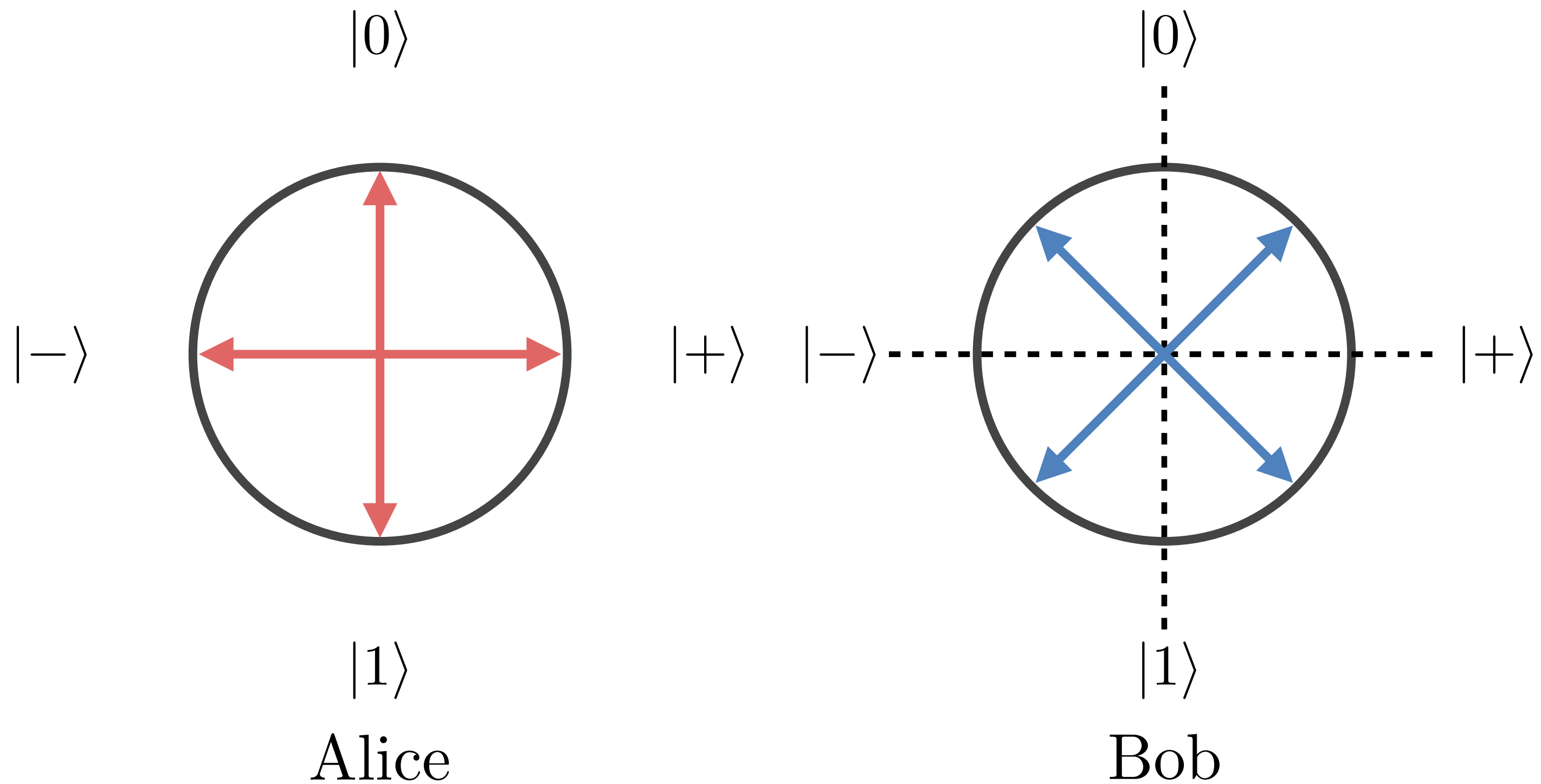
Quantum state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\rho = |\Phi^+\rangle\langle\Phi^+|$$

Quantum measurements

$$|m(\theta)\rangle = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$



The CHSH game

Quantum strategies

$$\rho = |\Phi^+\rangle\langle\Phi^+|$$

$$A_{0|0} = |m(0)\rangle\langle m(0)|, \quad A_{1|0} = \mathbb{1} - A_{0|0}$$

$$A_{0|1} = |m(\pi/4)\rangle\langle m(\pi/4)|, \quad A_{1|1} = \mathbb{1} - A_{0|1}$$

$$B_{0|0} = |m(\pi/8)\rangle\langle m(\pi/8)|, \quad B_{1|0} = \mathbb{1} - B_{0|0}$$

$$B_{0|1} = |m(-\pi/8)\rangle\langle m(-\pi/8)|, \quad B_{1|1} = \mathbb{1} - B_{0|1}$$

$$p(ab | xy) = \text{tr} [(A_{a|x} \otimes B_{b|y}) \rho] \quad \Longrightarrow \quad \mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy)$$

The CHSH game

Quantum strategies

$$\rho = |\Phi^+\rangle\langle\Phi^+|$$

$$A_{0|0} = |m(0)\rangle\langle m(0)|,$$

$$A_{1|0} = \mathbb{1} - A_{0|0}$$

$$A_{0|1} = |m(\pi/4)\rangle\langle m(\pi/4)|,$$

$$A_{1|1} = \mathbb{1} - A_{0|1}$$

$$B_{0|0} = |m(\pi/8)\rangle\langle m(\pi/8)|,$$

$$B_{1|0} = \mathbb{1} - B_{0|0}$$

$$B_{0|1} = |m(-\pi/8)\rangle\langle m(-\pi/8)|, \quad B_{1|1} = \mathbb{1} - B_{0|1}$$

$$p(ab | xy) = \text{tr} [(A_{a|x} \otimes B_{b|y}) \rho] \quad \Longrightarrow \quad \mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy)$$

assuming fair coins:

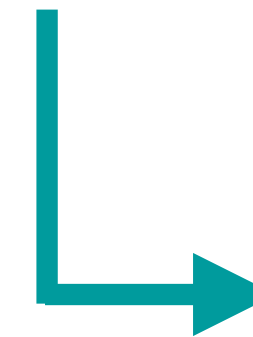
$$= \frac{2 + \sqrt{2}}{4} \approx 0.85 \dots > \frac{3}{4}$$

The CHSH game

$$\mathcal{S} = \frac{1}{4} \sum_{a,b,x,y} \delta_{(a \oplus b, x \cdot y)} p(ab | xy) \leq \frac{3}{4} < \frac{2 + \sqrt{2}}{4}$$



**for classical (= local)
correlations**



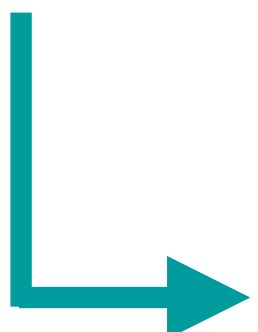
**achievable by
quantum correlations**

The CHSH game

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**for classical (= local)
correlations**

 **achievable by
quantum correlations**

Is there a strategy that can win the game **perfectly**?
($\mathcal{S} = 1$)

Nonsignalling scenarios

No constraints on the type of source and type of local processing function

Only constraint: **no communication** between Alice and Bob

Nonsignalling scenarios

No constraints on the type of source and type of local processing function

Only constraint: **no communication** between Alice and Bob

In scenarios **without communication**, all correlations satisfy the following constraint:

Alice's marginals are
independent of Bob's input:

$$q_A(a|x) := \sum_b p(ab | xy) = \sum_b p(ab | xy') \quad \forall a, x, y, y'$$

Bob's marginals are
independent of Alice's input:

$$q_B(b|y) := \sum_a p(ab | xy) = \sum_a p(ab | xy') \quad \forall b, y, x, x'$$

$$p(ab | xy) \in \text{NS}$$

Nonsignalling scenarios

Nonsignalling:

$$q_A(a|x) := \sum_b p(ab | xy) = \sum_b p(ab | xy') \quad \forall a, x, y, y'$$

$$q_B(b|y) := \sum_a p(ab | xy) = \sum_a p(ab | xy') \quad \forall b, y, x, x'$$

Quantum correlations are nonsignalling:

$$\begin{aligned} \sum_a p(ab|xy) &= \sum_a \text{tr} [(A_{a|x} \otimes B_{b|y})\rho] \\ &= \text{tr} \left[\left(\sum_a A_{a|x} \otimes B_{b|y} \right) \rho \right] \\ &= \text{tr} [(\mathbb{1} \otimes B_{b|y})\rho] = q_B(b|y) \quad \longrightarrow \quad \text{independent of } x \end{aligned}$$

Nonsignalling scenarios

Nonsignalling:

$$q_A(a|x) := \sum_b p(ab | xy) = \sum_b p(ab | xy') \quad \forall a, x, y, y'$$

$$q_B(b|y) := \sum_a p(ab | xy) = \sum_a p(ab | xy') \quad \forall b, y, x, x'$$

Classical correlations are nonsignalling:

$$\begin{aligned} \sum_a p(ab|xy) &= \sum_a \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda) \\ &= \sum_{\lambda} \pi(\lambda) \sum_a p_A(a|x, \lambda) p_B(b|y, \lambda) \\ &= \sum_{\lambda} \pi(\lambda) p_B(b|y, \lambda) = q_B(b|y) \longrightarrow \text{independent of } x \end{aligned}$$

Nonsignalling scenarios

$$p(ab \mid xy) \in \text{NS}$$

Nonsignalling:

$$q_A(a|x) := \sum_b p(ab \mid xy) = \sum_b p(ab \mid xy') \quad \forall a, x, y, y'$$

$$q_B(b|y) := \sum_a p(ab \mid xy) = \sum_a p(ab \mid xy') \quad \forall b, y, x, x'$$

The CHSH game

$$p(ab | xy) = \begin{cases} \frac{1}{2} & \text{if } a \oplus b = x \cdot y \\ 0 & \text{otherwise} \end{cases}$$

“PR box”

This strategy is **nonsignalling**.
This strategy wins the game perfectly.

The CHSH game

$$p(ab | xy) = \begin{cases} \frac{1}{2} & \text{if } a \oplus b = x \cdot y \\ 0 & \text{otherwise} \end{cases}$$

“PR box”

This strategy is **nonsignalling**.
This strategy wins the game perfectly.

However, this strategy is **not quantum**.

The CHSH game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy) \leq \mathcal{B}_L \leq \mathcal{B}_Q \leq \mathcal{B}_{\text{NS}}$$

local:

$$\mathcal{B}_L := \max_{p \in \mathcal{L}} \mathcal{S}$$

$$p(ab | xy) = \sum_{\lambda} \pi(\lambda) p_A(a | x, \lambda) p_B(b | y, \lambda)$$

quantum:

$$\mathcal{B}_Q := \max_{p \in \mathcal{Q}} \mathcal{S}$$

$$p(ab | xy) = \text{tr} [(A_{a|x} \otimes B_{b|y}) \rho]$$

nonsignalling:

$$\mathcal{B}_{\text{NS}} := \max_{p \in \text{NS}} \mathcal{S}$$

$$\begin{aligned} \sum_b p(ab | xy) &= \sum_b p(ab | xy') \\ \sum_a p(ab | xy) &= \sum_a p(ab | xy') \end{aligned}$$

The CHSH game

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy) \leq \mathcal{B}_L \leq \mathcal{B}_Q \leq \mathcal{B}_{\text{NS}}$$

assuming fair coins:

$$\qquad \qquad \qquad \qquad \qquad \qquad \frac{3}{4} \qquad \frac{2 + \sqrt{2}}{4} \qquad 1$$

local:

$$\mathcal{B}_L := \max_{p \in \text{L}} \mathcal{S}$$

$$p(ab | xy) = \sum_{\lambda} \pi(\lambda) p_A(a | x, \lambda) p_B(b | y, \lambda)$$

quantum:

