The power of catalytic local operations: State transformations and activation of Bell nonlocality

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QUICK SUMMARY

We study the power of **dimension-bounded catalytic local operations**, a class of operations dubbed CLO(d), where d upper-bounds the Schmidt number of the catalyst state. First, we show how they can be used to **activate Bell nonlocality**: we present a protocol where an entangled Bell-local state—which cannot violate any Bell inequality—combined with a catalyst, is transformed into a Bell-nonlocal state, while the catalyst is returned exactly in its initial state. Importantly, this **transformation is deterministic and based only on local operations**, without classical communication. Moreover, this procedure is possible even when the state of the **catalyst is itself Bell local**, demonstrating a new form of superactivation of Bell nonlocality. Second, we compare the class of operations CLO(d) with the class of local operations with classical communication LOCC and the class of stochastic local operations with classical communication and bounded quantum communication SLOCCQ(d), where d upper-bounds the total dimension of the quantum systems that are allowed to be communicated: here, we find that **CLO(d) is not a subset of SLOCCQ(d-1)**, and additionally that LOCC is not a subset of CLO(d), demonstrating the surprising **power of catalytic local operations bounded quantum communication and classical communication.**

DEFINITIONS

CLO(d): Catalytic Local Operations. The class of dimension-bounded catalytic local operations contains all transformations of a bipartite state ρ_{AB} into $\sigma_{A'B'}$ that can be implemented via local CPTP maps

$$\mathcal{A}: AC_A \to A'C_A \quad \text{and} \quad \mathcal{B}: BC_B \to B'C_B$$

that act locally also on a catalyst state $\omega_{C_AC_B}$ with Schmidt number

$$\mathrm{SN}(\omega_{C_AC_B}) \le d$$

such that

$$\mathcal{A}\otimes\mathcal{B}[\rho_{AB}\otimes\omega_{C_AC_B}] = \tau_{A'B'C_AC_B}$$

with

$$\mathrm{tr}_{C_AC_B}(\tau_{A'B'C_AC_B}) = \sigma_{A'B'} \quad \text{and} \quad \mathrm{tr}_{A'B'}(\tau_{A'B'C_AC_B}) = \omega_{C_AC_B}$$

preserving unchanged the marginal state of the catalyst.

SLOCCQ(d): Stochastic Local Operations with Classical Communication and Quantum communication. This class of transformations contains all sequences of stochastic local operations, classical communication, and dimension-bounded quantum communication, where the dimension of the quantum communication is bounded in the following way. Let d_i be the dimension of the quantum system sent from one party to another in round i. We require that

$$\prod_i d_i \le d\,.$$

Bell-local states: Quantum states that admit a local hidden variable model. A bipartite quantum state ρ_{AB} is called Bell local if, for all sets of local measurements $\{M^A_{a|x}\}$ and $\{M^B_{b|y}\}$, the correlations

$$p(ab|xy) = \mathrm{tr}[(M^A_{a|x}\otimes M^B_{b|y})\rho_{AB}]$$

admit a LHV model, according to

$$p(ab|xy) = \sum_\lambda \pi(\lambda)p_A(a|x,\lambda)p_B(b|y,\lambda)\,.$$

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RESULTS

A CATALYTIC TRANSFORMATION

Lemma 1. CLO(d) allows any (single-copy) state ρ_{AB} to be transformed into

resulting state

$$\tau_{A'B'} = \frac{1}{n}(\rho_{AB})^{\otimes n}\otimes[00]_{R_AR_B} + \frac{n-1}{n}(\sigma_A\otimes\sigma_B)^{\otimes n}\otimes[11]_{R_AR_B}$$

where $A' \equiv A_1\ldots A_nR_A$ and $B' \equiv B_1\ldots B_nR_B$ are composed of $n\in\mathbb{N}$ copies of A and B and classical bits R_A and R_B, using a catalysis state given by

catalyst

$$\omega_{C_AC_B} = \frac{1}{n}\sum_{i=0}^{n-1}(\rho_{AB})^{\otimes i}\otimes(\sigma_A\otimes\sigma_B)^{\otimes(n-1-i)}\otimes[ii]_{\tilde{R}_A\tilde{R}_B}.$$

BELL NONLOCALITY

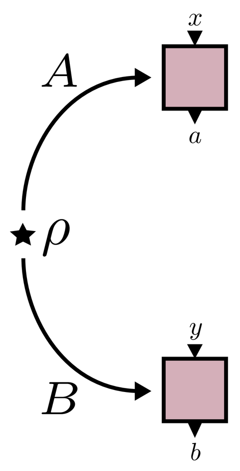

Fig. 01. Usual Bell scenario where Alice and Bob locally measure a quantum state.

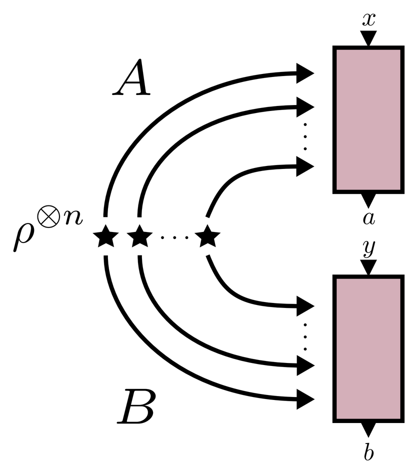

Fig. 02. Many-copy activation scenario where n copies of a Bell-local state, when measured together locally by Alice and Bob, violate a Bell inequality.

