Most incompatible measurements for robust steering tests

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THE PROBLEM

We address the problem of characterizing the steerability of quantum states under restrictive measurement scenarios, i.e., the problem of determining whether a quantum state can demonstrate steering when subjected to N measurements of k outcomes. We consider the cases of either general positive operator-valued measures (POVMs) or specific kinds of measurements (e.g., projective or symmetric). We propose general methods to calculate lower and upper bounds for the white-noise robustness of a d-dimensional quantum state under different measurement scenarios that are also applicable to the study of the noise robustness of the incompatibility of sets of unknown qudit measurements. We show that some mutually unbiased bases, symmetric informationally complete measurements, and other symmetric choices of measurements are not optimal for steering isotropic states and provide candidates to the most incompatible sets of measurements in each case. Finally, we provide numerical evidence that nonprojective POVMs do not improve over projective ones for this task.

PRELIMINARIES

In a semi-device independent approach, the main mathematical object is the assemblage $\{\sigma_{a|x}\}$:

$$\sigma_{a|x} = \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{I}\rho_{AB})$$

where $x \in \{1, \ldots, N\}$ labels measurements and $a \in \{1, \ldots, k\}$ labels outcomes of each measurement.

An assemblage is unsteerable if it admits a local hidden state (LHS) model of the form:

$$\sigma_{a|x} = \sum_{\lambda} \pi(\lambda) p(a|x, \lambda) \rho_\lambda, \ \ \lambda \in \Lambda$$

for all a, x. An assemblage is steerable if it violates a steering inequality:

$$\sum_{a,x} \mathrm{Tr}(F_{a|x}\sigma_{a|x}) \leq \beta^{\mathrm{uns}}.$$

The depolarizing channel

$$A \mapsto \Lambda_\eta(A) = \eta A + (1-\eta)\mathrm{Tr}(A)\frac{\mathbb{I}}{d}$$

can be used to define a steering quantifier, the white-noise robustness, when applied to the elements of an assemblage.

$$\eta(\{\sigma_{a|x}\}) = \max \{\eta \mid \{\Lambda_\eta(\sigma_{a|x})\} \in UNS\}$$

A quantifier of steering for quantum states can be defined as

$$\eta^*(\rho_{AB}, N, k) = \min_{\{M_{a|x}\}} \{\Lambda_\eta(\{\sigma_{a|x}\}) \mid \sigma_{a|x} = \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{I}\rho_{AB})\}$$

which is the critical visibility of a quantum state subjected to N k-outcome measurements.

We characterize steerability by calculating upper and lower bounds for $\eta^*(\rho_{AB}, N, k)$

RESULTS

We begin with the two-qubit Werner states under different measurement scenarios. **Planar projective measurements:** the optimal set is the set of equally spaced measurements. **General projective measurements:** we tested the Fibonacci spiral distribution and the Thomson problem distribution (which includes Platonic solids). Our methods showed that neither distributions are optimal and propose a nonintuitive pattern for the optimal sets of 2 to 6 projective measurements. **3- and 4-outcome symmetric POVMs:** both kinds of POVMs do not overperform projective measurements. **General POVMs:** also do not improve over projective