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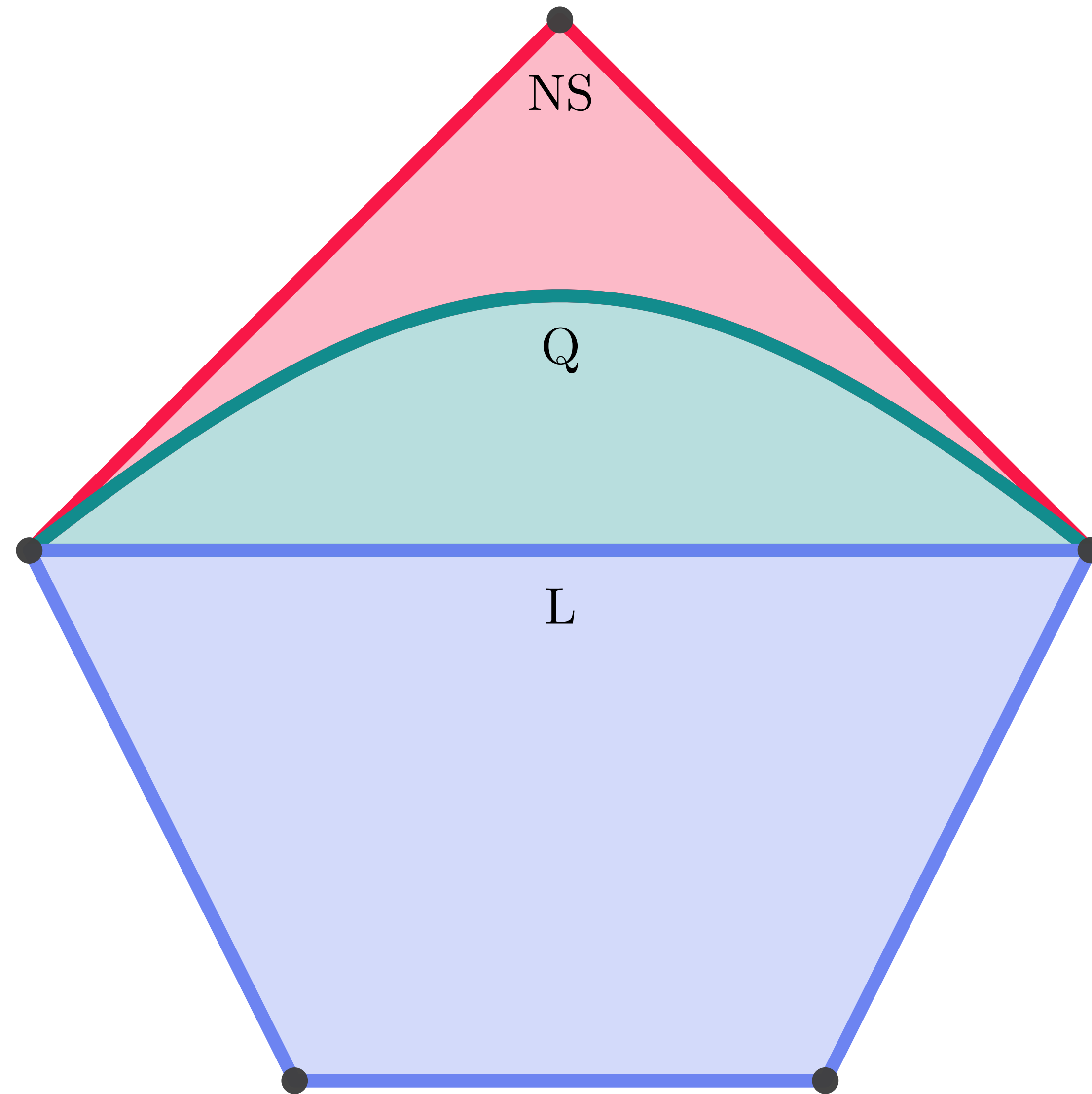
nonsignalling:

$$\mathcal{B}_{\text{NS}} := \max_{p \in \text{NS}} \mathcal{S}$$

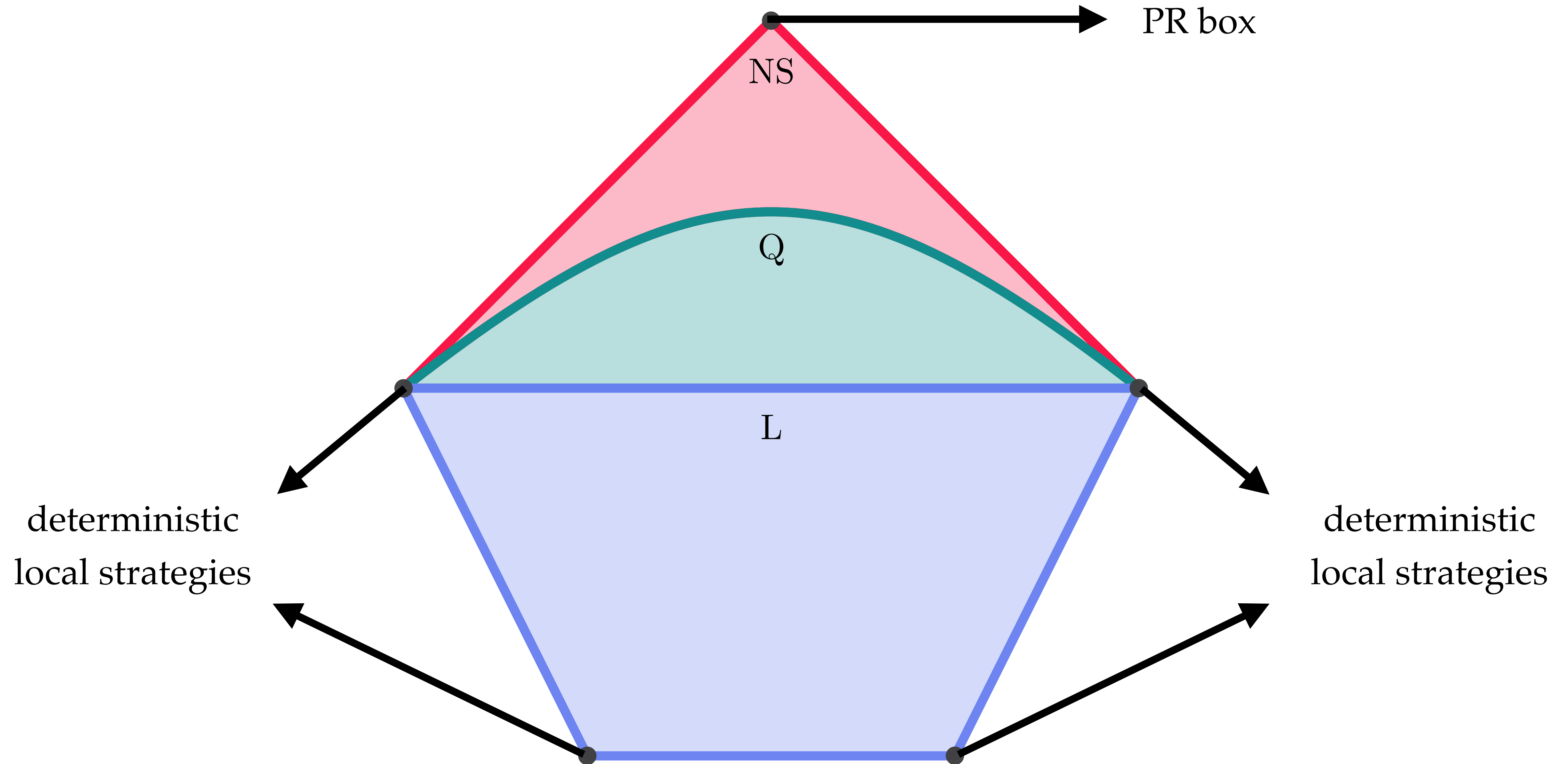
$$\sum_b p(ab | xy) = \sum_b p(ab | xy')$$

$$\sum_a p(ab | xy) = \sum_a p(ab | xy')$$

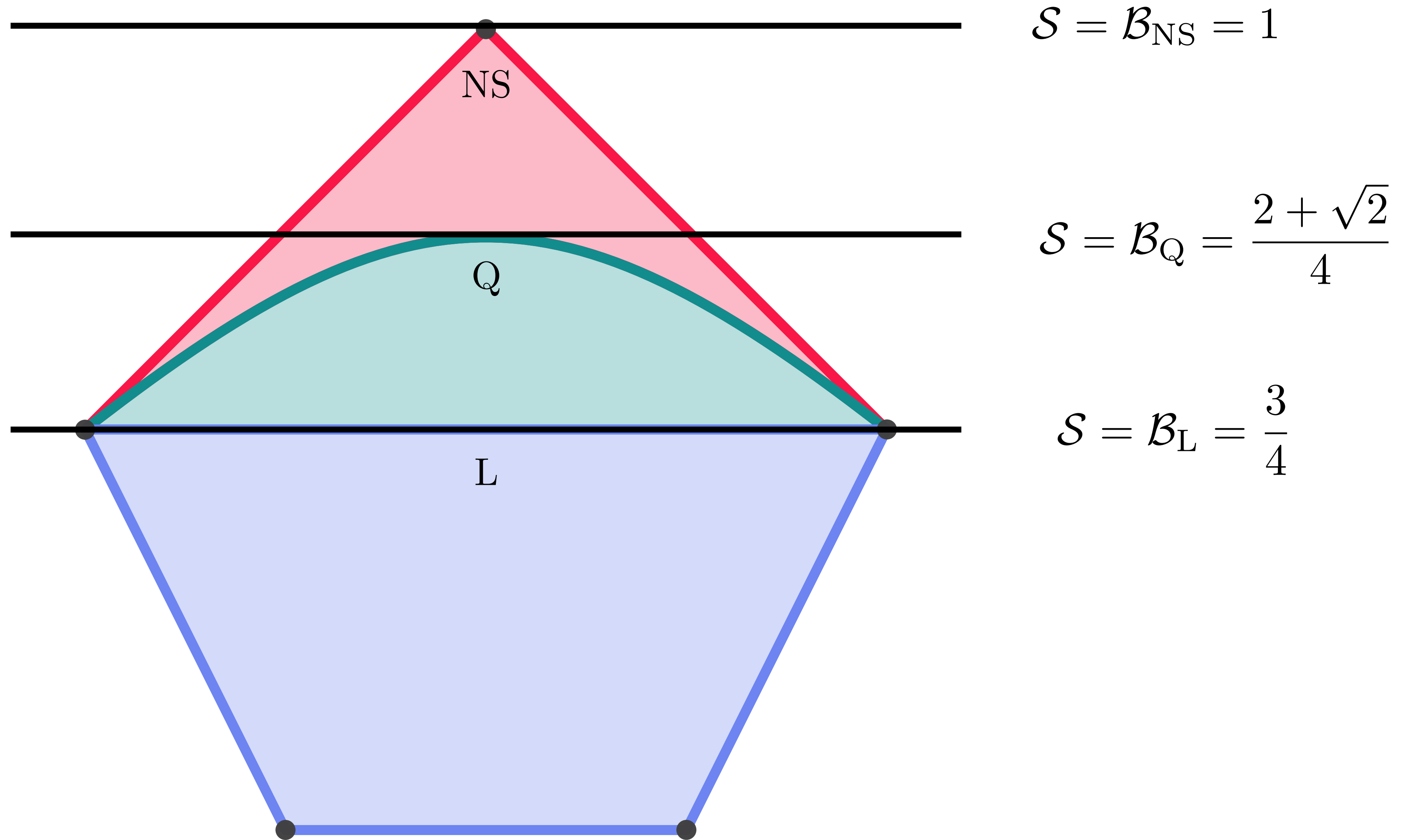
Geometric interpretation of CHSH



Geometric interpretation of CHSH



Geometric interpretation of CHSH



Bell inequalities

CHSH inequality:

$$\sum_{a,b,x,y} q(x,y) \delta_{(a \oplus b, x \cdot y)} p(ab | xy) \leq \frac{3}{4}$$

Bell inequalities

CHSH inequality:

$$\sum_{a,b,x,y} \underbrace{q(x,y) \delta_{(a \oplus b, x \cdot y)}}_{\text{coefficients can be generalized}} p(ab | xy) \leq \frac{3}{4}$$



coefficients can be generalized

Bell inequalities

CHSH inequality:

$$\sum_{a,b,x,y} \underbrace{q(x,y) \delta_{(a \oplus b, x \cdot y)}}_{\text{coefficients can be generalized}} p(ab | xy) \leq \frac{3}{4}$$

coefficients can be generalized



**General
Bell inequalities:**

$$\mathcal{S} = \sum_{a,b,x,y} \alpha_{xy}^{ab} p(ab | xy) \leq \mathcal{B}_L$$

Bell inequalities

CHSH inequality:

$$\sum_{a,b,x,y} \underbrace{q(x,y) \delta_{(a \oplus b, x \cdot y)}}_{\text{coefficients can be generalized}} p(ab | xy) \leq \frac{3}{4}$$

