Measurements in two basis are sufficient for certifying high-dim entanglement

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ADAPTIVE STRATEGY

1. Measure in the expected **Schmidt basis** $\{|m\rangle\}_{m=0}^{d-1}$ (first measurement).

2. Define **target-state** $|\Phi_\lambda\rangle = \sum_m \lambda_m |mm\rangle$, $\lambda_m = \sqrt{\frac{\langle mm|\rho|mm\rangle}{\sum_n \langle nn|\rho|nn\rangle}}$.

3. Upper-bound **fidelity** to the target-state:

$$F(\rho,\lambda) := \mathrm{Tr}(|\Phi_\lambda\rangle\langle\Phi_\lambda|\,\rho) \leq B_k(\lambda) := \sum_{m=0}^{k-1} \lambda_m^2.$$

4. Define **tilted basis**

$$|\tilde{j}\rangle = \frac{1}{\sqrt{\sum_n \lambda_n}} \sum_{m=0}^{d-1} \omega^{jm} \sqrt{\lambda_m}\,|m\rangle\,.$$

5. Measure in the tilted basis $\{|\tilde{j}\rangle\}_{j=0}^{d-1}$ (second measurement).

6. **Lower-bound** fidelity with measurement outcomes in the tilted basis.

7. Check Schmidt number k+1 **witness**:

$$\tilde{F}(\rho,\lambda) \leq B_k(\lambda).$$

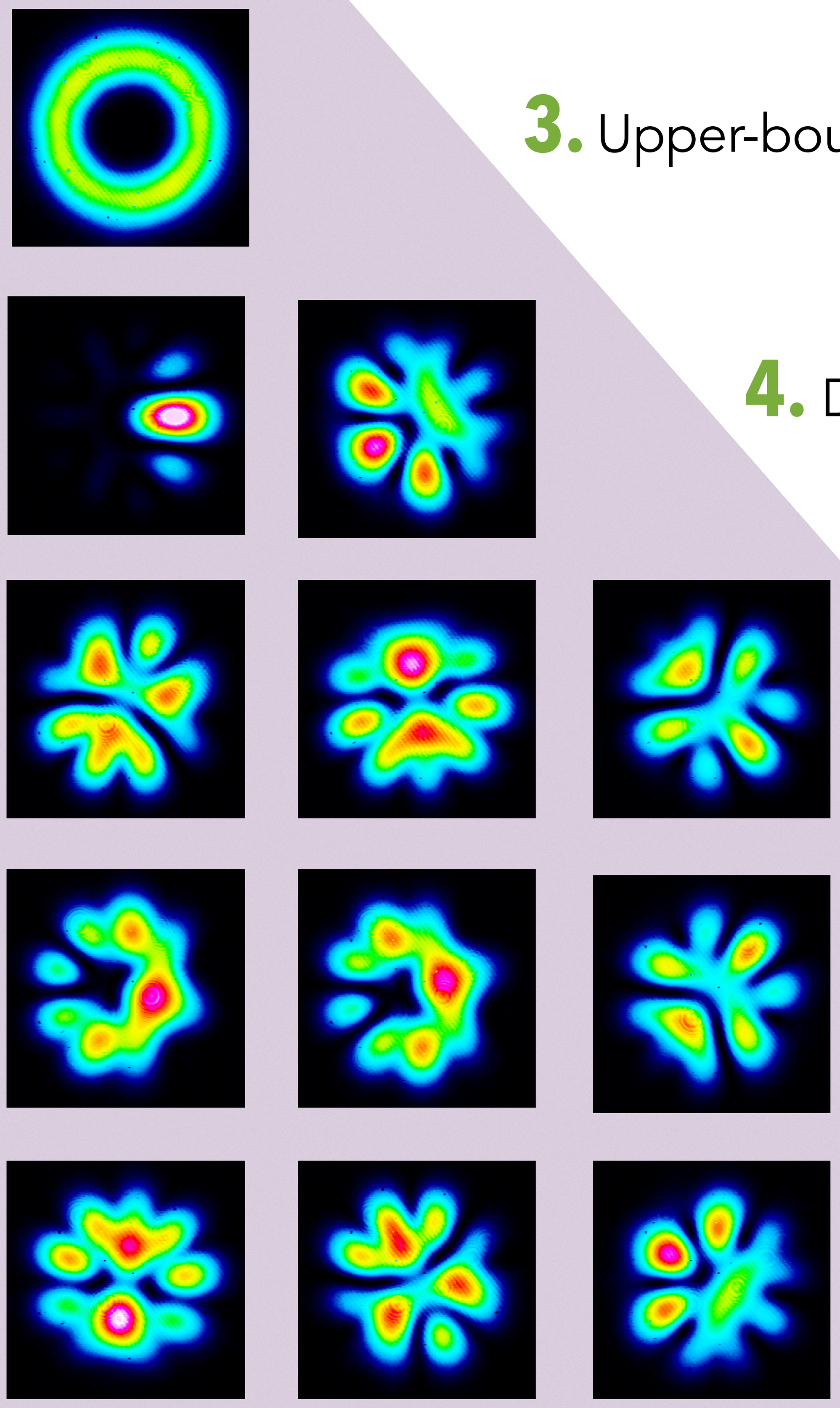


Fig. 01. First orbital angular momentum (OAM) mode of all 12 mutually unbiased basis (MUBs) in $d = 11$.

