Higher-order quantum transformations: From quantum circuit design to indefinite causal order

La Ricotta Summer School: The Quantum in the Haystack (June 29th – July 10th, 2026)

Course summary:

This course introduces the formalism of higher-order quantum transformations, a framework for describing transformations between quantum operations. We will see how higher-order transformations provide a natural characterization of quantum circuits while also enabling the study of quantum computation beyond the quantum circuit model, including scenarios with indefinite causal order. The course explores applications of this formalism to the design of optimal protocols and the derivation of fundamental no-go theorems for quantum information-processing tasks such as unitary transformations, channel discrimination, and parameter estimation. One lecture and one tutorial are devoted to semidefinite programming, a powerful analytical and numerical optimization tool that plays a central role in the higher-order framework and is widely used throughout quantum information science. We conclude by examining the problem of quantifying the resources required to simulate higher-order quantum operations using standard quantum circuits, and by surveying current challenges and open research directions in the field.

Lecturer:

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Support literature:

[1] **Tutorial and review:**
A Guide to Higher-order quantum operations
Philip Taranto, Simon Milz, Mio Murao, Marco Túlio Quintino, and Kavan Modi
PRX Quantum (2026)
arXiv:2503.09693 [quant-ph]

[2] **Book:**
Semidefinite Programming in Quantum Information Science
Paul Skrzypczyk and Daniel Cavalcanti
IOP Publishing (2023)
Chapter 3 in arXiv:2306.11637 [quant-ph]

[3] **Review:**
Indefinite Quantum Causality
Fabio Costa, Giulia Rubino, Cyril Branciard, Časlav Brukner, and Marco Túlio Quintino
arXiv:2606.19438 [quant-ph]

Course outline:

Lecture 1. The higher-order quantum formalism: Channels and superchannels

(Monday, July 6th, morning)

- Basic elements: 0th, 1st, and 2nd order quantum objects
- Course overview
- Quantum channels recap

  - CPTP maps and their Kraus, Choi, and Stinespring representations
  - The link product and the trace-and-replace map
  - Unitary channels
- Quantum superchannels
  - Definition
  - Characterization
  - Quantum circuit realization
- Quantum circuits as higher-order maps

Tutorial 1. Case study: Quantum channel discrimination as a higher-order problem
(Monday, July 6th, afternoon)

- Choi operators and link product
- The task of single-shot minimum-error quantum channel discrimination
- Quantum testers
  - Definition
  - Characterization
  - Quantum circuit realization

Lecture 2. General superchannels: computation with indefinite causal order
(Tuesday, July 7th, morning)

- Parallel (non-adaptive), sequential (adaptive), and general superchannels
- The quantum switch
- Superchannels with classical control of computational order
- Superchannels with quantum control of computational order
- Other forms of indefinite causal order in general superchannels

Lecture 3. Semidefinite programming
(Wednesday, July 8th, morning)

- Semidefinite programming (SDP)
- Non-canonical forms
- Feasibility and optimization problems
- Duality
- Strong duality
- Computer-assisted proofs

Tutorial 2. Case study: Quantum channel discrimination as a semidefinite programming problem
(Wednesday, July 8th, afternoon)

- Channel discrimination as an SDP
- Channel discrimination dual problem
- Numerical implementation
- But my problem is not an SDP:
  - Is it really not?
  - Seesaws of SDPs: noise robustness
  - Hierarchies of SDPs: entanglement detection

Lecture 4. Applications: Higher-order quantum computing tasks
(Thursday, July 9th, morning)

- Probabilistic exact and deterministic approximate strategies
- Achievability
  - Quantum circuits for unitary inversion, conjugation, transposition, and storage-and-retrieval
  - Strategies for quantum channel discrimination, Bayesian estimation, and metrology
- No-go theorems
  - Impossibility of unitary controlization and universal quantum channel purification
  - Upper bounds on probability of success
  - Lower bounds on quantum query complexity

Lecture 5. Simulating higher-order quantum computing with standard quantum circuits
(Friday, July 10th, morning)

- Simulating the unitary quantum switch
- Simulating classical control of computational order
- Simulating quantum control of computational order
  - Unitary inputs
  - Non-unitary inputs
- Open questions