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General
Bell inequalities:

$$\mathcal{S} = \sum_{a,b,x,y} \alpha_{xy}^{ab} p(ab | xy) \leq \mathcal{B}_L$$

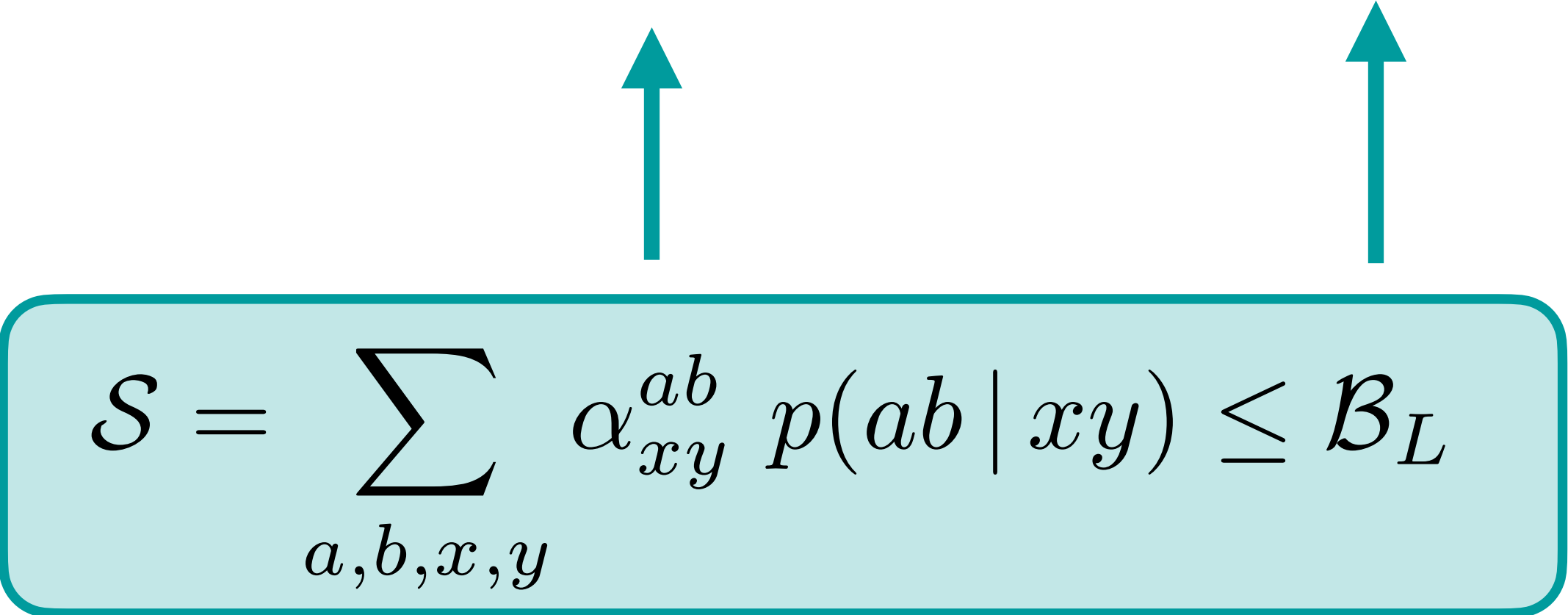
$$\alpha_{xy}^{ab} \in \mathbb{R} \quad \forall a, b, x, y$$

$$\mathcal{B}_L := \max_{p \in \mathcal{L}} \sum_{a,b,x,y} \alpha_{xy}^{ab} p(ab | xy)$$

similarly for **quantum**
and **nonsignalling**
bounds

Bell inequalities

linear combination
of probabilities + local bound = **Bell inequality**


$$\mathcal{S} = \sum_{a,b,x,y} \alpha_{xy}^{ab} p(ab | xy) \leq \mathcal{B}_L$$

“Useful” Bell inequalities are those that can be violated by quantum correlations,
that is, have a **classical** vs. **quantum** separation.

Relationship between entanglement and nonlocality

Nonlocality and entanglement

separable quantum state:

$$\rho = \sum_{\lambda} \pi(\lambda) \rho_A^{\lambda} \otimes \rho_B^{\lambda}$$

Let Alice and Bob locally measure a separable quantum state:

Nonlocality and entanglement

separable quantum state:

$$\rho = \sum_{\lambda} \pi(\lambda) \rho_A^{\lambda} \otimes \rho_B^{\lambda}$$

Let Alice and Bob locally measure a separable quantum state:

$$\begin{aligned} p(ab|xy) &= \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \rho \right] \\ &= \text{tr} \left[(A_{a|x} \otimes B_{b|y}) \sum_{\lambda} \pi(\lambda) (\rho_A^{\lambda} \otimes \rho_B^{\lambda}) \right] \\ &= \sum_{\lambda} \pi(\lambda) \text{tr} \left[(A_{a|x} \otimes B_{b|y}) (\rho_A^{\lambda} \otimes \rho_B^{\lambda}) \right] \\ &= \sum_{\lambda} \pi(\lambda) \text{tr} (A_{a|x} \rho_A^{\lambda}) \text{tr} (B_{b|y} \rho_B^{\lambda}) \\ &= \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda) \end{aligned}$$

Nonlocality and entanglement

separable quantum state:

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Let Alice and Bob locally measure a separable quantum state:

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Nonlocality and entanglement

**Separable quantum states always lead to local correlations
in Bell scenarios**

Nonlocality and entanglement

Separability of quantum states:

$$\rho = \sum_{\lambda} \pi(\lambda) \rho_A^{\lambda} \otimes \rho_B^{\lambda}$$

= convex combinations of product states

Separability of correlations:

$$p(ab | xy) = \sum_{\lambda} \pi(\lambda) p_A(a | x, \lambda) p_B(b | y, \lambda)$$

= convex combinations of product probabilities

Nonlocality and entanglement

separable quantum state $\implies$ **local correlations**

therefore

entangled quantum state $\impliedby$ **nonlocal quantum correlations**

Nonlocality and entanglement

separable quantum state $\implies$ **local correlations**

therefore

entangled quantum state $\Longleftarrow$ **nonlocal quantum correlations**

if

$$p(ab|xy) = \text{tr} [(A_{a|x} \otimes B_{b|y})\rho]$$

and

$\implies$

$$\rho \neq \sum_{\lambda} \pi(\lambda) \rho_A^{\lambda} \otimes \rho_B^{\lambda}$$

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$$

Nonlocality and entanglement

if

$$p(ab|xy) = \text{tr} [(A_{a|x} \otimes B_{b|y})\rho]$$

and

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$$

$\implies$

$$\rho \neq \sum_{\lambda} \pi(\lambda) \rho_A^{\lambda} \otimes \rho_B^{\lambda}$$

This implies that entanglement can be concluded directly from $p(ab|xy)$ without any need of information about the states and measurements that were implemented.

Nonlocality and entanglement

if

$$p(ab|xy) = \text{tr} [(A_{a|x} \otimes B_{b|y})\rho]$$

and

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$$

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This implies that entanglement can be concluded directly from $p(ab|xy)$ without any need of information about the states and measurements that were implemented.

This is known as **device-independent (DI) certification of entanglement**

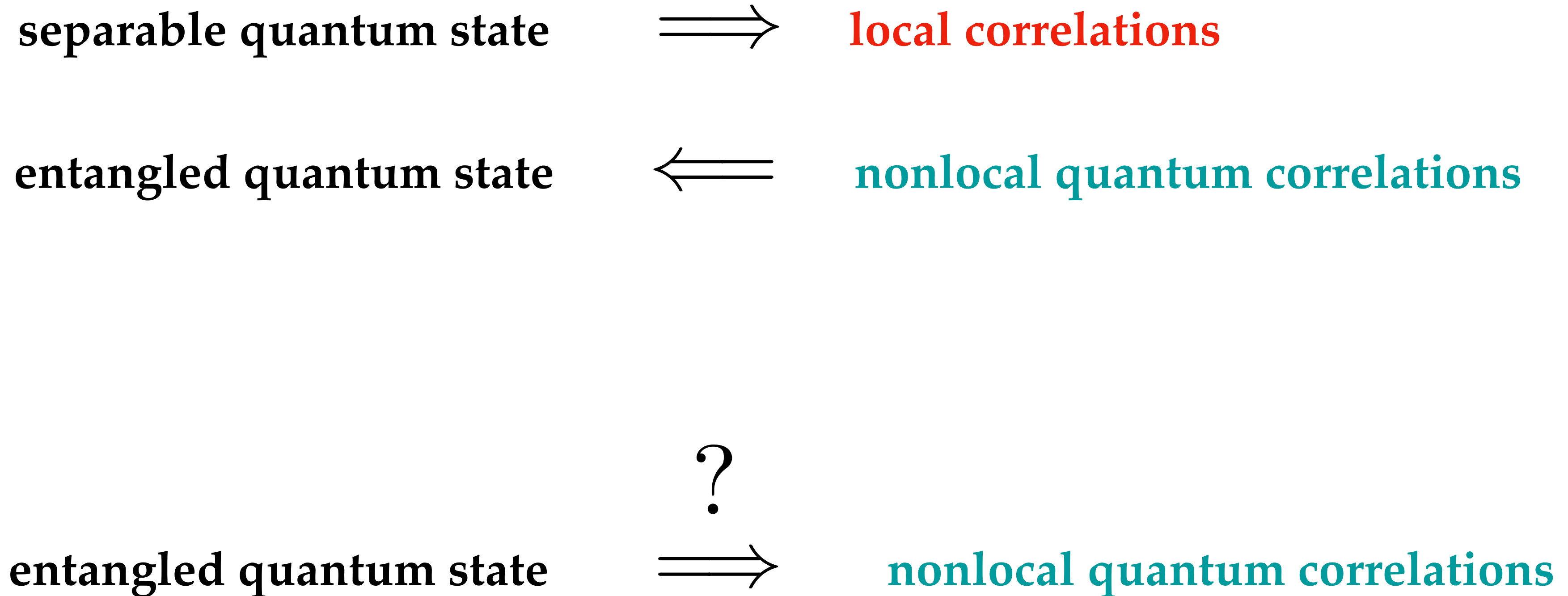
Nonlocality and entanglement

Applications of DI certification of entanglement

- Device-independent quantum key distribution
- Strong security proofs in quantum cryptography
- Randomness generation and amplification
- Self-testing of quantum states and measurements
- ...

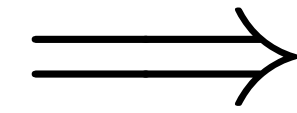
**minimal-assumption
certification**

Nonlocality and entanglement



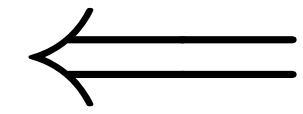
Nonlocality and entanglement

separable quantum state



local correlations

entangled quantum state



nonlocal quantum correlations

entangled quantum state



nonlocal quantum correlations

Nonlocality and entanglement

Some entangled quantum states always lead to local correlations, for all choices of measurements of Alice and Bob

Nonlocality and entanglement

Some entangled quantum states always lead to local correlations, for all choices of measurements of Alice and Bob

A quantum state ρ is **nonlocal** if

$$\exists \quad \{A_{a|x}\}, \{B_{b|y}\}; \quad \text{tr}[(A_{a|x} \otimes B_{b|y})\rho] \neq \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$$

A quantum state ρ is **local** if

$$\forall \quad \{A_{a|x}\}, \{B_{b|y}\}; \quad \text{tr}[(A_{a|x} \otimes B_{b|y})\rho] = \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$$

even infinitely many measurements with infinitely many outcomes

Nonlocality and entanglement

Some entangled quantum states always lead to local correlations, for all choices of measurements of Alice and Bob

A quantum state ρ is **local** if

$$\forall \quad \{A_{a|x}\}, \{B_{b|y}\}; \quad \text{tr}[(A_{a|x} \otimes B_{b|y})\rho] = \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$$

The statistics generated by a local entangled states in a Bell scenario can always be reproduced by a **separable state**, and hence, by a **classical/local hidden variable model**.

