Higher-order quantum transformations: From quantum circuit design to indefinite causal order

Tutorial II. Case study: Quantum channel discrimination as a semidefinite programming problem

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Exercise I. Channel discrimination

(1) Write the problem of finding the maximal average probability of success of channel discrimination of a given channel ensemble $\mathcal{E} = \{p_i, C_i\}_i$ as a function of the quantum state ρ and quantum measurement $\{M_i\}_i$ in the discrimination strategy. Is this problem in the form of an SDP?

(2) Write the problem of finding the maximal average probability of success of channel discrimination of a given channel ensemble $\mathcal{E} = \{p_i, C_i\}_i$ as an SDP.
Hint: Express the discrimination strategy as a quantum tester $\{T_i\}_i$.

(3) Derive the dual problem associated to this SDP.

(4) Prove that this problem is strictly feasible.

Exercise II. Robustness of signaling in superchannels

The robustness of signaling of $r(S)$ a superchannel S is defined as

$$\begin{aligned} \text{given} \quad & S \in \mathsf{General} \\ r(S) = \min \quad & t \\ \text{subject to} \quad & \frac{S + tN}{t+1} = S_{\text{PAR}} \\ & t \geq 0 \\ & N \in \mathsf{General} \\ & S_{\text{PAR}} \in \mathsf{Parallel}. \end{aligned}$$

That is, the minimal amount of a "noise" superchannel N must be mixed with S such that the result of this mixture is a parallel superchannel S_{PAR}.

(1) Is this optimization problem in the form of an SDP? Why?

(2) Can this optimization problem be written as an SDP? How?

Exercise III. Channel discrimination with no memory

(1) Write the optimization problem for single-shot channel discrimination without memory. Is this problem in the form of an SDP?

(2) Can this problem be approximated by an SDP? How?

(3) [Inner approximation of the set of memoryless testers]
Can this problem be tackled heuristically by a seesaw of SDPs? How?

(4) [Outer approximation of the set of memoryless testers]
Can this problem be tackled by a hierarchy of SDPs? How?