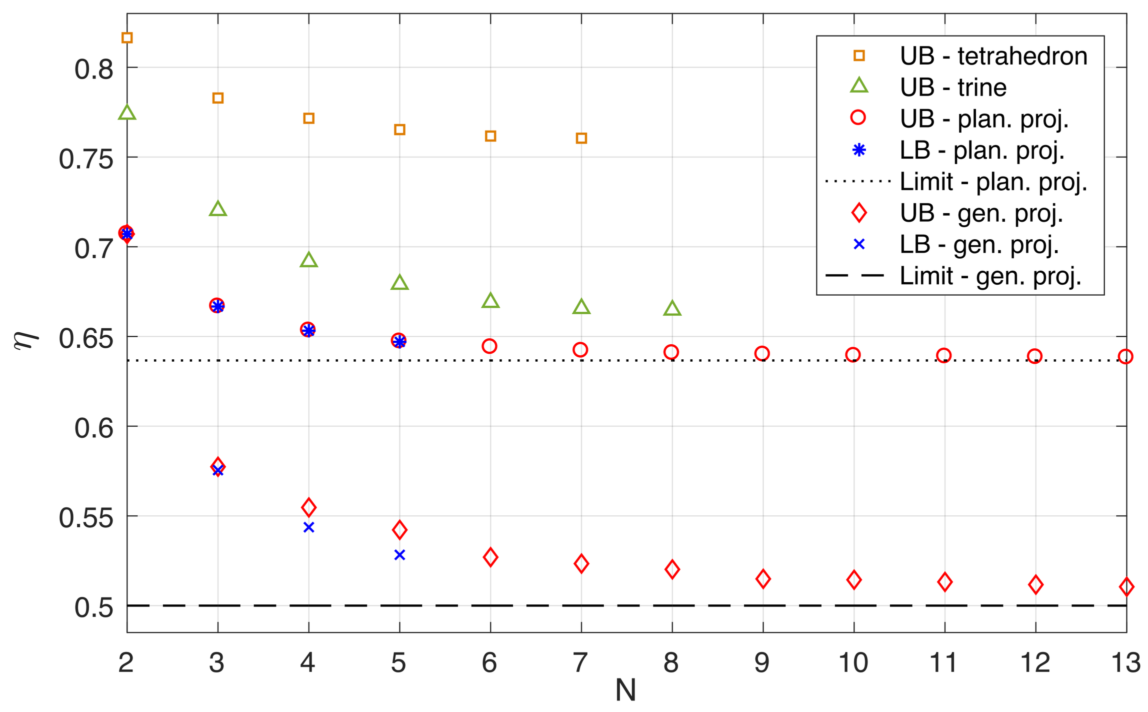

Fig. 04. Plot of upper and lower bounds for the critical visibility of two-qubit Werner states subjected to planar projective, general projective, symmetric, and general POVMs.