RESULTS

$$\tilde{F}(\rho,\lambda) = 0.5881$$
$$B_8(\lambda) = 0.5333$$
$$B_9(\lambda) = 0.6000$$

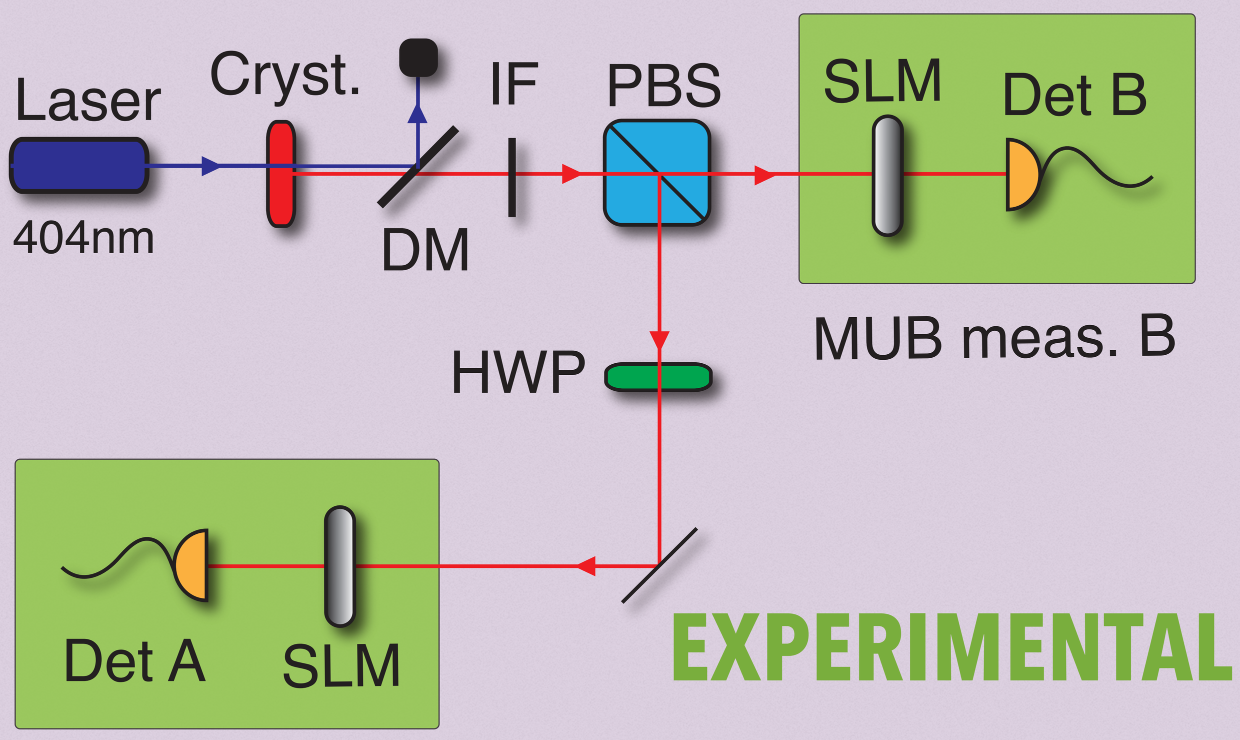
$$\Longrightarrow \quad k = 9$$

THE THEORY

High-dimensional encoding of quantum information provides a promising method of transcending current limitations in quantum communication. One of the central challenges in the pursuit of such an approach is the certification of high-dimensional entanglement. In particular, it is desirable to do so without resorting to inefficient full state tomography. Here, we show how carefully constructed **measurements in only two bases** can be used to efficiently quantify high-dimensional entanglement in realistic conditions. We considerably improve upon existing criteria and introduce new entanglement dimensionality witnesses which we put to the test for orbital angular momentum (OAM) entangled photons. In our experimental setup, we are able to verify 9-dimensional entanglement using only two measurements.

HIGHLIGHTS

- Proper certification of entanglement dimensionality (Schmidt number).
- Considerable improvement in the required number of measurements.
- Higher noise resistance compared to former methods.
- For max. entangled target-state, tilted bases become mutually unbiased (MUBs).
- Two MUBs are a necessary and sufficient criteria for max. entangled states under dephrasing noise.
- Tight witness for pure separable states.
- Can be extended to a larger number of measurements if necessary.