Nonlocality and entanglement

Some entangled quantum states always lead to local correlations, for all choices of measurements of Alice and Bob

Example: Two-qubit Werner states

$$\rho(v) = v |\Phi^+\rangle\langle\Phi^+| + (1 - v) \frac{\mathbb{1}}{4}$$



Nonlocality and entanglement

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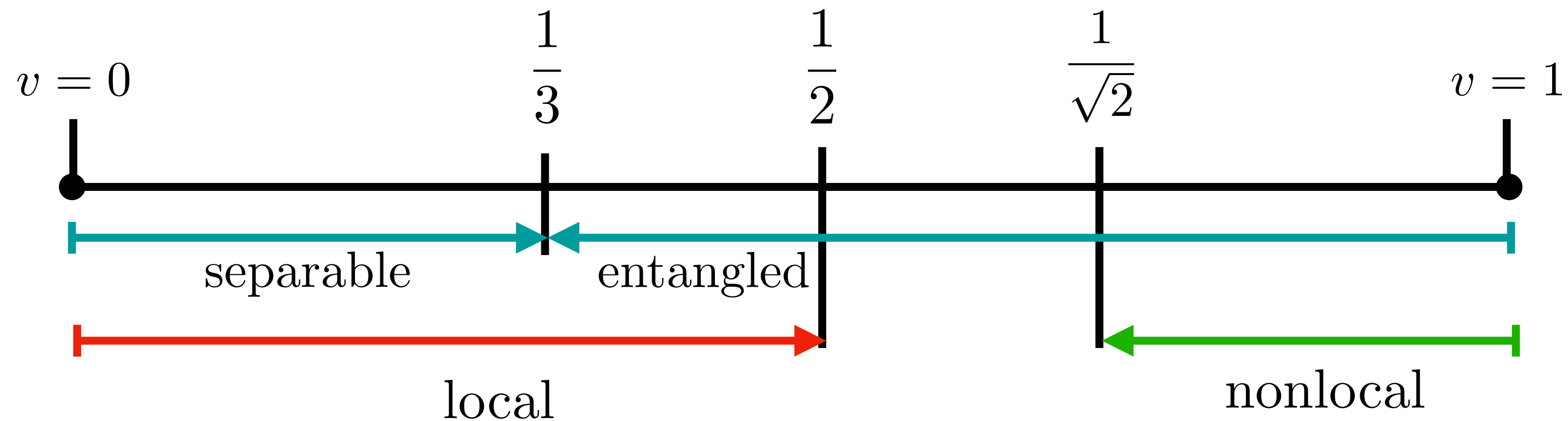


Nonlocality and entanglement

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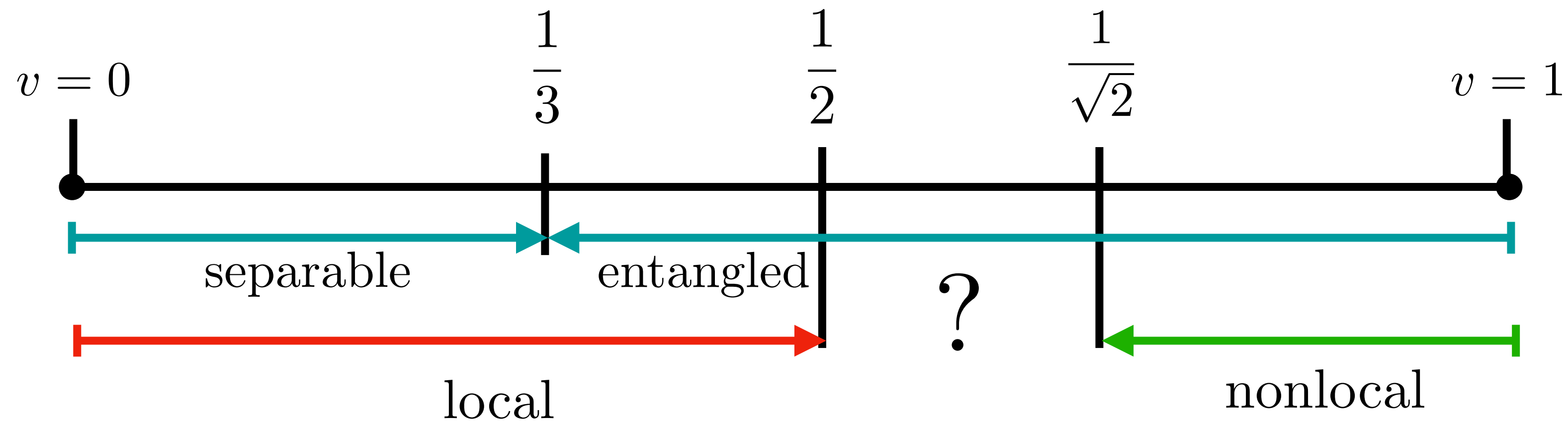


Nonlocality and entanglement

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Nonlocality and entanglement

entangled quantum state $\Leftarrow$ nonlocal quantum correlations

entangled quantum state $\nRightarrow$ nonlocal quantum correlations

Not every entangled quantum state is useful for DI quantum applications.

Nonlocality and entanglement

entangled quantum state $\Leftarrow$ nonlocal quantum correlations

entangled quantum state $\nRightarrow$ nonlocal quantum correlations

Not every entangled quantum state is useful for DI quantum applications.



Nonlocality activation:

- Hidden nonlocality (activation via LOCC)
- Many-copy activation
- Catalytic activation
- Network activation
- Activation via broadcasting

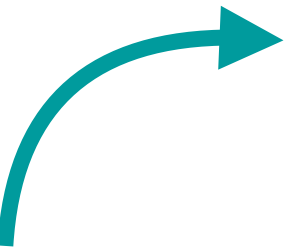
Single-shot games

Single-shot nonlocality

Nonlocal games

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) W(a,b,x,y) p(ab|xy)$$

$$W(a,b,x,y) = \begin{cases} 1 & \text{if } (a,b,x,y) \text{ satisfy winning condition} \\ 0 & \text{otherwise} \end{cases}$$

 **ex.:** $a \oplus b = x \cdot y$

Single-shot nonlocality

Nonlocal games

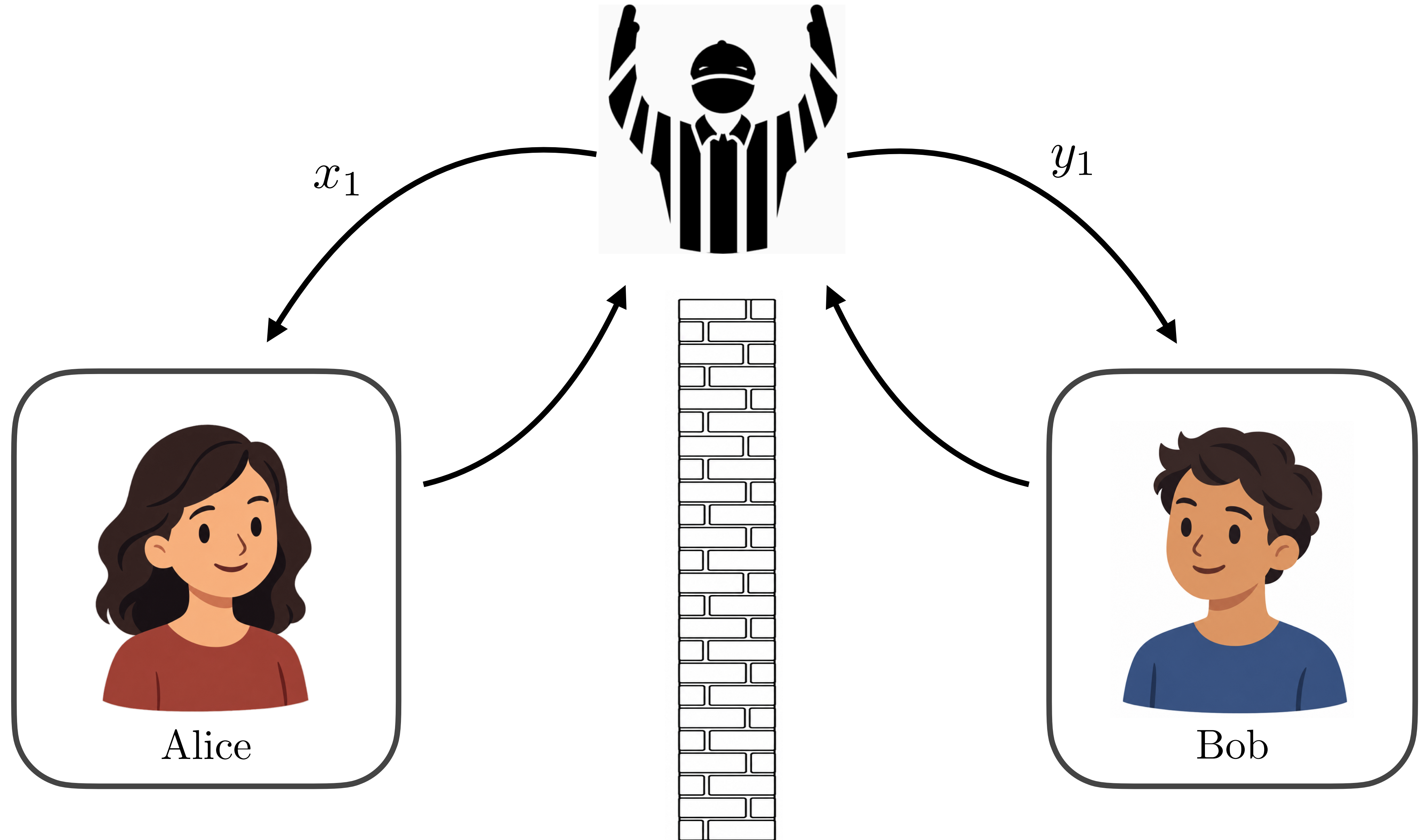
$$\mathcal{S} = \sum_{a,b,x,y} \underbrace{q(x,y) W(a,b,x,y) p(ab|xy)}_{\alpha_{xy}^{ab}}$$

$$W(a,b,x,y) = \begin{cases} 1 & \text{if } (a,b,x,y) \text{ satisfy winning condition} \\ 0 & \text{otherwise} \end{cases}$$