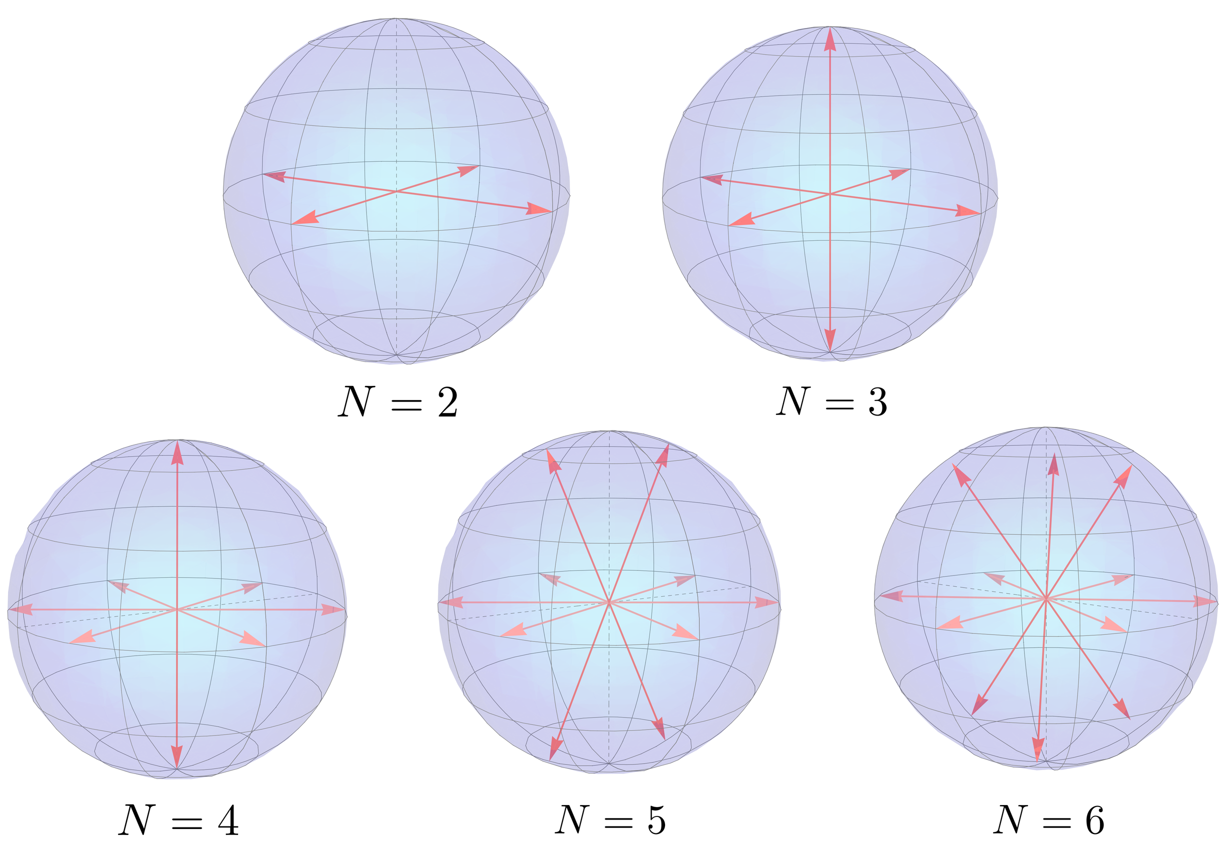

Fig. 05. Candidates for the optimal sets of 2 to 6 projective measurements for steering two-qubit Werner states.

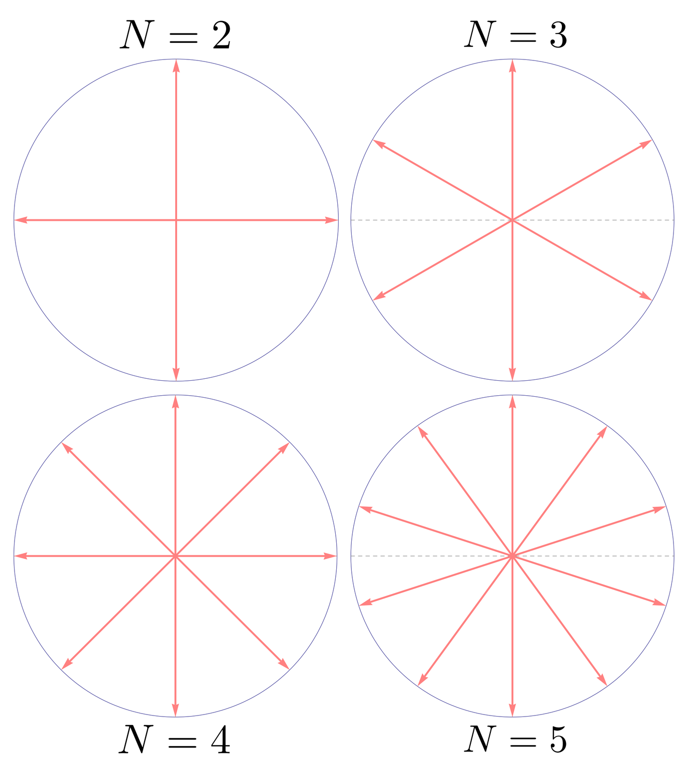

Fig. 06. Optimal sets of planar projective measurements for steering two-qubit Werner states.

In higher dimensions we studied the isotropic states subjected to 2 general POMVs. **From 2 to d outcomes:** the white noise robustness of the state improves with the increase in the number of outcomes. Optimal measurements are always projective. **$d+1$ outcomes:** the white-noise robustness of the state seems to stop improving. Optimal measurements are still projective with one null outcome. This result strengthens the idea that projective measurements are optimal for steering the isotropic states.

$N = 2$

k	$d=2$	3	4	5	6
2	0.7071	0.7000	0.6901	0.6812	0.6736
3	0.7071	0.6794	0.6722	0.6621	0.6527
4		0.6794	0.6665	0.6544	0.6448
5			0.6665	0.6483	0.6429
6				0.6483	0.6390
7					0.6390

Table 01. Upper bounds for the critical visibility of isotropic states subjected to sets of 2 general POVMs.