EXPERIMENTAL SETUP

Fig. 02. Schematic of the experimental setup: a type-II nonlinear ppKTP crystal is pumped with a 405 nm single-mode laser; spontaneous parametric down-conversion creates collinear photon pairs of 810 nm and orthogonal polarizations; photon pair is separated at PBS; spatial light modulators (SLM) and single mode fiber combination acts as a mode filter; photons are detected with avalanche photo-diode based single photon detectors and coincidence-logic.

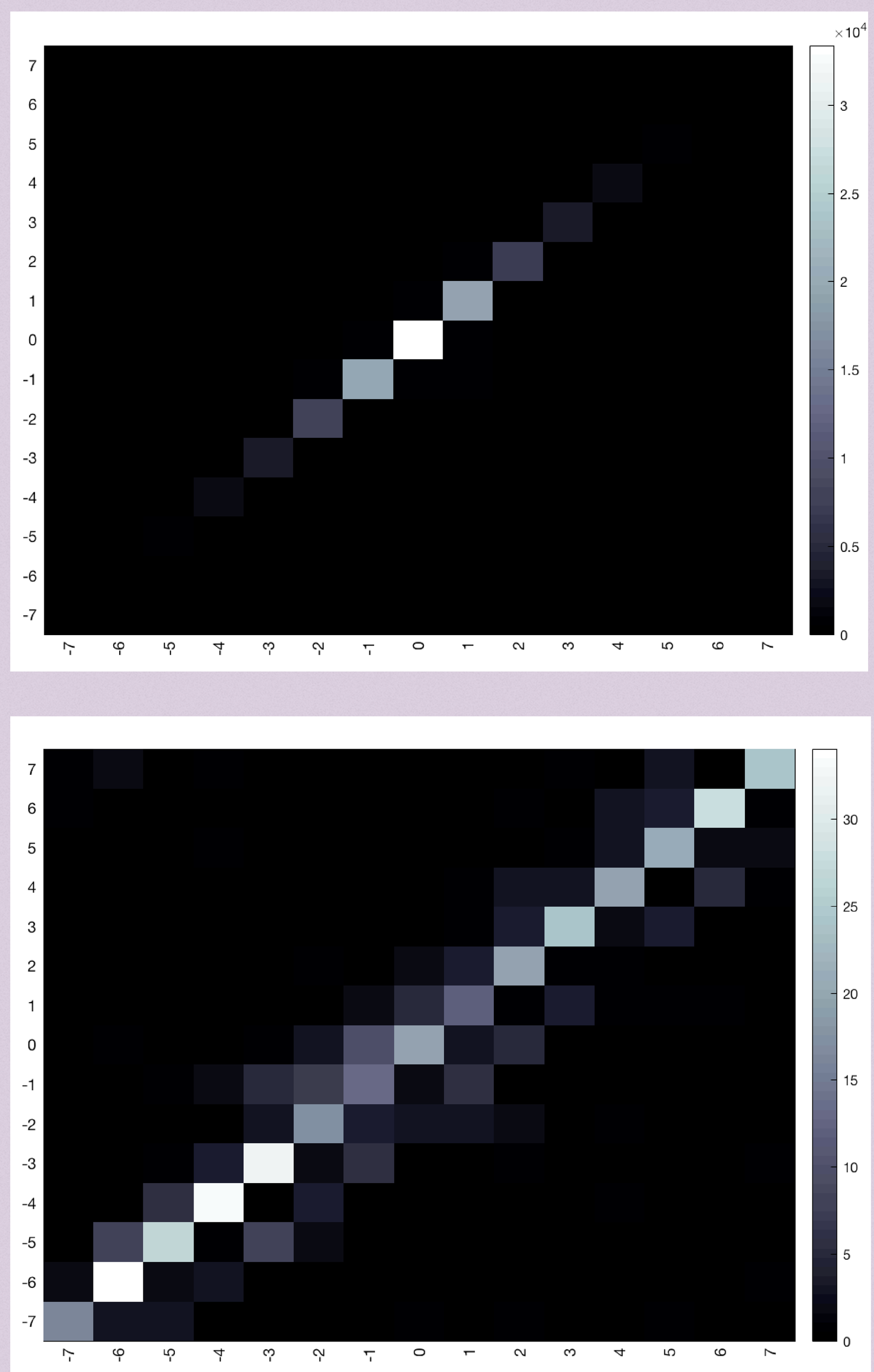


Fig. 03. Coincidence counts in Schmidt basis (top) and tilted basis (bottom) in $d = 15$.

EXPERIMENTAL CERTIFICATION OF $k = 9$ SCHMIDT NUMBER!

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THE EXPERIMENT