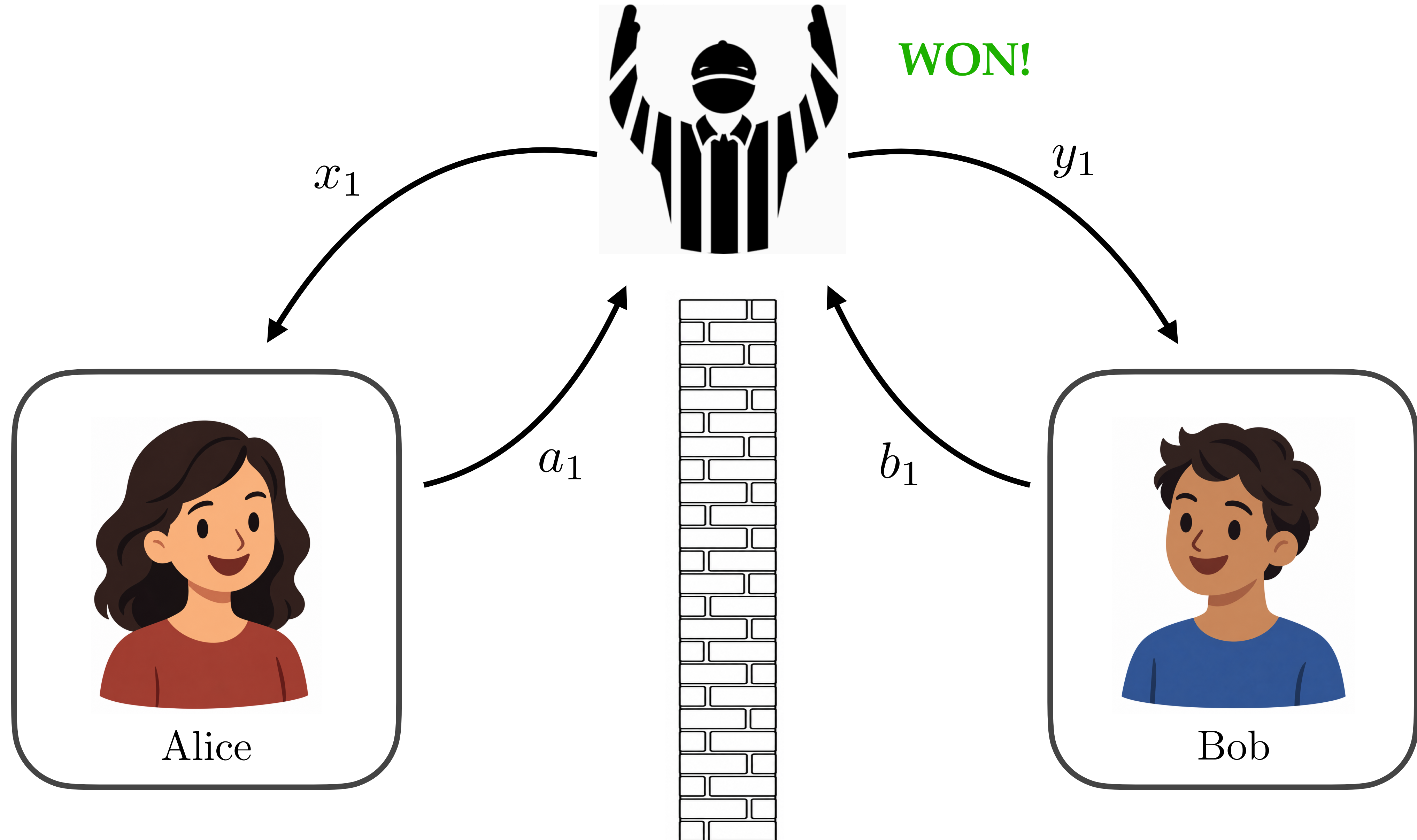
ex.: $a \oplus b = x \cdot y$

each (a,b,x,y) either **wins** or **loses**

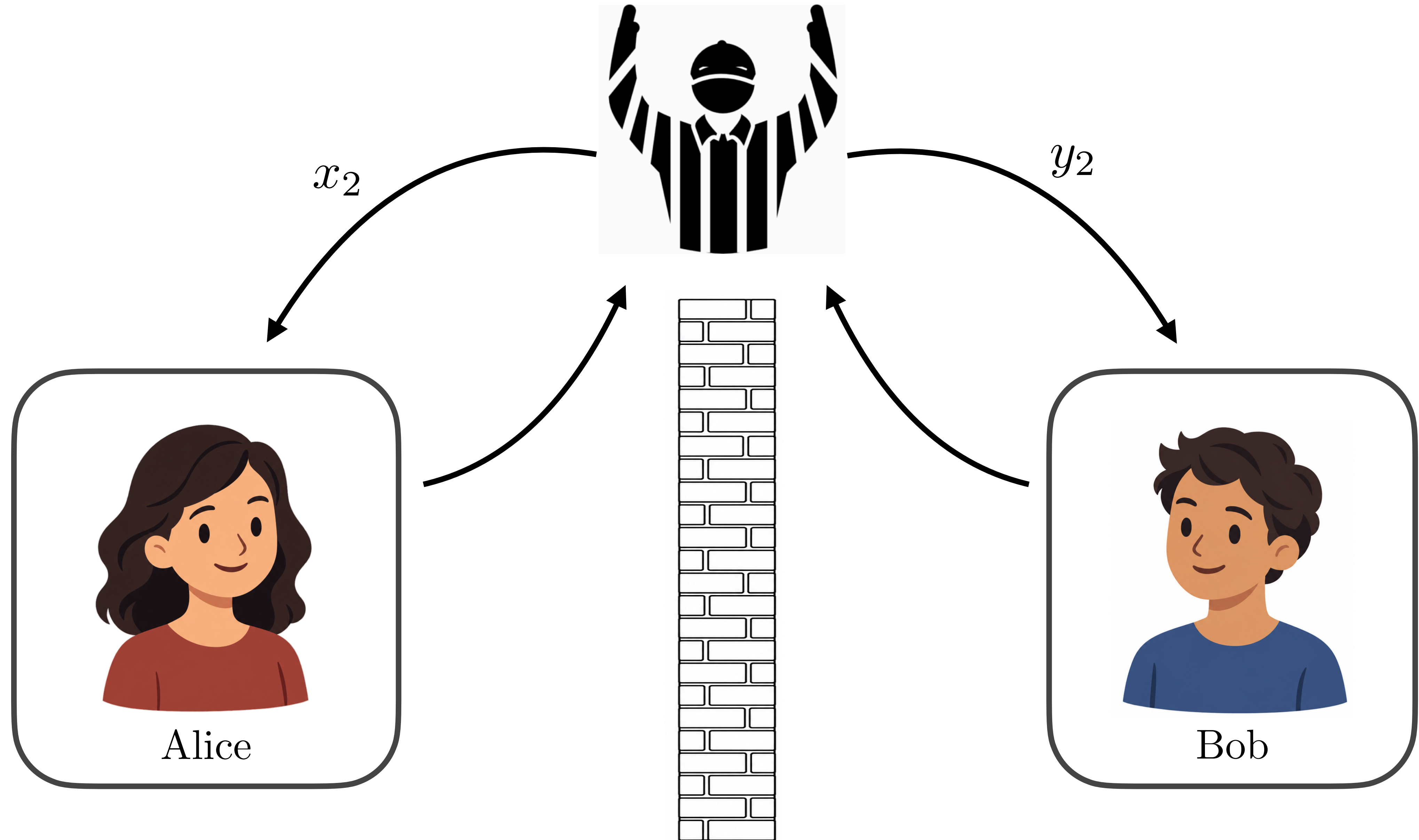
Single-shot nonlocality



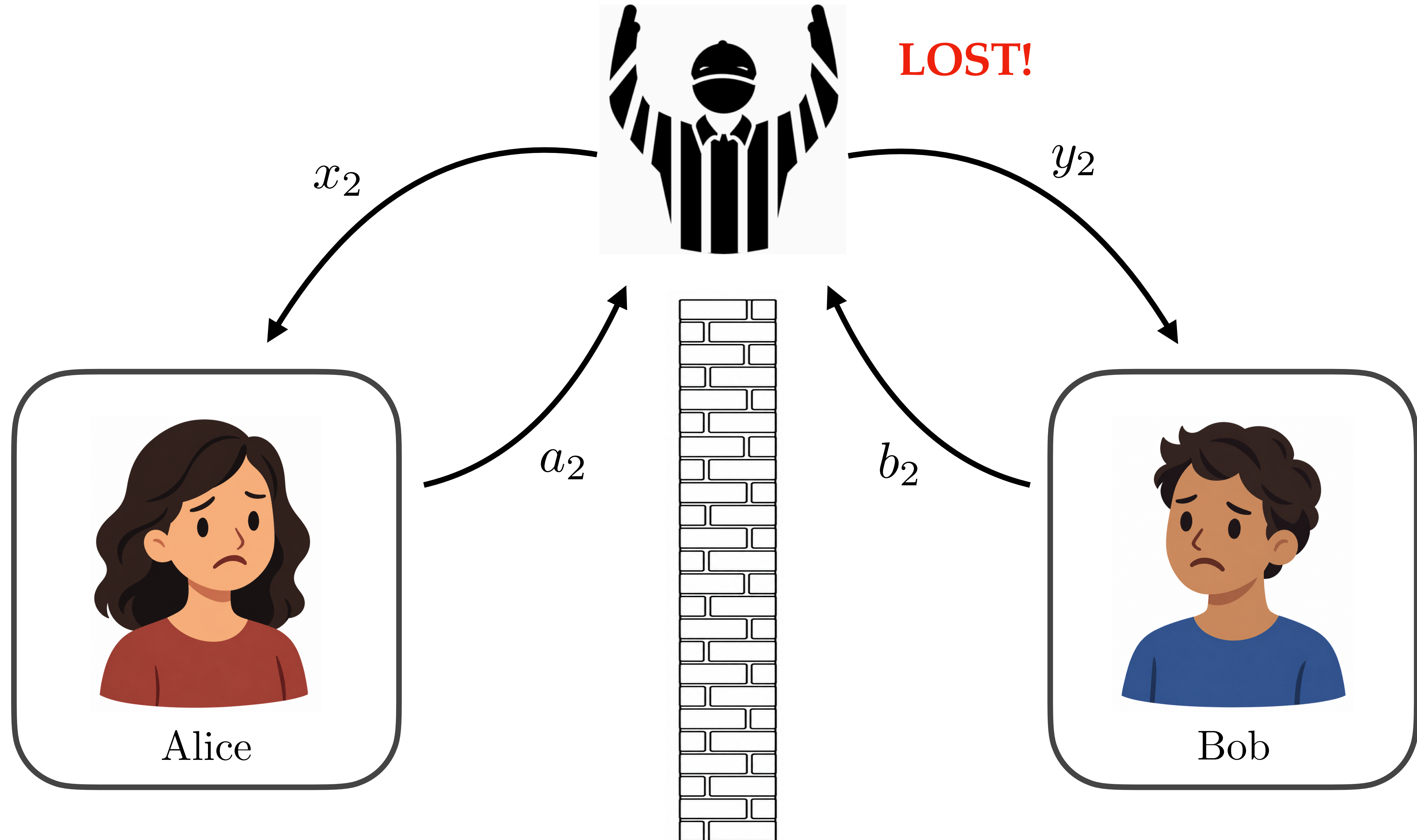
Single-shot nonlocality



Single-shot nonlocality



Single-shot nonlocality



Single-shot nonlocality

In a nonlocal game, the referee can decide if Alice and Bob won or lost in each individual round.

But in an individual round, Alice and Bob can always get lucky and win the game even with a purely classical strategy.

So why does this matter?

Single-shot quantum nonlocality

There exists a non local games for which

$$\exists \quad \{A_{a|x}\}, \{B_{b|y}\}, \rho$$

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) W(a,b,x,y) \operatorname{tr}[(A_{a|x} \otimes B_{b|y})\rho] = 1$$

There exists a **quantum strategy** that wins with probability 1

(not the case of the CHSH game)

Single-shot quantum nonlocality

There exists a non local games for which

$$\exists \quad \{A_{a|x}\}, \{B_{b|y}\}, \rho$$

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There exists a **quantum strategy** that wins with probability 1

$$\forall \quad \pi(\lambda), p_A(a|x, \lambda), p_B(b|y, \lambda)$$

N = cardinality of the inputs

$$\mathcal{S} = \sum_{a,b,x,y} q(x,y) W(a,b,x,y) \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda) \leq \frac{1}{O(N)} \xrightarrow{\lim N \rightarrow \infty} 0$$

All **classical strategies** win with a probability arbitrarily close to 0

Single-shot nonlocality

For every $\epsilon > 0$ there exists a nonlocal game which can be won by quantum strategy in every round with probability one and no classical strategy can win in any round with probability greater than ϵ .

Single-shot nonlocality

For every $\epsilon > 0$ there exists a nonlocal game which can be won by quantum strategy in every round with probability one and no classical strategy can win in any round with probability greater than ϵ .

Quantum and **classical** strategies can be distinguished in a **single round** of a game with a probability **arbitrarily close to 1**.

Part I. Quantum vs. classical correlations in nonsignalling scenarios

MAIN MESSAGE:

**Quantum and classical correlations
can be distinguished in scenarios where
communication is not allowed.**

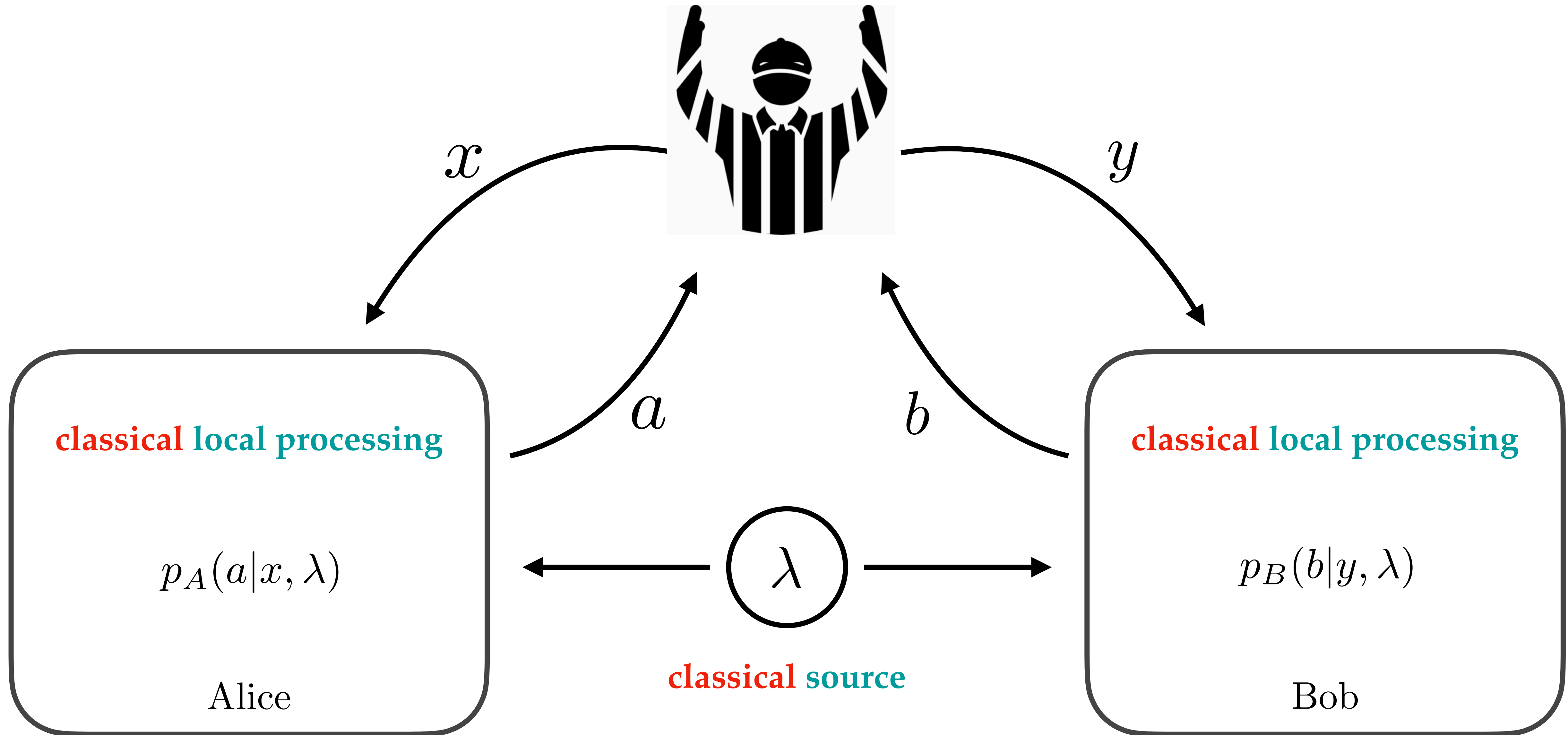
Quantum nonlocality guarantees the presence of entanglement
and can be certified device-independently and
in a single round of a game.

