Strict hierarchy between parallel, sequential, and indefinite-causal-order strategies for channel discrimination

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QUICK SUMMARY

We present an instance of a task of minimum-error discrimination of two qubit-qubit quantum channels for which a sequential strategy outperforms any parallel strategy. We then establish two new classes of strategies for channel discrimination that involve indefinite causal order and show that there exists a strict hierarchy among the performance of all four strategies. Our proof technique employs a general method of computer-assisted proofs. We also provide a systematic method for finding pairs of channels that showcase this phenomenon, demonstrating that the hierarchy between the strategies is not exclusive to our main example.

THE PROBLEM

The task of minimum-error channel discrimination works as follows: With some probability, Alice is given an unknown quantum channel, drawn from an ensemble of a finite number of channels that are known to her. Being allowed to use a finite number of copies (queries) of the channel, her task is to determine which of the ensemble channels she received. This problem is equivalent to Alice extracting the 'classical information' which is encoded in the 'label' of the unknown channel she received. In the simplest case of this task, when Alice is allowed to use one copy of this channel, the most general quantum operation that Alice could apply in her laboratory is to send part of a potentially entangled state through her copy of the unknown channel and jointly measure the output with a positive operator-valued measure, announcing the outcome of her measurement as her guess. Alice can improve her chances by optimizing over the operations she applies on the unknown channel based on her knowledge of the ensemble. Then, her maximal probability of successful discrimination is given by

$$P^S =: \max_{\rho,\{M_i\}} \sum_{i=1}^{N} p_i \operatorname{Tr}[(\widetilde{C}_i \otimes \widetilde{\mathbb{I}})(\rho)\, M_i].$$

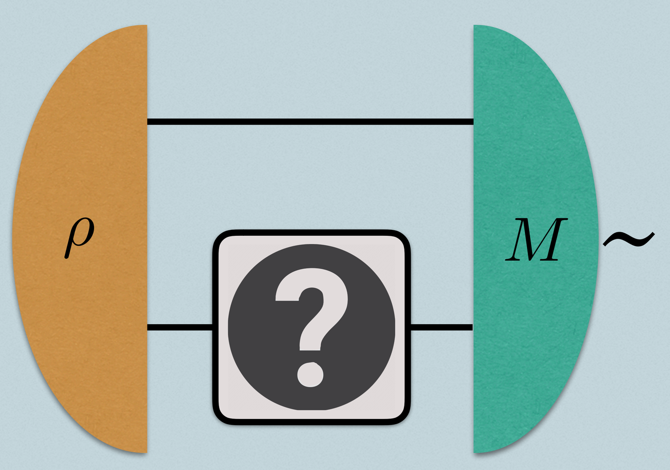

Fig. 01. Representation of Alice's strategy for a task of channel discrimination. The question mark denotes her single copy of an unknown channel.

RESULTS: THE TWO-COPY CASE

When Alice is allowed access to two copies of the unknown channel, she may explore different strategies for extracting the desired information from the channel, by playing around with the order in which these copies are applied.

Traditionally, Alice could apply her copies in a parallel strategy or, strictly better, in a sequential strategy.

More generally, she could apply her copies of the unknown channel in an indefinite causal order. We define two new classes of strategies which involve indefinite-causal-order, one called separable (not depicted) and another called general strategy.

Simply put, separable strategies are related to processes that obey a definite causal order when its future space is disregarded. One such example is the process known as the quantum switch. General strategies, on the other hand, are related to general process matrices.