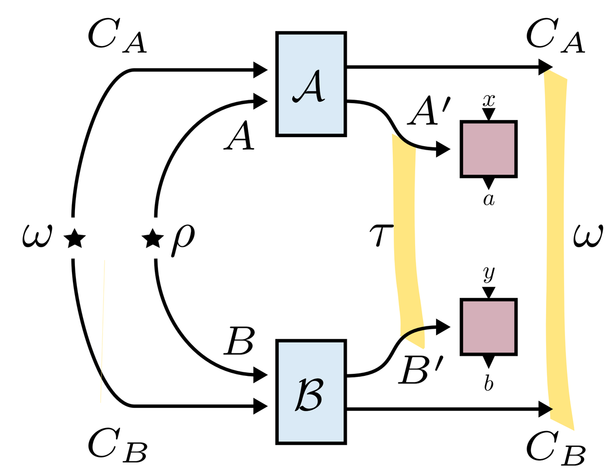

Fig. 03. Catalytic activation scenario, where Alice and Bob first locally transform a quantum state together with a shared catalyst, which is returned unchanged before the measurements of the Bell test.

Observation 1. If $\rho_{AB}^{\otimes n}$ is Bell nonlocal, $\tau_{A'B'}$ is also Bell nonlocal.

Observation 2. If $\rho_{AB}^{\otimes(n-1)}$ is Bell local, $\omega_{C_AC_B}$ is also Bell local.

Theorem 1. Let ρ_{AB} be a Bell-local entangled state, such that n copies exhibit Bell nonlocality, i.e., such that $\rho_{AB}^{\otimes n}$ violates a Bell inequality for some finite n. Then, the **Bell nonlocality** of one copy of ρ_{AB} **can be catalytically activated**. That is, a single copy of a Bell-local state ρ_{AB} can be catalytically transformed into $\tau_{A'B'}$, a Bell-nonlocal state, via the local transformations and catalyst state of Lemma 1. Moreover, the **state of the catalyst may itself be Bell-local**.

Many-copy activation of Bell nonlocality $\Longrightarrow$ Catalytic activation of Bell nonlocality

CLASSES OF OPERATIONS

Observation 3. $\mathrm{CLO}^{(d)} \subseteq \mathrm{LOSE}^{(d)} \subseteq \mathrm{SLOCCQ}^{(d)}$

Observation 4. $\mathrm{CLO}^{(1)} = \mathrm{LOSR}$

Observation 5. $\mathrm{LOCC} \not\subset \mathrm{CLO}^{(d)}$

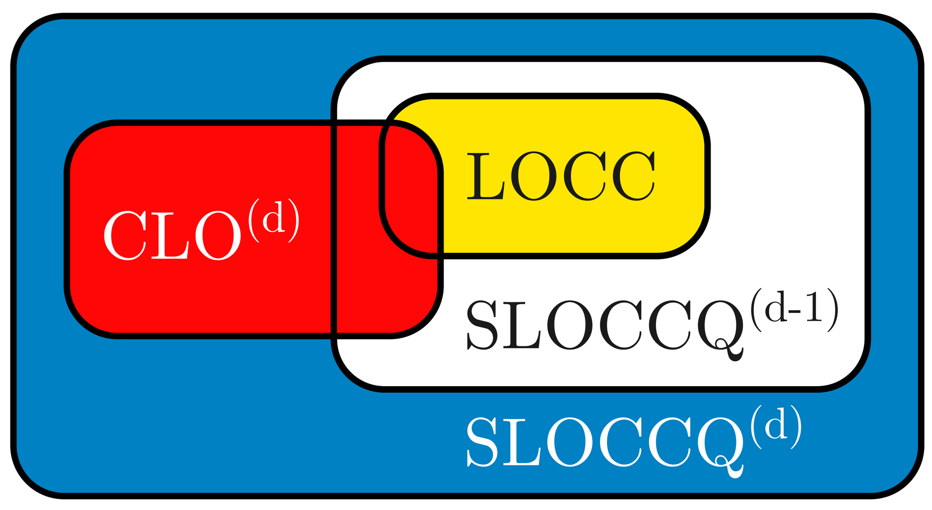

Fig. 04. The relation between the classes of transformations of quantum states CLO(d), LOCC and SLOCCQ(d).

Theorem 2. There exist **catalytic local operations CLO(d)** that can be realized with a catalyst state of Schmidt number at most $d = 2^n$, and which **cannot be realized via stochastic local operations with unlimited classical communication and quantum communication** of at most dimension d-1, i.e., **SLOCCQ(d-1)**. Concretely, take ρ_{AB} to be a maximally entangled two-qubit state, with Schmidt number 2. Via Lemma 1, it can be catalytically transformed into state $\tau_{A'B'}$, which has Schmidt number 2^{n+1}. On the other hand, SLOCC transformations cannot increase Schmidt number, and quantum communication of at least dimension 2^n is necessary to increase the Schmidt number of ρ_{AB} to 2^{n+1}, hence SLOCCQ(d-1) with $d = 2^n$ is not enough.

$$\mathrm{CLO}^{(d)} \not\subset \mathrm{SLOCCQ}^{(d-1)} \qquad \text{for } d = 2^n$$