Part II overview

Part II. Quantum vs. classical correlations in network scenarios

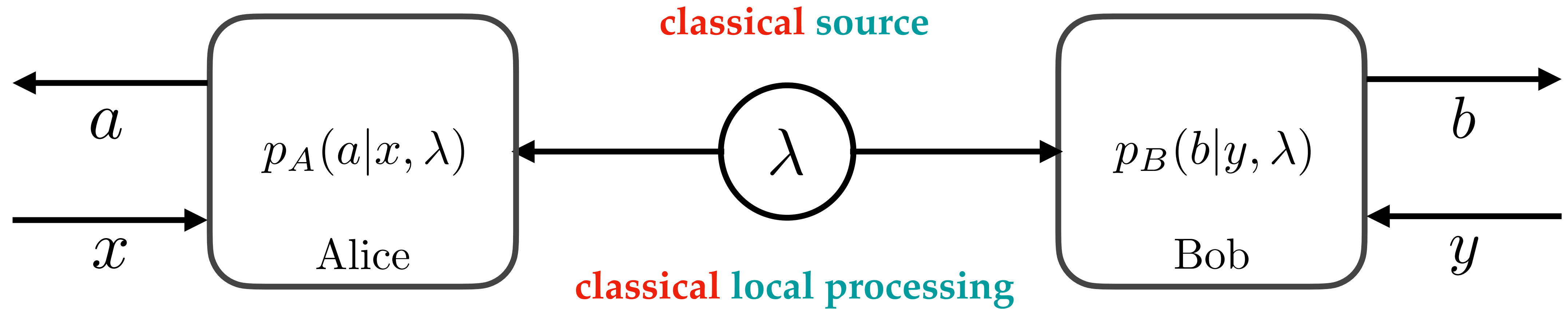
- The triangle network
 - Classical network models
 - Quantum network models
 - Nonlocality without inputs
 - Simplest example of quantum nonlocality

The triangle network



The triangle network

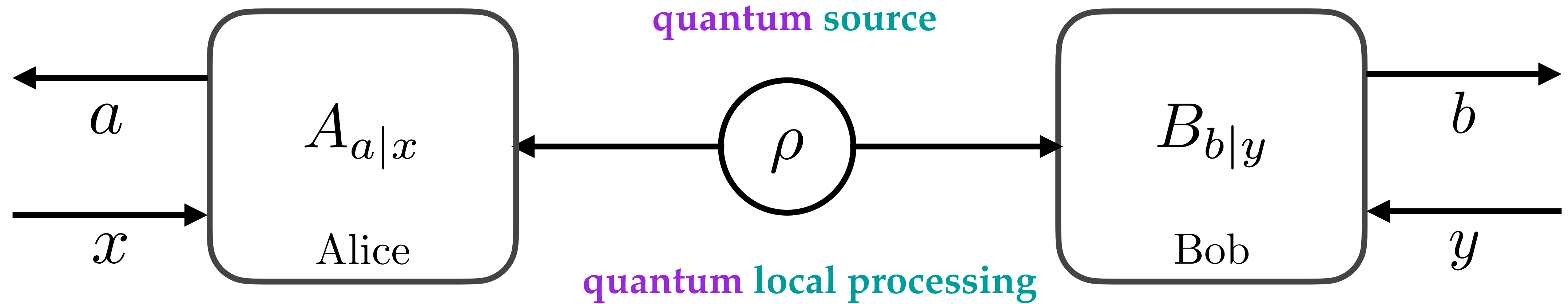
Standard bipartite Bell scenario



$$p(ab | xy) = \sum_{\lambda} \pi(\lambda) p_A(a | x, \lambda) p_B(b | y, \lambda)$$

The triangle network

Standard bipartite Bell scenario

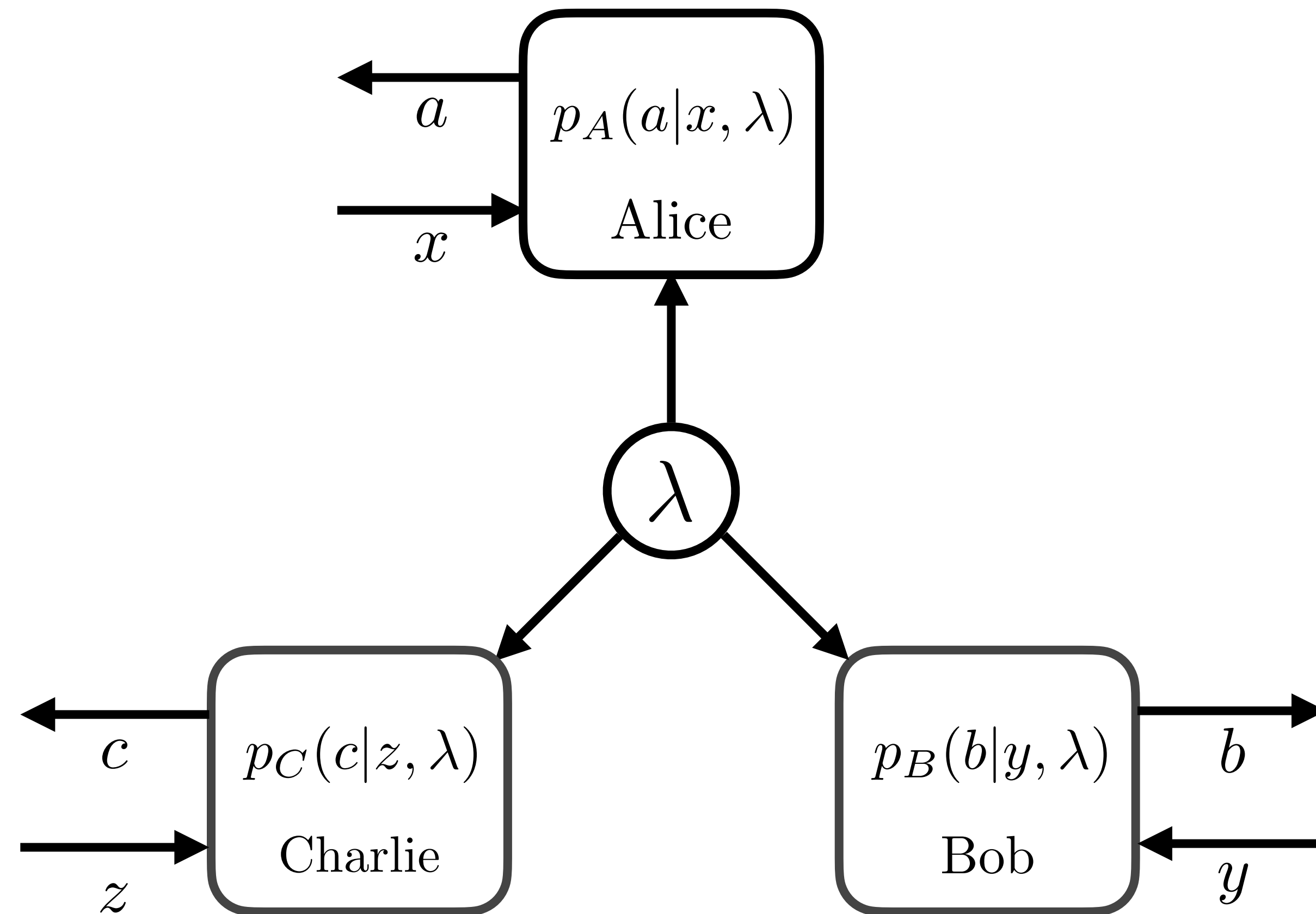


$$p(ab | xy) = \text{tr} [(A_{a|x} \otimes B_{b|y}) \rho]$$

The triangle network

Standard tripartite Bell scenario

classical source
classical local processing

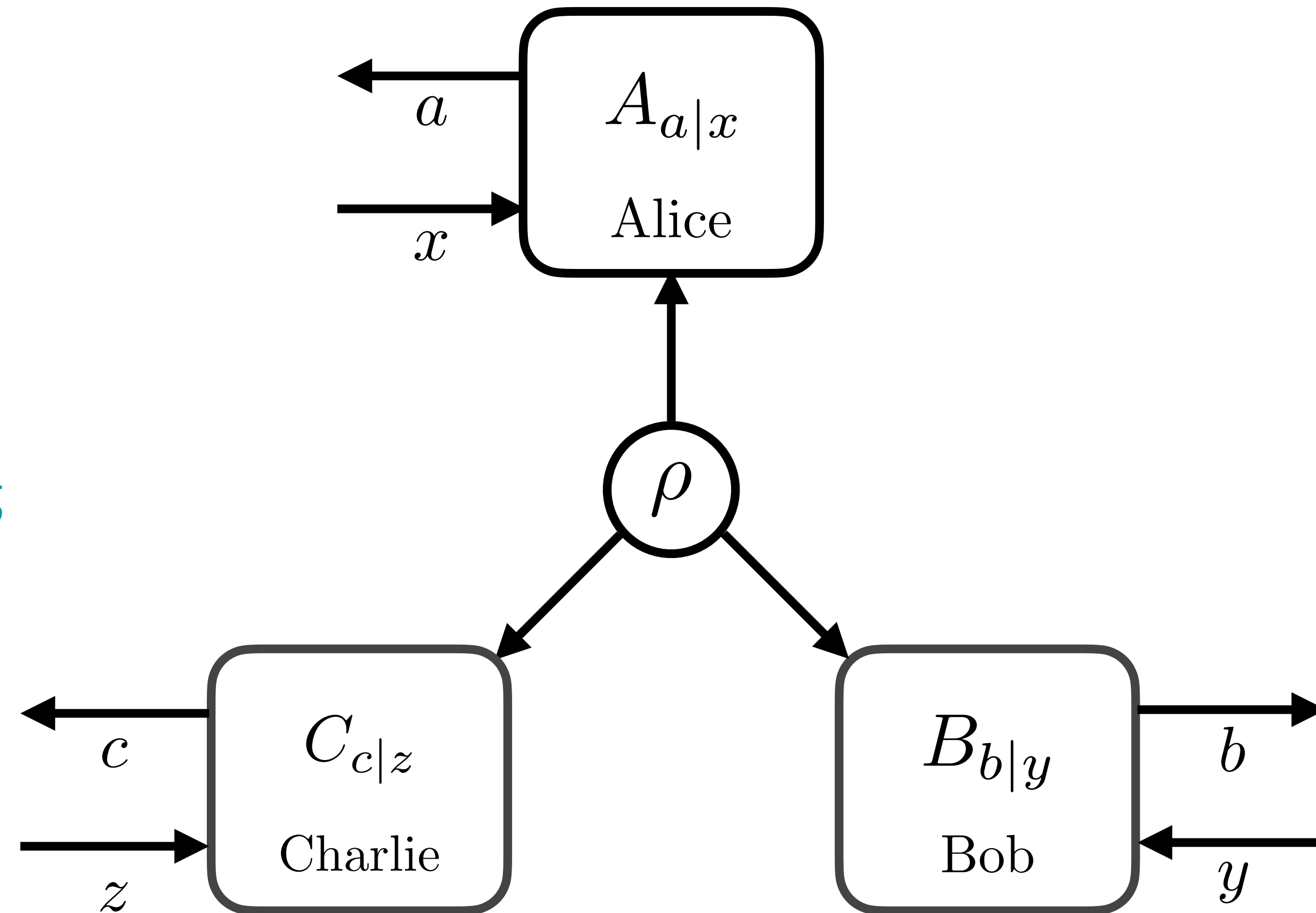


$$p(abc|xyz) = \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda) p_C(c|z, \lambda)$$