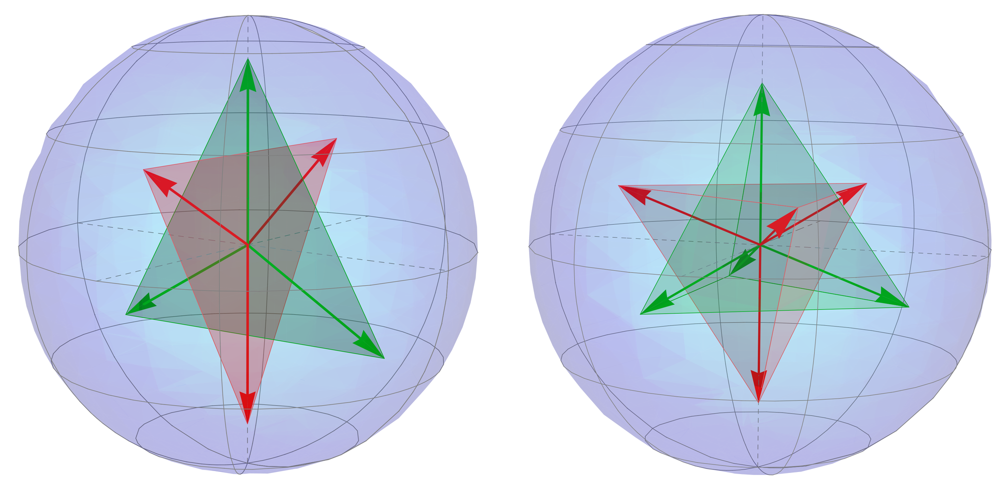

Fig. 07. Optimal sets of 2 trine POVMs (left) and 2 tetrahedron POVMs (right) for steering two-qubit Werner states.

We also studied measurements constructed from mutually unbiased bases (MUB). They are not optimal in many scenarios with up to d measurements, and the optimal measurements are also projective. However, MUB measurements seem to be the best for dimensions 2, 3, and 4 when a complete set of $d+1$ measurements is allowed.

MUBs

N	$d=2$	3	4	5	6
2	0.7071	0.6830	0.6667	0.6545	0.6449
3	0.5774	0.5686	0.5469	0.5393	0.5204
4		0.4818	0.5000	0.4615	
5			0.4309	0.4179	
6				0.3863	

General d-outcome POVMs

N	$d=2$	3	4	5	6
2	0.7071	0.6794	0.6665	0.6483	0.6395
3	0.5774	0.5572	0.5412	0.5266	0.5139
4		0.4818	0.4797	0.4615	
5			0.4309	–	
6				–	

Table 02. Upper bounds for the critical visibility of isotropic states subjected to MUB measurements and general d-outcome POVMs.

METHODS

All methods are based on semidefinite programming (SDP):

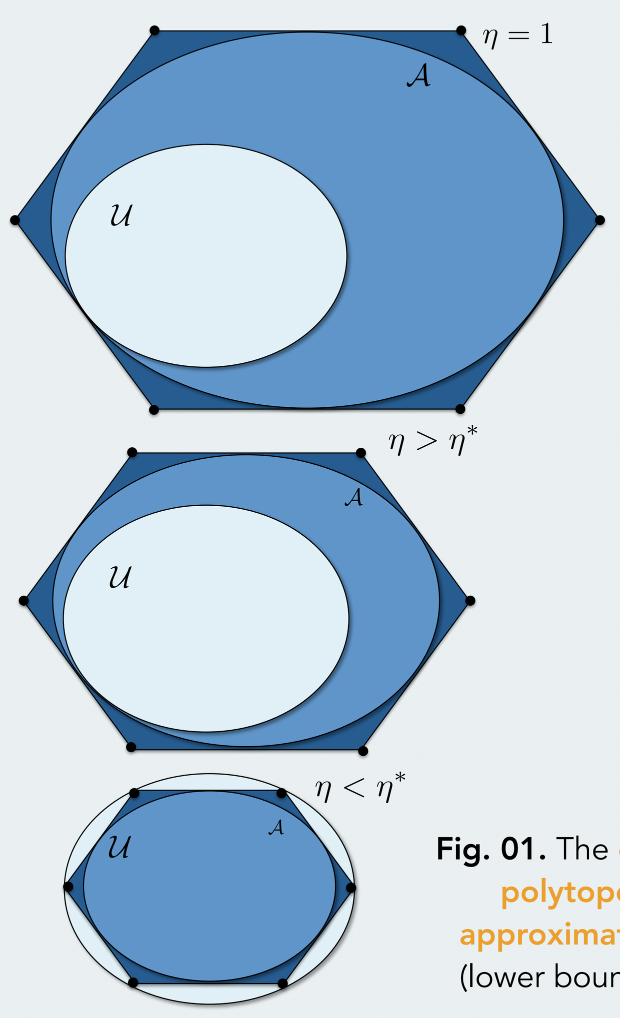

Fig. 01. The outer polytope approximation (lower bounds).