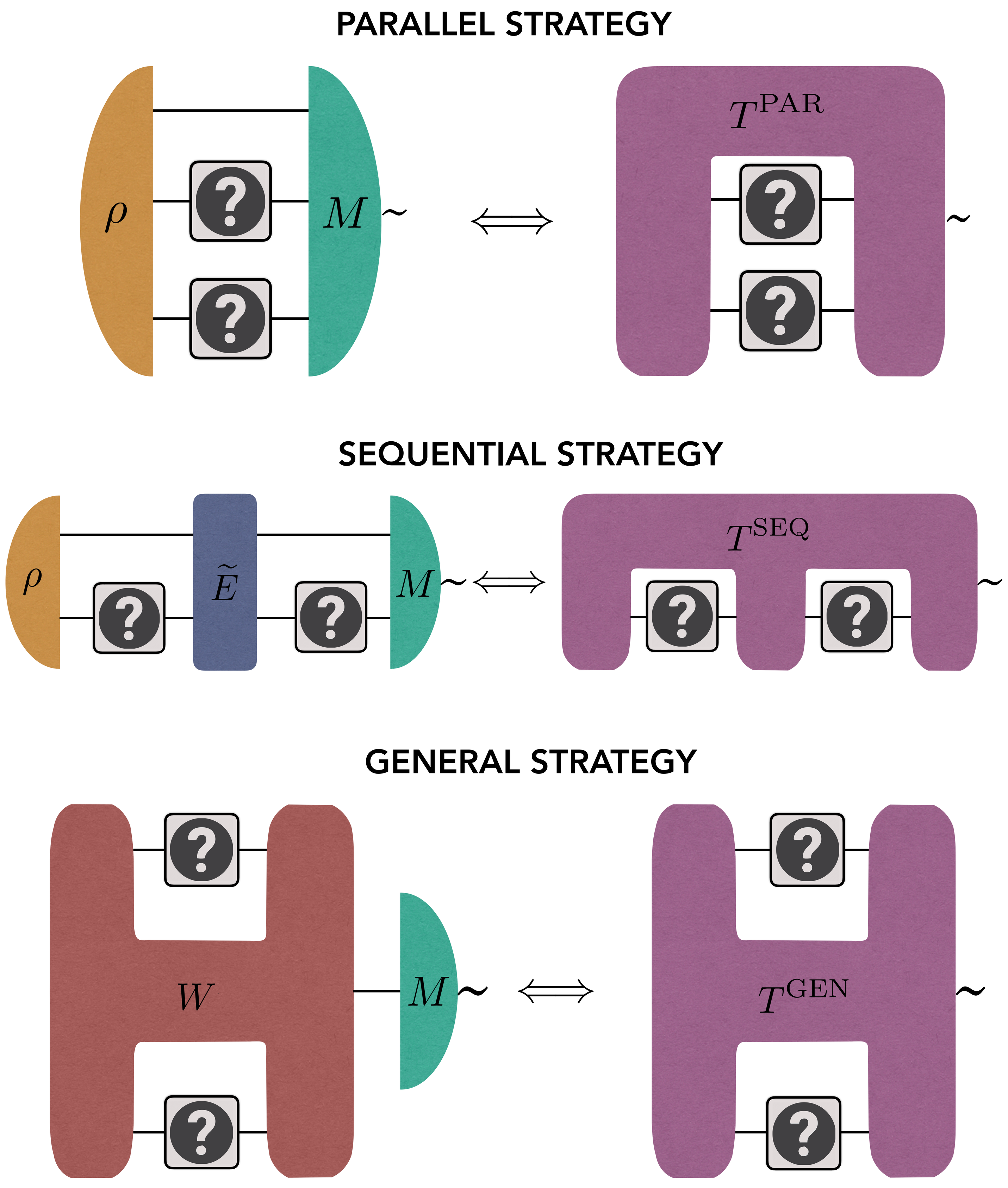

Fig. 02. Schematic representation of Alice's possible two-copy strategies.

We prove that all of our four classes of channel discrimination strategies form a strict hierarchy, by showing examples of channel discrimination tasks for which each strategy strictly outperforms the previous one. Our main example appears already at the simplest task of discriminating among two qubit-qubit channels using two copies. It involves an amplitude-damping channel and a bit-flip channel, and it yields maximum probabilities of success that follow:

$$P^{\mathrm{PAR}} < P^{\mathrm{SEQ}} < P^{\mathrm{SEP}} < P^{\mathrm{GEN}}$$

SAMPLING CHANNELS

Strategy gap	Number of pairs of channels (out of 100 000)
$P^{\mathrm{PAR}} < P^{\mathrm{SEQ}}$	99 955
$P^{\mathrm{SEQ}} < P^{\mathrm{SEP}}$	99 781
$P^{\mathrm{SEP}} < P^{\mathrm{GEN}}$	94 026
$P^{\mathrm{PAR}} < P^{\mathrm{SEQ}} < P^{\mathrm{SEP}} < P^{\mathrm{GEN}}$	94 015

To show that this strict hierarchy is not particular to our main example, we developed a method of sampling pairs of quantum channel that display this phenomenon with a very high probability (see table).

METHODS

PRIMAL

$$\begin{aligned} &\textbf{given} && \{p_i, C_i\} \\ &\textbf{maximize} && \sum_i p_i \operatorname{Tr}(T_i^S\, C_i^{\otimes 2}) \\ &\textbf{subject to} && \{T_i^S\} \text{ is a tester with strategy } \mathcal{S}. \end{aligned}$$

DUAL

$$\begin{aligned} &\textbf{given} && \{p_i, C_i\} \\ &\textbf{minimize} && \lambda \\ &\textbf{subject to} && p_i C_i^{\otimes 2} \le \lambda \overline{W}^S \quad \forall i, \end{aligned}$$

To prove our results, we apply a method of computer-assisted proofs. We first formulate the maximal probability of success under any of our four strategies using semi-definite programming (SDP).

Using the SDP output as ansatz, we rigorously recompute the solution without using floating-point variables.

Applying our method to the primal problem returns precise upper bounds, and to the dual, lower bounds. The gap between bounds can be made arbitrarily small!

FOR MORE DETAILS

Pre-print: arXiv:2011.08300 [quant-ph]

Code: https://github.com/mtcq/

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