The triangle network

Standard tripartite Bell scenario

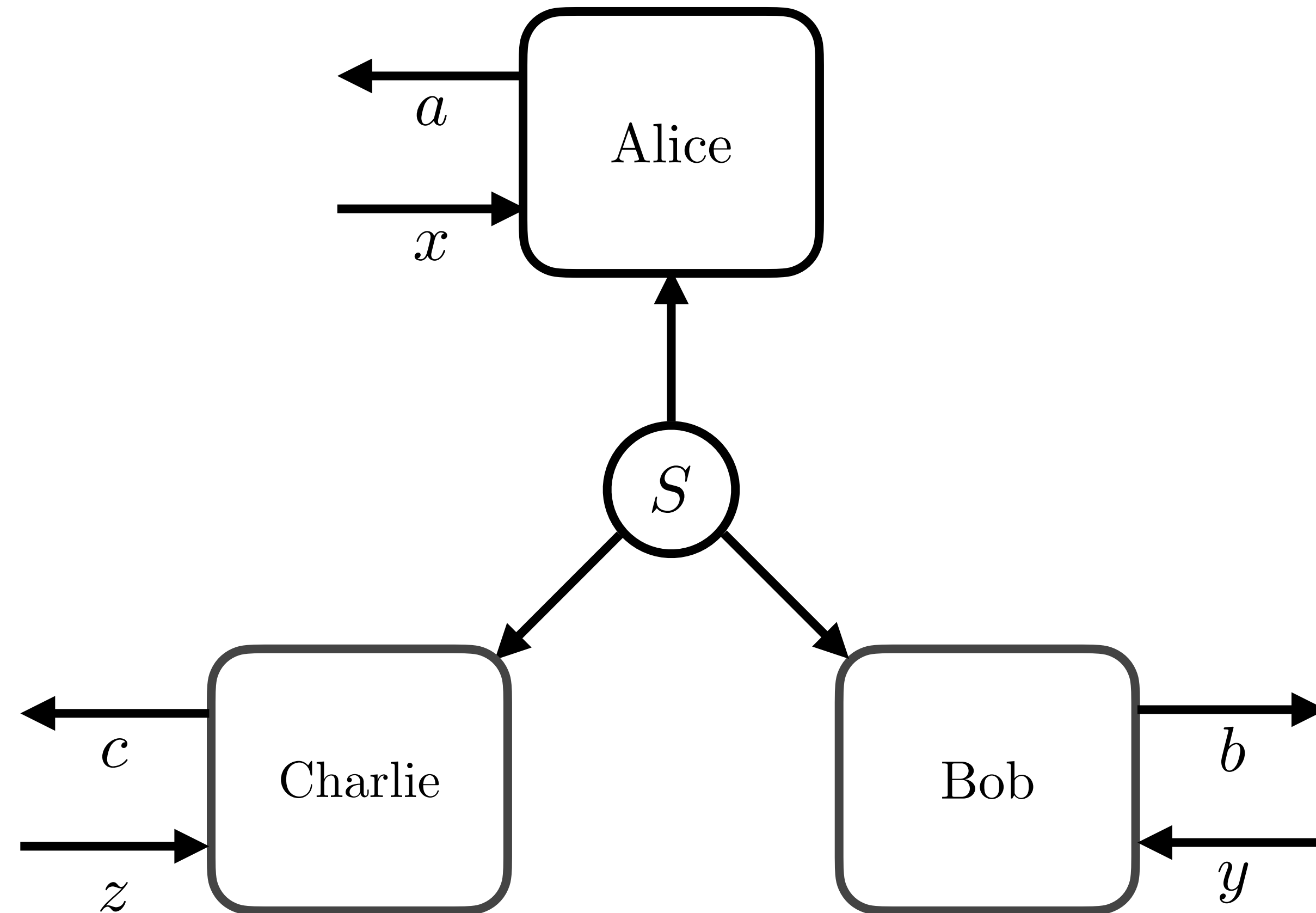
quantum source
quantum local processing



$$p(abc|xyz) = \text{tr} \left[(A_{a|x} \otimes B_{b|y} \otimes C_{c|z}) \rho \right]$$