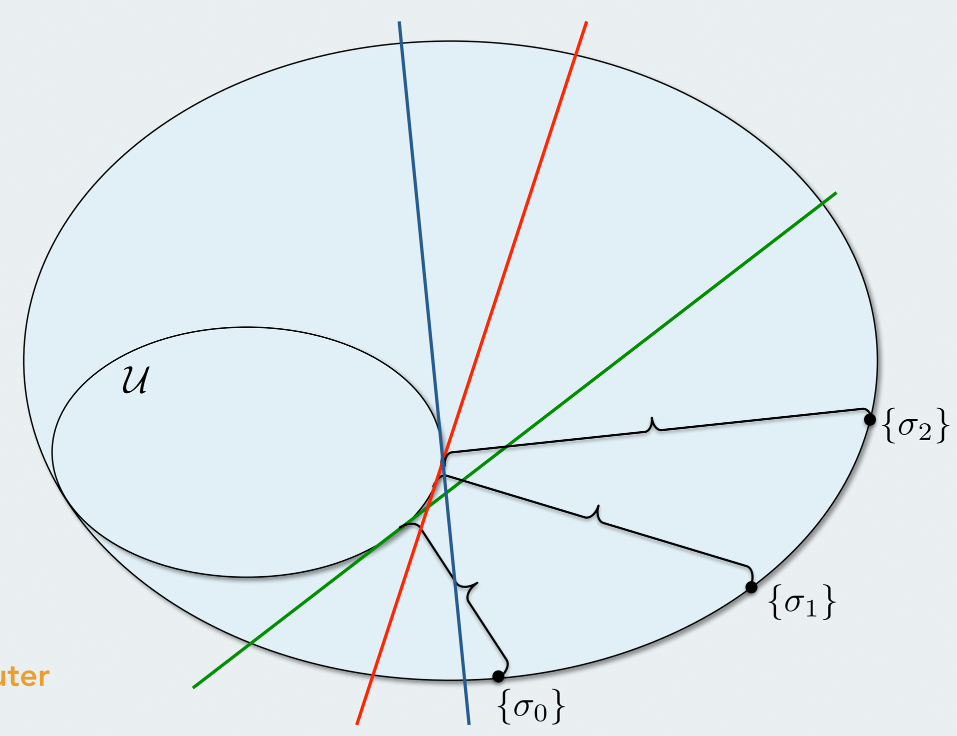

Fig. 02. The see-saw algorithm (upper bounds).

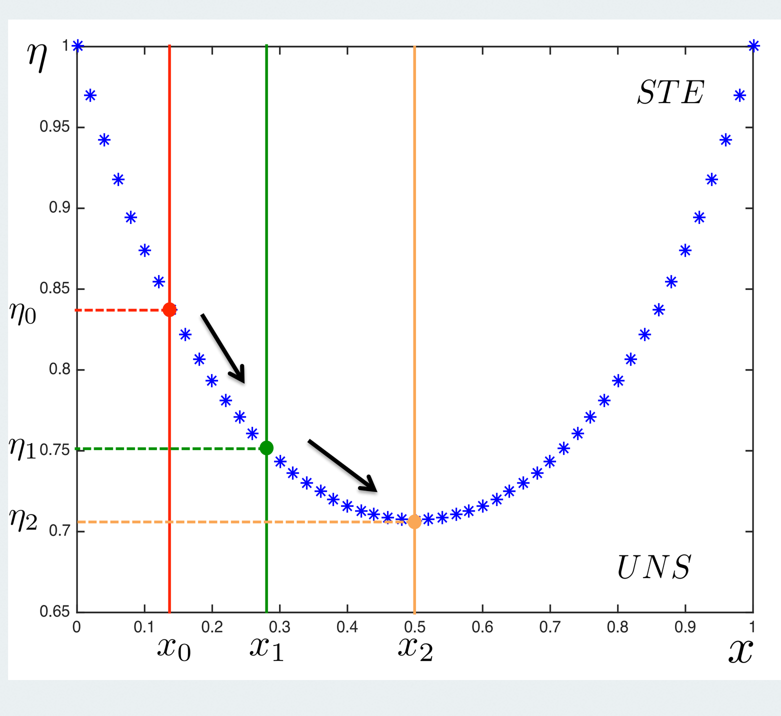

Fig. 03. The search algorithm (upper bounds).

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Pre-print: arXiv:1704.02994 [quant-ph]
Code at: https://git.io/v9znv

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