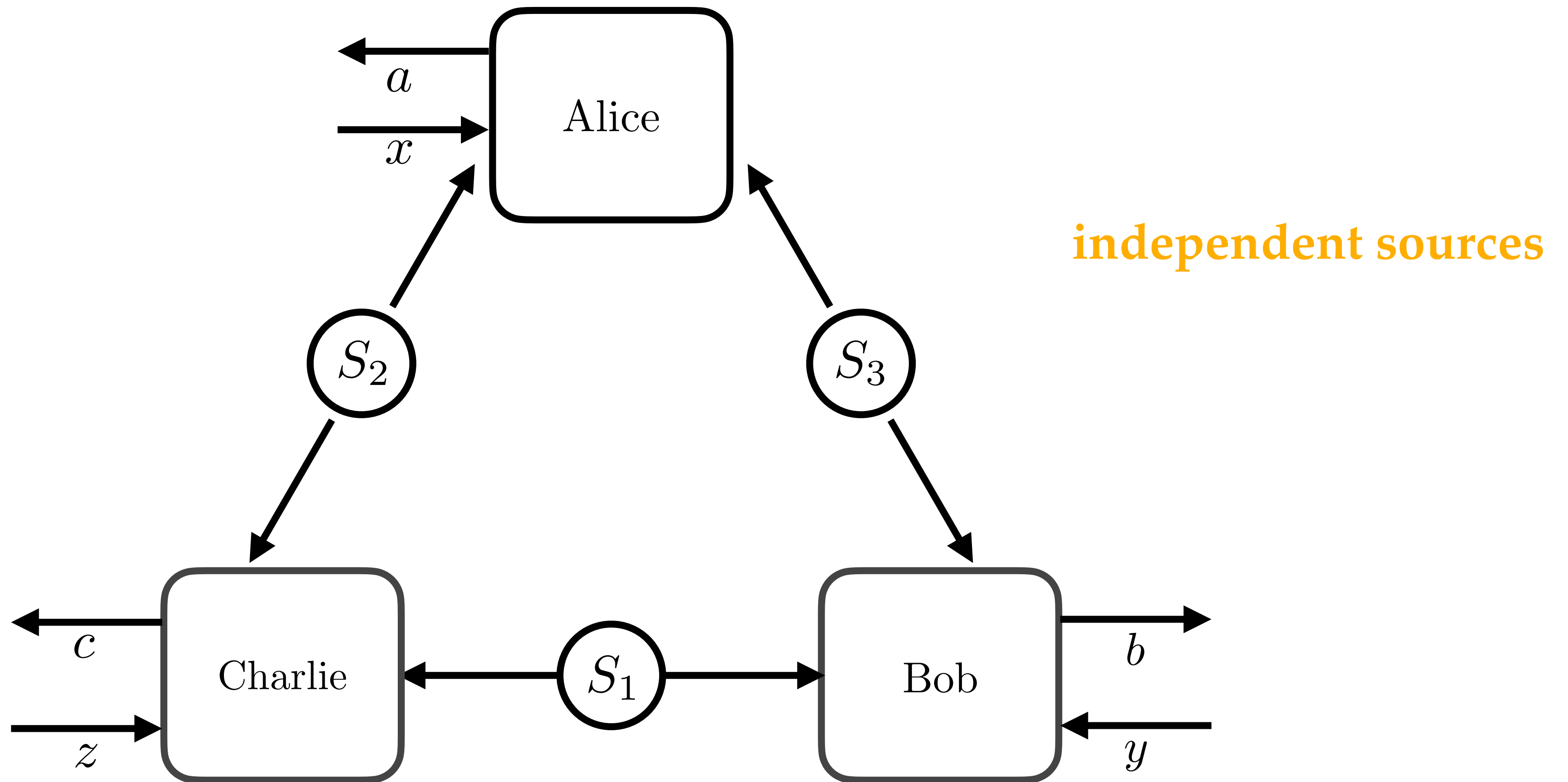
The triangle network

Standard tripartite Bell scenario



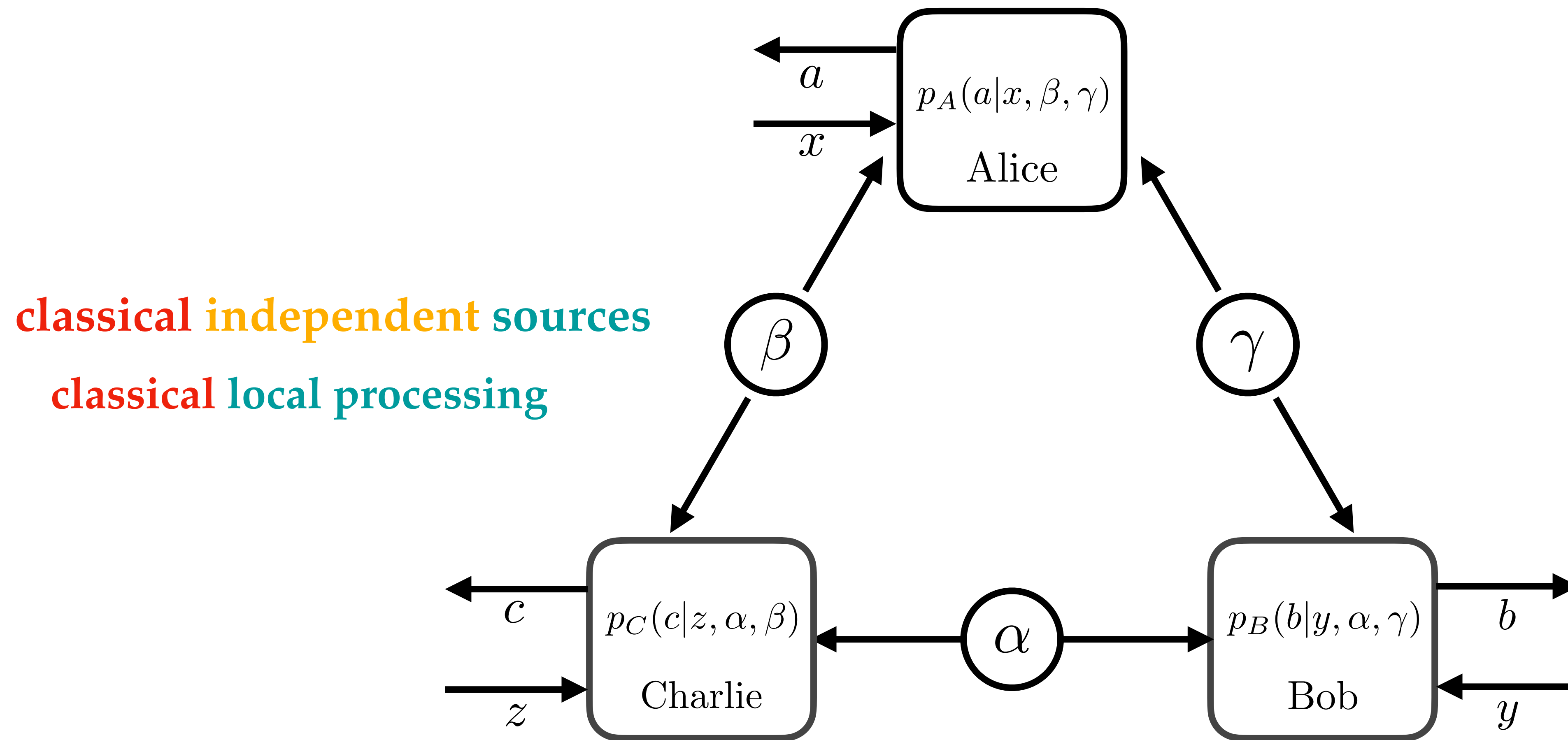
The triangle network

Network Bell scenario



The triangle network

Network Bell scenario

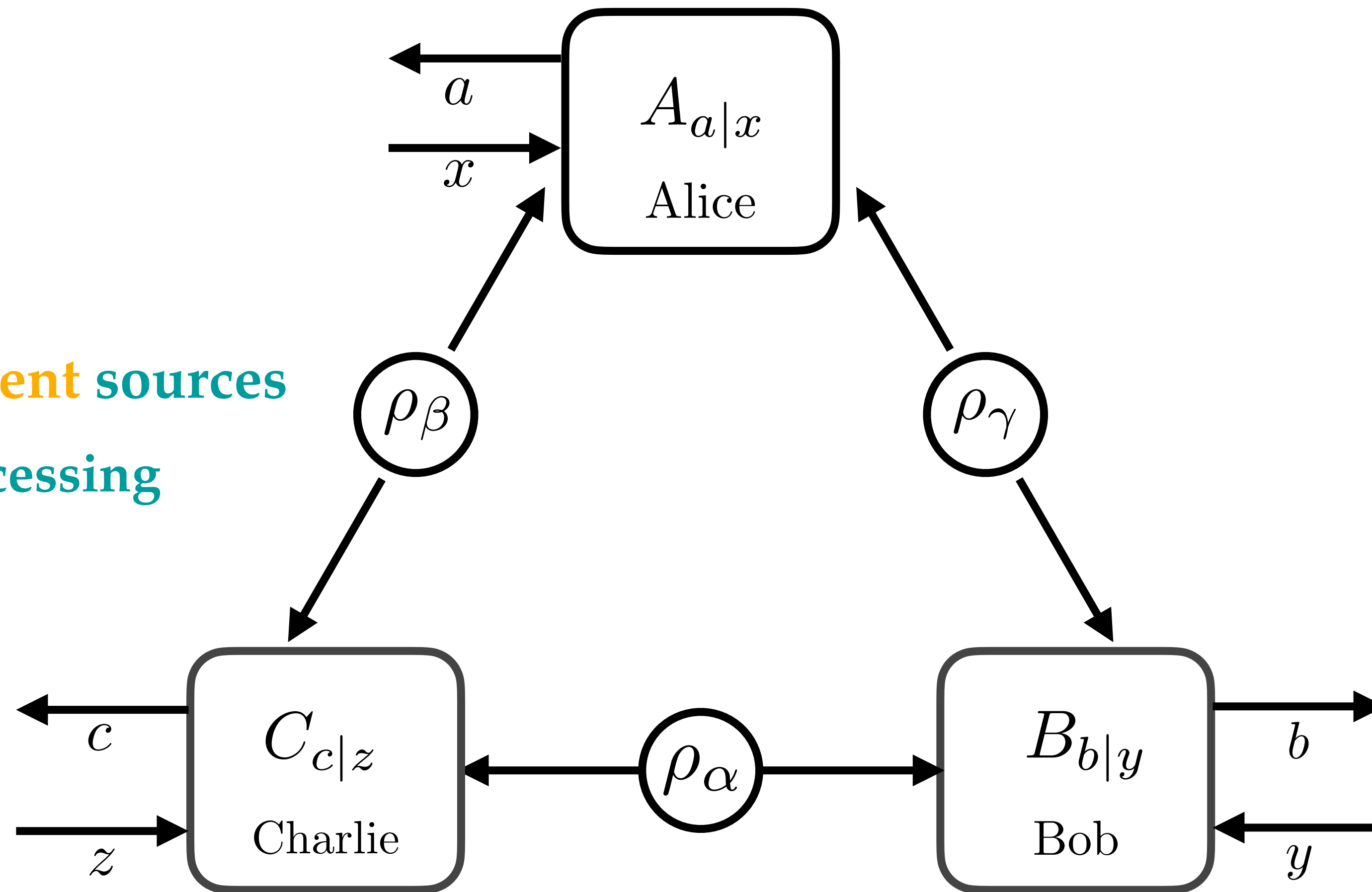


$$p(abc|xyz) = \sum_{\lambda} q(\alpha) r(\beta) s(\gamma) p_A(a|x, \beta, \gamma) p_B(b|y, \alpha, \gamma) p_C(c|z, \alpha, \beta)$$

The triangle network

Network Bell scenario

quantum independent sources
quantum local processing



$$p(abc|xyz) = \text{tr} \left[(A_{a|x} \otimes B_{b|y} \otimes C_{c|z}) (\rho_\alpha \otimes \rho_\beta \otimes \rho_\gamma) \right]$$

The triangle network

Network Bell scenario

Which sets of correlations, either classical or quantum, can be created in any given network structure?

Complicated question, usually form non convex sets.

The triangle network

In a standard, **bipartite** Bell scenarios, correlations where

$$|x| = |y| = 1$$

that is, where Alice and Bob have a fixed, constant input value,
also called the “**no-input**” scenario,
there is no quantum vs. classical separation:

$$p(ab) = \text{tr} [(A_a \otimes B_b) \rho] = \sum_{\lambda} \pi(\lambda) p_A(a|\lambda) p_B(b|\lambda)$$

The triangle network

Network Bell scenario

$$|x| = |y| = |z| = 1$$

But this is not true for “no-input” network scenarios:

The triangle network

Network Bell scenario

$$|x| = |y| = |z| = 1$$

But this is not true for “no-input” network scenarios:

$$p(abc) = \text{tr} [(A_a \otimes B_b \otimes C_c)(\rho_\alpha \otimes \rho_\beta \otimes \rho_\gamma)]$$

$$\neq$$

$$\sum_{\lambda} q(\alpha) r(\beta) s(\gamma) p_A(a|\beta, \gamma) p_B(b|\alpha, \gamma) p_C(c|\alpha, \beta)$$

The triangle network

Network Bell scenario

Minimal scenario for a **classical** vs. **quantum** correlation separation

$$|x| = |y| = |z| = 1 \qquad |a| = |b| = |c| = 2$$

There exists correlations between three binary variables $p(abc)$ that are both
quantum and **nonlocal**



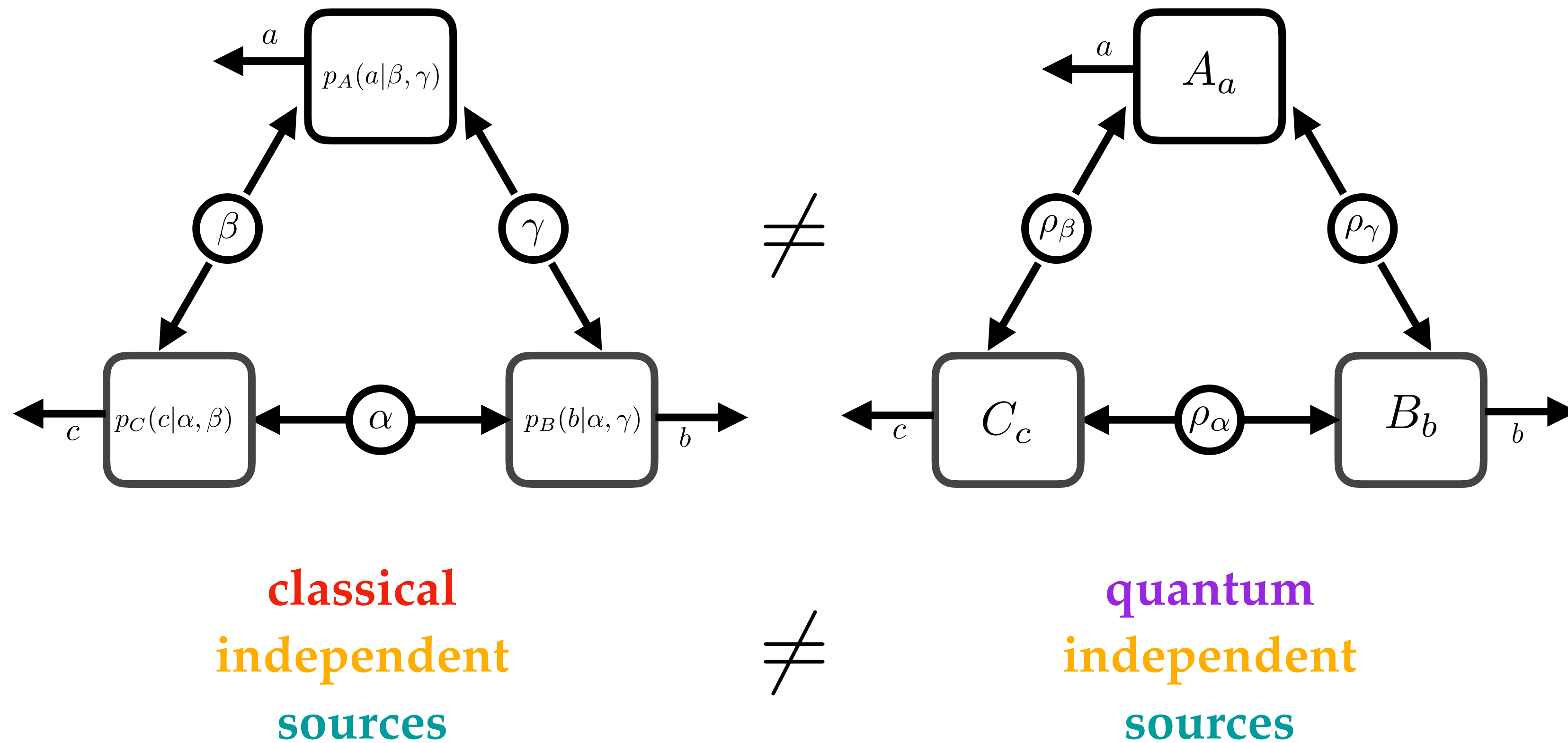
Simplest quantum nonlocality scenario

The triangle network

Network Bell scenario

$$|a| = |b| = |c| = 2$$

$$p(abc)$$



The triangle network

Network Bell scenario

$$|a| = |b| = |c| = 2$$

There exists correlations between three binary variables $p(abc)$ that are both
quantum and **nonlocal**.

However, these correlations are local in a single classical source scenario

$$\exists p(abc) = \text{tr} [(A_a \otimes B_b \otimes C_c)(\rho_\alpha \otimes \rho_\beta \otimes \rho_\gamma)]$$

$\neq$

$=$

$$\sum_{\lambda} q(\alpha) r(\beta) s(\gamma) p_A(a|\beta, \gamma) p_B(b|\alpha, \gamma) p_C(c|\alpha, \beta)$$

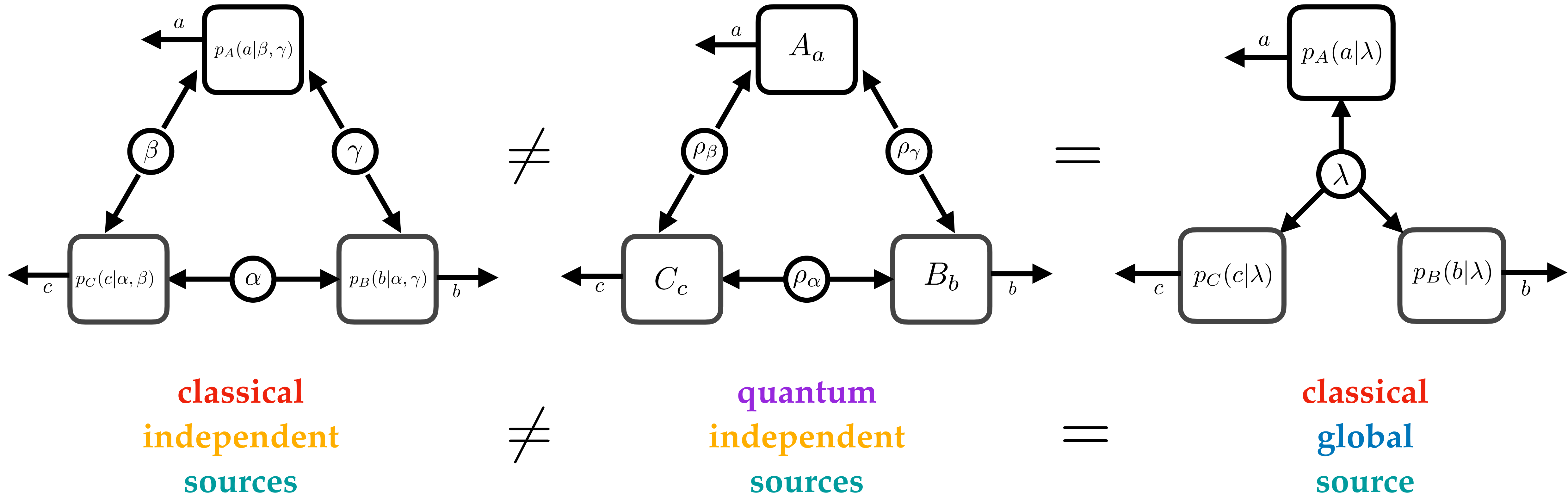
$$\sum_{\lambda} \pi(\lambda) p_A(a|\lambda) p_B(b|\lambda) p_C(c|\lambda)$$

The triangle network

Network Bell scenario

$$|a| = |b| = |c| = 2$$

$$p(abc)$$



Part II. Quantum vs. classical correlations in network scenarios

MAIN MESSAGE:

**Quantum and classical correlations
can be distinguished in network scenarios
with independent sources.**

The simplest scenario of quantum vs. classical separation
is the binary triangle network.